2011 ALL INDIA ENGINEERING ENTRANCE EXAMINATION

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This Booklet contains 24 printed pages

Test Booklet Code

S

PAPER - 1 : CHEMISTRY, MATHEMATICS & PHYSICS

Do not open this Test Booklet until you are asked to do so

Read carefully the instruction on the Back Cover of this Test Booklet.

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/ Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are three parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics**, having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question. 1/ 4 (one fourth) marks will deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use **Blue/ Black Ball Point Pen only** for writing particulars/ marking responses on Side-1 and Side-2 of the Answer Sheet. **Use of pencil is strictly prohibited.**
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/ room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **3** pages (Pages **21 - 23)** at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/ Hall. **However, the candidates are allowed to take away this Test Booklet with them.**
12. The CODE for this Booklet is **S**. Make sure that the CODE printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital letters) : _____

Roll Number : in figures ☐☐☐☐☐☐☐☐

: in words _____

Examination Centre Number : ☐☐☐☐☐☐

Name of Examination Centre (in Capital letters) : _____

Candidate's Signature : _____ Invigilator's Signature : _____

CHEMISTRY

1. Silver Mirror test is given by which one of the following compounds ?

   1) Formaldehyde 2) Benzophenone

   3) Acetaldehyde 4) Acetone

Ans : 3

Sol : Organic compounds containing $-\overset{\overset{O}{\|}}{C}-H$ group can give silver mirror test

$CH_3 - CHO$

2. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution ?

   1) 0.086 2) 0.050 3) 0.100 4) 0.190

Ans : 1

Sol : $m = \frac{X_2}{X_1} \times \frac{1000}{MW_1}$

3. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :

   1) 2, 2, 2 - Trichloropropanol 2) Chloroform

   3) 2, 2, 2 - Trichloroethanol 4) Trichloromethanol

Ans : 2

Sol : $2CCl_3 - CHO \xrightarrow{NaOH} CCl_3 - COONa + CHCl_3$

$CCl_3 - CHO + NaOH \rightarrow CHCl_3 + HCOONa$

4. The rate of a chemical reaction doubles for every $10^0 C$ rise of temperature. If the temperature is raised by $50^0 C$, the rate of the reaction increases by about :

   1) 32 times 2) 64 times 3) 10 times 4) 24 times

Ans : 1

Sol : $r_2 = r_1 . 2^n$

where $n = \frac{\Delta T}{10}$

5. 'a' and 'b' are van der Waal's constants for gases. Chlorine is more easily liquefied than ethane because

1) a for $Cl_2 < a$ for C_2H_6 but b for $Cl_2 > b$ for C_2H_6

2) a for $Cl_2 > a$ for C_2H_6 but b for $Cl_2 < b$ for C_2H_6

3) a and b for $Cl_2 > a$ and b for C_2H_6

4) a and b for $Cl_2 < a$ and b for C_2H_6

Ans : 2

Sol : Inter molecular forces and molecular weight is more for Cl_2, hence Cl_2 has high boiling point than C_2H_6.

But the size of molecule C_2H_6 is greater than Cl_2 hence for C_2H_6 b is greater than Cl_2.

$a_{Cl_2} > a_{C_2H_6}$ *but* $b_{C_2H_6} > b_{Cl_2}$

6. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of $10\,dm^3$ to a volume of $100\,dm^3$ at 27^0C is :

1) $32.3\,J\,mol^{-1}K^{-1}$ 2) $42.3\,J\,mol^{-1}K^{-1}$ 3) $38.3\,J\,mol^{-1}K^{-1}$ 4) $35.8\,J\,mol^{-1}K^{-1}$

Ans : 3

Sol : $\Delta S = 2.303\,nR\log\left(\frac{V_2}{V_1}\right)$

7. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is :

1) 0.3 atm 2) 0.18 atm 3) 1.8 atm 4) 3 atm

Ans : 3

Sol : $CO_2(g) + C(s) \rightleftharpoons CO(g)$

at eq. pressure 0.5 - x - 2x

$P_T = 0.5 + x = 0.8$

So,

$\therefore K_p = 1.8$

8. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at :

1) 743 nm 2) 518 nm 3) 1035 nm 4) 325 nm

Ans : 1

Sol : $E_T = E_1 + E_2$

i.e. $\frac{hc}{\lambda_T} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$

9. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is :

1) A_2B_3 2) A_2B_5 3) A_2B 4) AB_2

Ans : 2

Sol : After removing atoms, formula is $AB_{\frac{5}{2}}$

i.e A_2B_5

10. Among the following the maximum covalent character is shown by the compound

1) $AlCl_3$ 2) $MgCl_2$ 3) $FeCl_2$ 4) $SnCl_2$

Ans : 1

Sol : According to Fajan's rule

1) Polarisation α charge on cation

in turn polarisation α covalent character

$Al^{+3}Cl_3, Mg^{+2}Cl_2, Fe^{+2}Cl_2, Sn^{+2}Cl_2$

$AlCl_3$ is most covalent

11. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ?

1) $Na_2O < K_2O < MgO < Al_2O_3$ 2) $K_2O < Na_2O < Al_2O_3 < MgO$

3) $Al_2O_3 < MgO < Na_2O < K_2O$ 4) $MgO < K_2O < Al_2O_3 < Na_2O$

Ans : 3

Sol : As electropositive character increases basic nature metal oxides increases

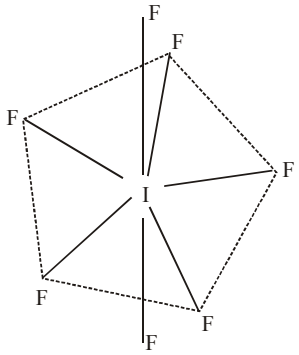
$Al_2O_3 < MgO < Na_2O < K_2O$

12. The structure of IF_7 is :

1) octahedral 2) pentagonal bipyramid
3) square pyramid 4) trigonal bipyramid

Ans : 2

Sol : IF_7 exhibits Pentagonal bipyramidal shape



13. Identify the compound that exhibits tautomerism

1) 2 - Pentanone 2) Phenol 3) 2 - Butene 4) Lactic acid

Ans : 1

Sol : 2 - Pentanone $CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-CH_3$

$$CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-CH_3 \rightleftharpoons CH_2=\overset{\overset{OH}{|}}{C}-CH_2-CH_2-CH_3$$

2 - Pentanone present in the liquid state hence it exhibits tautomerism

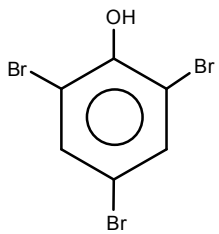
$\left(C_6H_5O_{11}\ is\ solid\ MP=42^0C\right)$

14. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is :

1) 4 - Bromophenol 2) 2, 4, 6 - Tribromophenol
3) 2 - Bromophenol 4) 3 - Bromophenol

Ans : 2

Sol : In aqueous medium phenol with Br_2 gives tribromo phenol



15. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at -6^0C will be : (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62 g mol^{-1})

1) 400. 0 g 2) 304. 60 g 3) 804. 32 g 4) 204. 30 g

Ans : 3

Sol : $\Delta T_f = i.K_f.m$

16. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression :

1) $\alpha = \frac{x+y-1}{i-1}$ 2) $\alpha = \frac{x+y+1}{i-1}$ 3) $\alpha = \frac{i-1}{(x+y-1)}$ 4) $\alpha = \frac{i-1}{(x+y+1)}$

Ans : 3

Sol : $A_xB_y \rightarrow xA^{+y} + y.B^{-x}$

$1-\alpha \quad x\alpha \quad y\alpha$

$i = \frac{i-1}{(x+y-1)}$

17. Boron cannot form which one of the following anions ?

1) $B(OH)_4^-$ 2) BO_2^- 3) BF_6^{3-} 4) BH_4^-

Ans : 3

Sol : Maximum covalency for any 2nd period element possible is only 4. hence BF_6^{-3} are not possible

18. The strongest acid amongest the following compounds is :

1) $CH_3CH_2CH(Cl)CO_2H$ 2) $ClCH_2CH_2CH_2COOH$

3) CH_3COOH 4) $HCOOH$

Ans : 1

Sol : $CH_3 - CH_2 - \underset{\underset{Cl}{|}}{CH} - COOH$ is the strongest acid due to - I inductive effect at α – carbon

19. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is :

1) Ethyl chloride 2) Ethyl ethanoate

3) Diethyl ether 4) 2 - Butanone

Ans : 2

Sol : $CH_3-\overset{\overset{O}{\|}}{C}-Cl+NaO-CH_2-CH_3 \rightarrow CH_3-\overset{\overset{O}{\|}}{C}-O-CH_2-CH_3+NaCl$

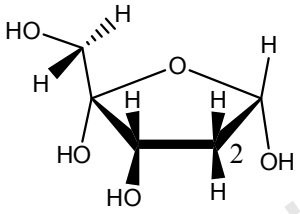
Nucleophilic substitution of $CH_3-CH_2-O^-$ on $CH_3-\overset{\overset{O}{\|}}{C}-Cl$ results esterification

20. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA ?

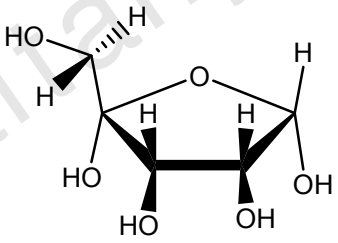
1) 3rd 2) 4th 3) 1st 4) 2nd

Ans : 4

Sol :



Deoxy Ribose



Ribose

In DNA Deoxy ribose is present which has 2 Hydrogen atoms on the 2nd carbon. In RNA Ribose is present.

21. The outer electron configuration of Gd (Atomic No. : 64) is :

1) $4f^4\ 5d^4\ 6s^2$ 2) $4f^7\ 5d^1\ 6s^2$ 3) $4f^3\ 5d^5\ 6s^2$ 4) $4f^8\ 5d^0\ 6s^2$

Ans : 2

Sol : Gadalonium exhibits half filled F^7 configuration

$Gd(64): [Xe]4f^7 5d^1 6s^2$

22. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is :

1) 2.82 BM 2) 1.41 BM 3) 1.82 BM 4) 5.46 BM

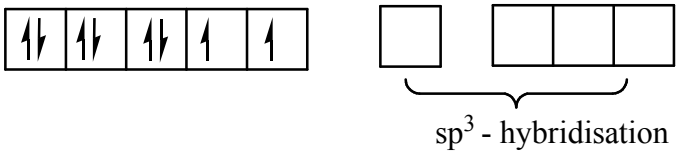
Ans : 1

Sol : $[NiCl_4]^{-2}$

$Ni^{+2}: 3d^8 4s^0$

Cl^- is a weak ligand

have no pairinig of non bonding electrons takes place



sp^3 - hybridisation

(n = 2)

hence $\mu = \sqrt{n(n+2)}\ B.M$

$= \sqrt{2(2+2)} = 2.82\,BM$

23. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively :

1) sp, sp^3, sp^2 2) sp^2, sp^3, sp 3) sp, sp^2, sp^3 4) sp^2, sp, sp^3

Ans :4

Sol : $NO_3^- = S.No. = \frac{5+0-(-1)}{2} = 3(sp^2)$

$NO_2^{\oplus} = S.No. = \frac{5+0-1}{2} = 2(sp)$

$NH_4^{\oplus} = S.No. = \frac{5+4-1}{2} = 4(sp^3)$

24. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of :

1) an isopropyl group 2) an acetylenic triple bond

3) two ethylenic double bonds 4) a vinyl group

Ans : 4

Sol : Ozonalysis of an organic compound give formaldehyde when it containing atleast one vinylic double bond. $H_2C = CH - R$

25. Which of the following facts about the complex $\left[Cr\left(NH_3\right)_6\right]Cl_3$ is wrong ?

1) The complex is an outer orbital complex.

2) The complex gives white precipitate with silver nitrate solution.

3) The complex involves d^2sp^3 hybridisation and is octahedral in shape

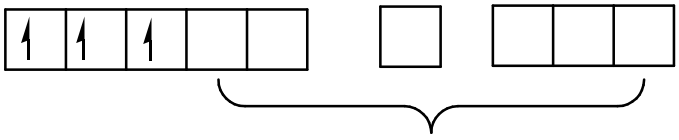
4) The complex is paramagnetic

Ans : 1

Sol : $\left[Cr\left(NH_3\right)_6\right]Cl_3$

NH_3 acts as strong ligand

$Cr^{+3} = 3d^3 4s^0$



d^2sp^3

Its is inner orbital complex

26. The reduction potential of hydrogen half-cell will be negative if :

1) $p\left(H_2\right) = 2$ atm and $\left[H^+\right] = 1.0$ M

2) $p\left(H_2\right) = 2$ atm and $\left[H^+\right] = 2.0$ M

3) $p\left(H_2\right) = 1$ atm and $\left[H^+\right] = 2.0$ M

4) $p\left(H_2\right) = 1$ atm and $\left[H^+\right] = 1.0$ M

Ans : 1

Sol : $E_{H^+/H_2,Pt} = E^0_{H^+/H_2,Pt} + \frac{0.0591}{2}\log\frac{\left[H^+\right]^2}{P_{H_2}}$

27. In context of the lanthanoids, which of the following statements is not correct ?

1) Because of similar properties the separation of lanthanoids is not easy.

2) Availability of 4f electrons results in the formation of compounds in +4 state for all the memebers of the series.

3) There is a gradual decrease in the radii of the members with increasing atomic number in the series.

4) All the members exhibit +3 oxidation state

Ans : 2

Sol : All the Lanthanides cannot form + 4 oxidation state

28. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?

1) Molisch reagent 2) Neutral $FeCl_3$ 3) Aqueous NaOH 4) Tollen's reagent

Ans : 2

Sol : $6C_6H_5OH \rightarrow 3H^+ + \underset{\text{Bloodred colour}}{\left[Fe\left(OC_6H_5\right)_6\right]^{-3}} + 3HCl$

29. Which of the following statements regarding sulphur is incorrect ?

1) At 600°C the gas mainly consists of S_2 molecules.

2) The oxidation state of sulphur is never less than +4 in its compounds.

3) S_2 molecule is paramagnetic.

4) The vapour at 200°C consists mostly of S_8 rigns.

Ans : 2

Sol : Sulphur can exhibit -2, +2, +1 oxidation states also

30. Which of the following statements is wrong ?

1) Single N–N bond is weaker than the single P–P bond.

2) N_2O_4 has two resonance structures.

3) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.

4) Nitrogen cannot form $d\pi - p\pi$ bond.

Ans : 3

Sol : Stability of VA group hydrides decreases from NH_3 to BiH_3 due to the decrease in M - H bond energyt

$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

MATHEMATICS

31. If $\vec{a}=\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\vec{b}=\frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\vec{a}-\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is :

1) 5 2) 3 3) – 5 4) – 3

Sol. $\left(2\bar{a}-\bar{b}\right)\left\{\left(\bar{a}\times\bar{b}\right)\times\left(\bar{a}+2\bar{b}\right)\right\}$

$=-5\left|\bar{a}\right|^2\left|\bar{b}\right|^2=-5$

$\because\left|\bar{a}\right|=1 \qquad \left|\bar{b}\right|=1 \qquad \bar{a}.\bar{b}=0$

32. Coefficient of x^7 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is :

1) – 144 2) 132 3) 144 4) – 132

Sol. $\left(1-x-x^2+x^3\right)^6=(1-x)^6\left(1-x^2\right)^6$

coefficient of x^7 in $\left(1-6c_1x+6c_2x^2-6c_3x^3+6c_4x^4-6c_5x^5+6c_6x^6)\right)$

$\left(1-6c_1x^2+6c_2x^4-6c_3x^6+6c_4x^8-6c_5x^{10}+6c_6x^{12})\right)$

$=\left(-6c_1\times-6c_3\right)+\left(-6c_3\times-6c_2\right)+\left(-6c_5\times-6c_1\right)$

$=120-300+36$

$=-144$

Ans : (1)

33. Let α,β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line

Re z = 1, then it is necessary that :

1) $|\beta|=1$ 2) $\beta\in(1,\infty)$ 3) $\beta\in(0,1)$ 4) $\beta\in(-1,0)$

Sol. $z^2+\alpha z+\beta=0$

$z=x+iy$

$x=1\Rightarrow z=1+iy\Rightarrow(1+iy)^2+\alpha(1+iy)+\beta=0$

$\Rightarrow\left(1-y^2+\alpha+\beta\right)+i(2y+\alpha y)=0$

$\Rightarrow 2y+\alpha y=0$

$\Rightarrow \alpha = -2$

$1 - y^2 + \alpha + \beta = 0$

Put $\alpha = -2$

$\Rightarrow 1 + y2 = \beta \geq 1$

$\Rightarrow \beta \in [1, \infty)$

34. Consider the following statements

P : Suman is brilliant Q : Suman is rich R : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only is Suman is rich" can be expressed as :

1) $\sim Q \leftrightarrow \sim P \wedge R$ 2) $\sim (P \wedge \sim R) \leftrightarrow Q$ 3) $\sim P \wedge (Q \leftrightarrow \sim R)$ 4) $\sim (Q \leftrightarrow (P \wedge \sim R))$

Sol. $\sim \{P \wedge \sim R \Leftrightarrow Q\}$

$= \sim \{Q \Leftrightarrow P \wedge \sim R\}$

35. $\frac{d^2x}{dy^2}$ equals :

1) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ 2) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ 3) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ 4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

Sol. $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right)$

$-\frac{1}{\left(\frac{dy}{dx}\right)^2} \times \frac{d}{dy}\left(\frac{dy}{dx}\right)$

$= -\frac{1}{\left(\frac{dy}{dx}\right)^2} \times \frac{d^2y}{dx^2} \times \frac{dx}{dy}$

$= -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

36. **Statement–1 :**

The point A (1, 0, 7) is the mirror images of the point B(1, 6, 3) in the line :

$$\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$$

Statement–2 : The line segment joining A(1, 0, 7) and B(1, 6, 3)

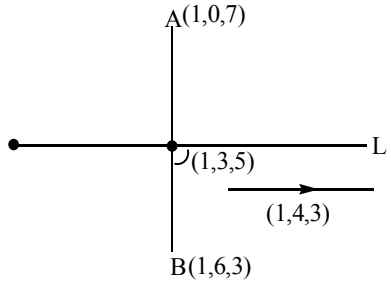
1) Statement–1 is true, Statement–2 is false

2) Statements–1is false, Statement–2 is true

3) Statement–1 is true, Statement–2 is true ;
Statement–2 is a correct explanation for Statement–1

4) Statement–1 is true, Statement–2 is true ;
Statement–2 is not a correct explanation for Statement–1.

Sol.



Dr's of AB $\perp$ Dr's of L

B is Image of A

$\therefore (1,3,5)$ is mid point of AB

37. If C and D are two events such that $C \subset D$ and

1) $P(C|D) < P(C)$ 2) $P(C|D)=\frac{P(D)}{P(C)}$ 3) $P(C|D)=P(C)$ 4) $P(C|D)\geq P(C)$

Sol. $C \subset D \Rightarrow P(C \cap D)=P(C)$

$P(D)\neq 0$

$$P(C/D)=\frac{P(C)}{P(D)}\geq P(C)$$

38. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure in greater than or equal to $\frac{31}{32}$, then p lies in the interval :

1) $\left[0,\frac{1}{2}\right]$ 2) $\left[\frac{11}{12},1\right]$ 3) $\left(\frac{1}{2},\frac{3}{4}\right]$ 4) $\left(\frac{3}{4},\frac{11}{12}\right]$

Sol. Pb of at least one failure = 1 – Pb of no failure

$$= 1-5c_5\left(p^n\right)\left(q^0\right)=\frac{31}{32}$$

$$1-P^5=\frac{31}{35}$$

$$\Rightarrow p=\frac{1}{2}$$

39. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}\,dx$ *is* :

(1) $\frac{\pi}{2}\log 2$ (2) $\log 2$ (3) $\pi\log 2$ (4) $\frac{\pi}{8}\log 2$

Sol. $\int_0^1 \frac{8\log(1+x)}{\left(1+x^2\right)}dx$

put x= $\tan\theta$

$$\Rightarrow \int_0^{\pi/4} \frac{8\log(1+\tan\theta)}{\cancel{\sin^2\theta}}\times \cancel{\sin^2\theta}\,d\theta=\frac{\cancel{8}\pi\log 2}{\cancel{8}}=\pi\log 2$$

40. For $x\in\left(0,\frac{5\pi}{2}\right)$, define $f(x)=\int_0^x \sqrt{t}\ \sin t\,dt$ Then f has:

1) local minimum at π and local maximum at 2π

2) local maximum at π and local minimum at 2π

3) local maximum at π and 2π 4) local minimum at π and 2π

Sol. $xt\left(0,\frac{5\pi}{2}\right)$

$$f(x)=\int_0^x \sqrt{t}\sin t\,dt$$

$f^1(x) = \sqrt{x}\sin x = 0 \Rightarrow x = 0,\ \pi,\ 2\pi$

$f^{11}(x) < 0\ ;\ f^{11}(2\pi) > 0$

$\Rightarrow f(x)$ has maximum at $x = \pi$

minimum at $x = 2\pi$

41. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d}=0$. The the vector $\vec{d}$ is equal to :

1) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ 2) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ 3) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ 4) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

Sol. $\bar{a} \not\perp \bar{b}$ $\bar{a}.\bar{b} = 0 \Rightarrow \bar{a} \perp \bar{d}$

$\bar{b}\times\bar{c} = \bar{b}\times\bar{d}$

take cross product with $\bar{a}$

$\bar{a}\times(\bar{b}\times\bar{c}) = \bar{a}\times(\bar{b}\times\bar{d})$

$\Rightarrow \bar{d} = \bar{c} - \frac{(\bar{a}.\bar{c})}{(\bar{a}.\bar{b})}\bar{b}$

42. Let R be the set of real numbers .

Statement-1: A= $\{(x,y) \in R\times R : y - x$ is an integer$\}$ is an ewquivalence relation on R.

Statement-2: B = $\{(x,y) \in R\times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation of R.

1) Statement-1 is true, Statement-2 is false.

2) Statement-1 is false, Statement-2 is true

3) Statement-1 is true, Statement-2 is true;

Statement-2 is a correct explanation for Statement-1.

4) Statement-1 is true, Statement-2 is true;

Statement-2 is not a correct explanation for Statement-1.

Sol. St (1) is Reflexive, symmetric & Trensitive

$\Rightarrow$ It is equivalence relation on R

St (2) is Reflexive, symmetric do not transitive

$\therefore$ not equivalence relation on R

43. Let A and B be two symmetric matrices of order 3.

Statement-1: A(BA) and (AB)A are symmetric matrices.

Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative.

1) Statement-1 is true, Statement-2 is false.

2) Statement-1 is false, Statement-2 is true

3) Statement-1 is true, Statement-2 is true;

Statement-2 is a correct explanation for Statement-1.

4) Statement-1 is true, Statement-2 is true;

Statement-2 is not a correct explanation for Statement-1.

Sol. Given $A^T = A$

$B^T = B$

$\left(A(BA)\right)^T = (BA)^T A^T = \left(A^T B^T\right) A^T$

$= (AB)A = A(BA)$

$\left((AB)A\right)^T = A^T (AB)^T = A^T \left(B^T A^T\right)$

$= A(BA) = (AB)A$

$\Rightarrow$ ST (1) is true

$(AB)^T = B^T A^T = BA$

$= AB$ if commutative

$\Rightarrow$ ST (2) is true

44. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if:

1) a = 2c 2) | a| = 2c 3) 2|a| = c 4) |a| = c

Sol. $x^2 + y^2 - ax = 0$ $x^2 + y^2 = c^2$

$c_1 = \left(\frac{a}{2}, 0\right)$ $c_2 = (0,0)$

$r_1 = \frac{a}{2}$ $r_2 = c$

$c_1c_2 = |r_1 - r_2| \Rightarrow \frac{a}{2} = \left(\frac{a}{2} - c\right)$ $\Rightarrow |a| = c$

45. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

1) equals - $\sqrt{2}$ 2) equals $\frac{1}{\sqrt{2}}$ 3) does not exist 4) equals $\sqrt{2}$

Sol. $\lim_{x\to 2}\frac{\sqrt{1-\cos 2(x-2)}}{x-2}$

$= \lim_{x\to 2}\frac{\sqrt{2}|\sin(x-2)|}{x-2}$

LHL $=-\sqrt{2}$ RHL $=+\sqrt{2}$

does not exist

46. If A = $\sin^2 x + \cos^4 x$, then for all real x:

1) $1 \le A \le 2$ 2) $\frac{3}{4} \le A \le \frac{13}{16}$ 3) $\frac{3}{4} \le A \le 1$ 4) $\frac{13}{16} \le A \le 1$

Sol. $A = \sin^2 x + \cos^4 x$

$= 1 - \cos^2 x \sin^2 x = 1 - \frac{\sin^2 2x}{4}$

$\because 0 \le \sin^2 x \le 1$

$\Rightarrow 3/4 \le A \le 1$

47. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement-2 : In any triangle into two similar triangles.

1) Statement-1 is true, Statement-2 is false.

2) Statement-1 is false, Statement-2 is true

3) Statement-1 is true, Statement-2 is true;

Statement-2 is a correct explanation for Statement-1.

4) Statement-1 is true, Statement-2 is true;

Statement-2 is not a correct explanation for Statement-1.

Sol. $p(-2,-2)\ Q(1,-2)\ R\left(\frac{2\sqrt{2}-2\sqrt{5}}{2\sqrt{2}+\sqrt{5}},-2\right)$

$|PR|:|RQ| = 2\sqrt{2}:\sqrt{5}$

Statement 1 is true

Statement 2 is false

48. The domain of the function $f(x)=\frac{1}{\sqrt{|x|-x}}$ is :

1) $(-\infty,0)$ 2) $(-\infty,\infty)-\{0\}$ 3) $(-\infty,\infty)$ 4) $(0,\infty)$

Sol. $|x|-x>0 \quad \Rightarrow x<0$

49. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals :

1) $2/5$ 2) $5/3$ 3) $2/3$ 4) $3/2$

Sol. $\frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}}=\frac{3}{\sqrt{14}}$

$\Rightarrow \lambda=\frac{2}{3}$

50. The shortest distance between line $y-x=1$ and curve $x=y^2$ is :

1) $\frac{8}{3\sqrt{2}}$ 2) $\frac{4}{\sqrt{3}}$ 3) $\frac{\sqrt{3}}{4}$ 4) $\frac{3\sqrt{2}}{8}$

Sol. $p\left(t^2,t\right)\ \lambda=\frac{|t^2-t+1|}{\sqrt{2}}=\frac{t^2-t+1}{\sqrt{2}}$

$\lambda_{\min}=\frac{3}{4\sqrt{2}}=\frac{3\sqrt{2}}{8}$

51. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. his total saving from the start of service will be Rs. 11040 after :

1) 20 months 2) 21 months 3) 18 months 4) 19 months

Sol. $600+\frac{n-3}{2}\left[480+(n-4)40\right]=11040$

$\Rightarrow n=21$

52. If the mean deviation abour the median of the numbers a, 2a, ..., 50a is 50, then | a | equals :

1) 4 2) 5 3) 2 4) 3

Sol. $MD=\frac{\sum|xi-M|}{n}$ $M=\frac{51a}{2}$

$|a|=4$ n = 50

53. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7$ =A+ B ω. Then (A, B) equals :

1) (1, 0) 2) (-1, 1) 3) (0, 1) 4) (1, 1)

Sol. $(1+\omega)^7=A+B\omega$

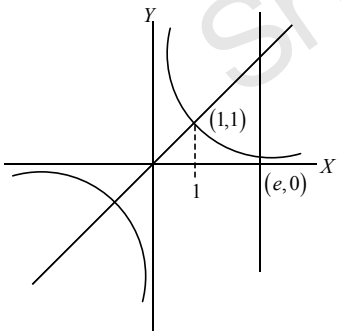
$\Rightarrow 1+\omega=A+B\omega$

$\Rightarrow A,B=(1,1)$

54. The area of the region enclosedby the curves $y=x, x=e, y=1/x$ and the positive x-axis is :

1) $3/2$ square units 2) $5/2$ square units 3) $1/2$ square units 4) 1 square units

Sol.



$A=\frac{1}{2}+\int_1^e 1/x\,dx=3/2$

55. The number of values of k for which the linear equations

$4x+ky+2z=0$

$kx+4y+z=0$

$2x+2y+z=0$

posses a non-zero solution is :

1) 1 2) zero 3) 3 4) 2

Sol. $\begin{vmatrix}4 & k & 2\\ k & 4 & 1\\ 2 & 2 & 1\end{vmatrix}=0\Rightarrow k=2,4$

Ans: 4

56. The values of p and q for which the function $f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$ is continuous

for all x in R, are:

1) $p=-\frac{3}{2},q=\frac{1}{2}$ 2) $p=\frac{1}{2},q=\frac{3}{2}$ 3) $p=\frac{1}{2},q=-\frac{3}{2}$ 4) $p=\frac{5}{2},q=\frac{1}{2}$

Sol. $\lim\limits_{x\to 0^+}\dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}=1/2$

$\lim\limits_{x\to 0^-}\dfrac{\sin(p+1)x+\sin x}{x}=p+2$

$f(0)=q\Rightarrow q=p+2=\dfrac{1}{2}$

$(p,q)=\left(\dfrac{-3}{2},\dfrac{1}{2}\right)$

Ans: 1

57. **Statement - 1**

The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is emtpty is $^9C_3$

Statement-2

The number of ways of choosing any 3 places from 9 different places is $^9C_3$

1) Statement-1 is true, Statement-2 is false

2) Statement-1 is false, Sttement-2 is true

3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

4) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

Sol. $ST(1):{}^{n-1}C_{r-1} = {}^{10-1}C_{4-1} = {}^{9}C_{3}$

$ST(2):{}^{n}C_{r} = {}^{9}C_{3}$

Ans : 4

58. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (-3,1) and has eccentricity $\sqrt{2/5}$ is:

1) $3x^2+5y^2-15=0$ 2) $5x^2+3y^2-32=0$ 3) $3x^2+5y^2-32=0$ 4) $5x^2+3y^2-48=0$

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Passes through (-3, 1)

$\Rightarrow \frac{9}{a^2}+\frac{1}{b^2}=1(1)$

$(1)b^2=a^2(1-e^2)=\frac{3a^2}{5}-(2)$

$(2)in(1)\Rightarrow a^2=\frac{32}{3};b^2=\frac{32}{5}\Rightarrow 3x^2+5y^2-32=0$

Ans: 3

59. Let I be the purchase value of an equipment an d V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt}=-k(T-1)$, where k>0 is a constant and t is the total life in years of the equipment. Then the scrap value V (T) of the equipment is

1) $I-\frac{k(T-t)^2}{2}$ 2) e^{-kT} 3) $T^2-\frac{I}{k}$ 4) $I-\frac{kT^2}{2}$

Sol. $\frac{d(v(t))}{dt}=-k(T-t);k>0$

Integrating

$v(t)=-kTt+\frac{kt^2}{2}+c$

$t=0,v(0)=I\Rightarrow c=I$

$v(t)=-kTt+\frac{k}{2}t^2+I$

$t = T \Rightarrow v(t)v(T)$

$v(T) = I - \frac{1}{2}kT^2$

Ans: 4

60. If $\frac{dy}{dx} = y + 3 > 0 \; and \; y(0) = 2$, then y(In2) is equal to:

1) 13 2) 2 3) 7 4) 5

Sol. $\int \frac{dy}{y+3} = \int dx \Rightarrow \ln(y+3) = x + \ln 5$

$\Rightarrow y + 3 = 10$

y = 7

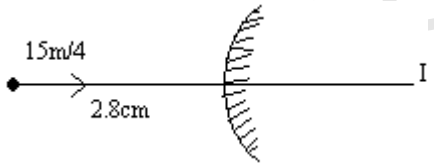
Ans: 3

PHYSICS

61. A car is fitted with a convex side-view mirror of focal length 20cm. A second car 2.8m behind the first car is overtaking the first car at a relative speed of 15m/s. The speed of the image of the second car as seen in the mirror of the first one is

1) 10 m/s 2) 15m/s 3) $\frac{1}{10} m/s$ 4) $\frac{1}{15} m/s$

Sol. convex f = 20cm



$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow -1/u^2 \cdot \frac{du}{dt} - \frac{1}{v^2}\left(\frac{dv}{dt}\right) = 0$

$\left(\frac{dv}{dt}\right) = -\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$

$= -\left(\frac{u}{f} - 1\right)^{-2}\frac{du}{dt}$

$= -\frac{1}{15^2} \times 15$

62. The half life of a radioactive substance is 20minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is

1) 20 min 2) 28 min 3) 7 min 4) 14 min

Sol. $T_{1/2} = 20\,\text{min}\quad N = \frac{N_0}{2^n} \Rightarrow \frac{N}{N_0} = \frac{1}{2^n} = \frac{2}{3}$

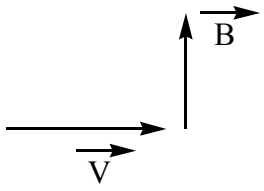
$$\frac{N^1}{N} = \frac{1}{2^n} = \frac{1}{3}$$

The substance has decayed by half $\Rightarrow \triangle t = T_{1/2} = 20\,\text{min}$

63. A boat is moving due east in a region where the earth's magnetic field is $5.0\times10^{-5}\ NA^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is $1.50\,ms^{-1}$, the magnitude of the induced emf in the wire of aerial is

1) 0.50 mV 2) 0.15 mV 3) 1 mV 4) 0.75 mV

Sol.



$B = 5\times10^{-5}\ N/Am$

l=2m $\Rightarrow \varepsilon = Blv = 5\times10^{-5}\times2\times1.5$

$= 15\times10^{-5} = 0.15mV$

64. The transverse displacement y(x,t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$. This represents a:

1) Standing wave of frequency $\sqrt{b}$

2) Standing wave of requency $\frac{1}{\sqrt{b}}$

3) Wave moving in + x direction with speed $\sqrt{\frac{a}{b}}$

4) Wave moving in - x direction with

Sol. $y = e^{-}\left(\sqrt{a}x + \sqrt{bt}\right)^2 = f\left(x+vt\right)$

$v = \sqrt{b/a}$ and -ve x direction

65. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the foutain is υ, the total area around the fountain that gets wet is

1) $\frac{\pi}{2}\frac{\upsilon^4}{g^2}$ 2) $\pi\frac{\upsilon^2}{g^2}$ 3) $\pi\frac{\upsilon^2}{g}$ 4) $\pi\frac{\upsilon^4}{g^2}$

Sol. $R_{max}=\frac{u^2}{g}\Rightarrow$ and $A=\pi R_{max}^2=\frac{\pi v^4}{g^2}$

66. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $x_0(x_0>A)$. If the maximum separation between them is (X_0+A), the phase difference between their motion is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$

Sol. **Ans :** $\frac{\pi}{2}$

67. This question has Statement - 1 and Statement - 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement - 1

A metallic surface is irradiated by a monochromatic light of requency $v>v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement - 2

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light

1) Statement - 1 is true, Statement - 2 is true, Statement -2 is not the correct explanation of Statement - 1

2) Statement - 1 is false, Statement - 2 is true.

3) Statement - 1 is true, Statement - 2 is false.

4) Statement - 1 is true, Statement - 2 is true, Statement - 2 is the correct explanation of Statement - 1.

Sol. $\upsilon>\upsilon_0$

$h\upsilon=k_{max}+\phi$

$2h\upsilon=k_{max}^1+\phi\Rightarrow k_{max}^1>2k_{max}$

$k_{max}=eV_0$

Statements 1 is incorrect

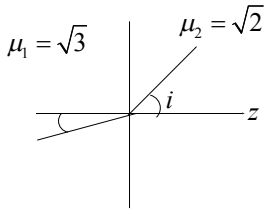
Statements 2 is correct

Ans : (2)

68. Let the x-z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with z<0 has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle o refraction in medium 2 is

1) 60^0 2) 75^0 3) 30^0 4) 45^0

Sol.



$\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$

angle made by the incident ray with z axis

$$\cos i = \frac{\vec{A}.\left(-\hat{K}\right)}{\left|\vec{A}\right|} = \frac{10}{\sqrt{36x3 + 64x3 + 100}} = \frac{10}{20}$$

$\therefore i = 60°$

$\sqrt{2}.\sin 60 = \sqrt{3}.\sin r$

$\sqrt{2}.\ \frac{\sqrt{3}}{2} = \sqrt{3}.\ \sin r \Rightarrow r = 45°$

Ans : (4)

69. A carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively:

1) 330 K and 268 K 2) 310 K and 248 K

3) 372 K and 310 K 4) 372 K and 330 K

Sol. $1 - \frac{T_2}{T_1} = \frac{1}{6}$

$\frac{T_2}{T_1} = \frac{5}{6}$

$1 - \left(\frac{T_2 - 62}{T_1}\right) = \frac{1}{3}$

$$1-\frac{T_2}{T_1}+\frac{62}{T_1}=\frac{1}{3}\Rightarrow\frac{1}{6}+\frac{62}{T_1}=\frac{1}{3}\Rightarrow\frac{62}{T_1}=\frac{1}{6}\Rightarrow T_1=62\text{x}6=372k$$

$$T_2=62\text{x}5=310K$$

Ans : (3)

70. Energy required for the electron excitation in Li^{++} from the first to the third-Bohr orbit is

1) 108.8eV 2) 122.4 eV 3) 12.1 eV 4) 36.3 eV

Sol. $\Delta E=13.6\text{x}3^2\left[1-\frac{1}{3^2}\right]$

$$=13.6\left[9-1\right]=13.6\text{x}8=108.8\ eV$$

Ans (1)

71. A resistor 'R' and $2\mu F$ capacitor in series is connected through a switch to 200V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10}2.5=0.4$)

1) $2.7\times10^6\Omega$ 2) $3.3\times10^7\Omega$ 3) $1.3\times10^4\Omega$ 4) $1.7\times10^5\Omega$

Sol. $q=120\times2=240\mu C$

$$q_0=400\mu C$$

$$q=q_0\left(1-e^{-t/\tau}\right)$$

$$40=400\left(1-e^{-t/\tau}\right)$$

$$\frac{6}{10}=\frac{3}{5}=\left(1-e^{-t/\tau}\right)$$

$$1-\frac{3}{5}=e^{-t/\tau}$$

$$\frac{2}{5}=e^{-t/\tau}\Rightarrow l\mu\left(\frac{5}{2}\right)=\frac{t}{\tau}$$

$$t=\tau\times l\mu\left(2.5\right)=$$

$$\text{RC}\ RC=\frac{5}{l\mu\left(2.5\right)}=\frac{5}{\log\left(2.5\right)}\times\log e$$

$$R=\frac{2.5\times\log e}{\log\left(2.5\right)}=\frac{5}{\log\left(2.5\right)}\times\log e$$

$$\frac{2.5\times0.43}{0.4}\times10^6 \approx 2.7\times10^6$$

Ans : (1)

72. A thrmally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed υ and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

1) $\frac{\gamma M\upsilon^2}{2R}K$ 2) $\frac{(\gamma-1)}{2R}M\upsilon^2K$ 3) $\frac{(\gamma-1)}{2(\gamma+1)R}M\upsilon^2K$ 4) $\frac{(\gamma-1)}{2\gamma R}M\upsilon^2K$

Sol. $\frac{C_p}{C_v}=v$

$$\frac{1}{2}\left(n \text{ x } N_0M\right)\upsilon^2 = nC_v\Delta T$$

$$\Rightarrow \Delta T=\frac{N_0M\upsilon^2}{2C_v}=\frac{N_0M\upsilon^2}{2\left(\frac{R}{r-1}\right)}$$

Ans : (2)

73. Work done in increasing the size of a soap bubble from a radius of 3cm to 5cm is nearly (Surface tension of soap solution = 0.03Nm^{-1}):

1) $2\pi\ mJ$ 2) $0.4\pi\ mJ$ 3) $4\pi\ mJ$ 4) $0.2\pi\ mJ$

Sol. Surface energy : $8\pi r^2T$

$$W=\Delta V_s=8\pi T\left(6^2-9^2\right)=8\times\pi0.03\left(25-9\right)\times10^{-2}$$

$$\simeq 0.4\pi\text{mJ}$$

Ans : (2)

74. A fully charged capacitor C with inital charge q_0 is connected to a coil of self inductance L at t = 0 . The time at which the energy is stored equally between the electric and the magnetic field s is

1) $2\pi\sqrt{LC}$ 2) $\sqrt{LC}$ 3) $\pi\sqrt{LC}$ 4) $\frac{\pi}{4}\sqrt{LC}$

Sol. $U_E=\frac{q^2}{2C},\quad U_L=\frac{1}{2}LI^2,$

$$U_E=U_L\Rightarrow\frac{q^2}{2C}=\frac{1}{2}\frac{q_0^2}{2C}\Rightarrow q=\frac{q_0}{\sqrt{2}}$$

$$\therefore \omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$$

Ans : (4)

75. Direction :

The question has a paragraph followed by two statements, **statement_1** and **statement_2** of the given four alternateves after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

statement_1: When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

statement_2: The centre of the interference pattern is dark.

1) statement_1 is true, statement_2 is true and statement_2 is **not** the correct explanation of statement_1.

2) statement_1 is false, statement_2 is true.

3) statement_1 is true, statement_2 is false

4) statement_1 is true, statement_2 is true and statement_2 is the correct explanation of statement_1.

Sol. Statement : 1 is true

($\because$ rarer to denser propatation)

Statement : 2 is true

at the centre $\Delta x = 0$

$\Rightarrow \Delta\phi = \pi$

Ans : (4)

76. A screw gauge gives following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm.

Circular scale reading : 52 divisions

Given that 1mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is

1) 0.26 cm 2) 0.005cm 3) 0.52cm 4) 0.052cm

Sol. $\text{Reading} = 0 + 52 \times \frac{1}{100} mm = 0.52mm = 0.052cm$

Ans : (4)

77. Three perfect gases at absolute temparatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1,n_2 and n_3 respectively. Assuming no loss of energy, the final temparature of the mixture is

1) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ 2) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

3) $\frac{(T_1+T_2+T_3)}{3}$ 4) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

Sol. $n_1C_{v_1}T_1+n_2C_{v_2}T_2+n_3C_{v_3}T_3=(n_1+n_2+n_3)C_{v_{mix}}.T$

Ans : (4)

78. The electrostatic potential inside a charged spherical ball is given by $\phi=ar^2+b$ where r is the distance from the centre; a,b are constants. Then the charge density inside the ball is

1) $-24\pi a\varepsilon_0$ 2) $-6a\varepsilon_0$ 3) $-24\pi a\varepsilon_0 r$ 4) $-6a\varepsilon_0 r$

Sol. $\phi=ar^2+b$

$$E=\frac{-\partial\phi}{dr}=-2ar$$

$$\oint\vec{E}.d\vec{s}=\frac{q_{in}}{\in_0}$$

$$-2ar\times4\pi r^2=\frac{\int f\,4\pi r^2dr}{\in_0}$$

$$=-2ar^3\in_0=\int fr^2dr$$

$$=-6ar^3\in_0=sr^2\Rightarrow f=-6a\in_0$$

Ans : (2)

79. The question has **statement_1** and **statement_2** of the four choices given after the statements, choose the one that best describes the statements.

statement_1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

statement_2 : The state of ionosphere varies from hour to hour, day to day and season to season.

1) statement_1 is true, statement_2 is true and statement_2 is **not** the correct explanation of statement_1.

2) statement_1 is false, statement_2 is true. 3) statement_1 is true, statement_2 is false

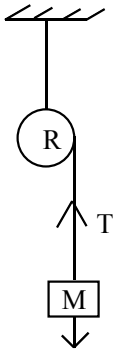
4) statement_1 is true, statement_2 is true and statement_2 is the correct explanation of statement_1.

Sol. **Ans (4)**

80. A mass m hangs with the help of a string wrapped around a pulley on frictionless beraing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc. the acceleration of the mass m, if the string does not slip on the pulley, is

1) $\frac{2}{3}g$ 2) $\frac{g}{3}$ 3) $\frac{3}{2}g$ 4) g

Sol.



$F_{Net} = ma$

$Mg - T = Ma$ ------(1)

$T_{net} = I \propto$

$TR = \frac{MR^2}{2}\frac{a}{R}$

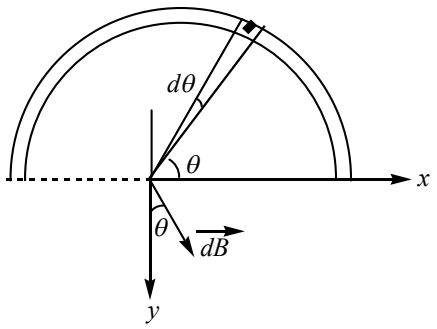
$\Rightarrow T = \frac{Ma}{2}$ ---------(2)

solving 1 and 2 $\Rightarrow a = \frac{2g}{3}$

81. A current I flows in a inifinitely long wire with cross section in the form of a semicicular ring of radius R. The magnitude of the magnetic induction along its axis is

1) $\frac{\mu_0 I}{2\pi R}$ 2) $\frac{\mu_0 I}{4\pi R}$ 3) $\frac{\mu_0 I}{\pi^2 R}$ 4) $\frac{\mu_0 I}{2\pi^2 R}$

Sol. $dI = \frac{I}{\pi \cancel{R}}.\cancel{R}d\theta = \frac{I}{\pi}d\theta$



$dB = \frac{\mu_0 dI}{2\pi R} = \frac{\mu_0}{2\pi R}.\left(\frac{I}{\pi}\right)d\theta$

$$= \frac{\mu_0 I}{2\pi^2 R}.d\theta$$

$$dB_y = dB\ Cos = \frac{\mu_0 I}{2\pi^2 R}.Cos\theta.d\theta\ ,\ dB_x = \frac{\mu_0 I}{2\pi^2 R}\sin\theta.d\theta$$

$$B_y = 2\int_{\theta}^{\pi}\left(\frac{\mu_0 I}{2\pi^2 R}\right)Cos\theta d\theta = zero$$

$$B_x = \int dB_x = 2\frac{\mu_0 I}{2\pi^2 R}\int_0^{\pi/2}\sin\theta.d\theta = \left(\frac{\mu_0 I}{2\pi^2 R}\right)\times 2\times\left(Cos\theta\right)\int_0^{\pi/2} = \frac{\mu_0 I}{\pi^2 R}$$

$$B = \sqrt{Bx^2 + By^2} = \frac{\mu_0 I}{\pi^2 R}$$

82. If a wire is streched to make it 0.1% longer, its resistance will :

1) decrese by 0.2 % 2) decrese by 0.05 %

3) increase by 0.05 % 4) increase by 0.2 %

Sol. $R = \frac{\ell L}{A} = \frac{\ell L^2}{v}$ where v is the volume of wire

$R \propto L^2$

$\frac{\Delta R}{R} = 2\frac{\Delta L}{L}$

$\Rightarrow$ % change in resistance of wire = 2 (% change in L)

= 2 (0.1 %)

= + 0.2 %

83. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other ene. During the journey of the insect, the angular speed of the disc:

1) continuously increase 2) first increase and then decrease

3) remains unchanged 4) continuously decrease

Sol. Since there is no external torque acting on system, angular momentum of the system is constant. As the insect moves from A to B, moment of inertia of system first decreases then increases. Since $\omega = \frac{L}{I}$, ω first increases then decreases.

84. 100 g of water is heated from 30^0C to 50^0C Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 418 J/Kg/K)

1) 84kJ 2) 2.1kJ 3) 4.2kJ 4) 8.4kJ

Sol. M = 0.1 kg

$d\theta = 20$

$du = MCd\theta$

$= 0.1(4184)(20)$

= 8368J

= 8.4 KJ

85. An object, moving with a speed of 6.25 m/s, is declerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$. where v is the instantaneous speed. The time taken by the object, to come to rest, would be

1) 4s 2) 8s 3) 1s 4) 2s

Sol. $u = 6.25ms^{-1}$

$\frac{dv}{dt} = -2.5\sqrt{V}$

$\int \frac{du}{\sqrt{V}} = -2.5\int dt$

$2\left[V^{\frac{1}{2}}\right]_{V=6.25}^{0} = -2.5[t]_{t=0}^{t=T}$

$2[0-2.5] = -2.5[T]$

T = 2 sec

86. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance $2\times10^{-3}m$ below the tap is close to

1) $9.6\times10^{-3}m$ 2) $3.6\times10^{-3}m$ 3) $5.0\times10^{-3}m$ 4) $7.5\times10^{-3}m$

Sol. $r_1 = 8\times10^{-3}$ m

$v_1 = 0.4ms^{-1}$

h = 0.2m

$\ell gh + \frac{1}{2}\ell v_1^2 = \frac{1}{2}\ell V_2^2$

$V_2^2 = V_1^2 + 2gh = 4.16$

from principle of continuity $A_1V_1 = A_2V_2$

$r_1^2 v_1 = r_2^2 v_2$

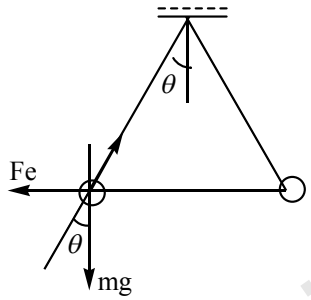
$$\frac{r_1\sqrt{V_1}}{\sqrt{V_2}} = r_2$$

$\Rightarrow r_2 = 3.6\times10^{-3}$ m

87. Two identical charged spheres suspended from a common point by two massless strings of length *l* are initially a distance d (d << *l*) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity u. Then as a function fo distance x between them,

1) $v \propto x^{\frac{1}{2}}$ 2) $v \propto x$ 3) $v \propto x^{-\frac{1}{2}}$ 4) $v \propto x^{-}$

Sol. $TCos\theta = mg$



$TSin\theta = Fe$

$$\tan\theta = \frac{Fe}{mg} \Rightarrow Fe = mg\tan\theta$$

$$\frac{kq^2}{x^2} = mg = \frac{x/2}{l}$$

$\therefore q^2 \propto x^3$

$$-\left(\frac{dq}{dt}\right).2q \propto 3x^2\left(-\frac{dx}{dt}\right)$$

$$V \propto \left(-\frac{dq}{dt}\right).\frac{q}{x^2} \propto x^{3/2-2} \propto x^{-1/2}$$

88. A mass M, attached to a horizontal spring, excutes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is

1) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$ 2) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$ 3) $\frac{M}{M+m}$ 4) $\frac{M+m}{M}$

Sol. $\omega = \sqrt{\frac{K}{M}}$

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{m+M}{M}}$$

$$P_1 = P_2$$

$$M_1 A_1 W_1 = M_2 A_2 W_2$$

$$\frac{A_1}{A_2} = \frac{M+m}{M}\sqrt{\frac{M}{M+m}} = \sqrt{\frac{M+m}{M}}$$

89. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

1) $-\frac{6Gm}{r}$ 2) $-\frac{9Gm}{r}$ 3) zero 4) $-\frac{4Gm}{r}$

Sol. $\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$

$$\frac{1}{x} = \frac{2}{r-x}$$

x = r/3

$$v_{net\ at\ p} = \frac{-Gm}{r/3} - \frac{G(4m)}{2r/3}$$

$$= \frac{-9Gm}{r}$$

90. A pulley of radius 2m is rotated about its axis by a force $F = \left(20t - 5t^2\right)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is

1) more than 6 but less than 9 2) more than 9

3) less than 3 4) more than 3 but less than 6

Sol. Direction of motion is reversed $\Rightarrow$ the pulley comes to momentary rest

$$\tau = FR = \left(20t - 5t^2\right) \times 2 = I\alpha$$

$$t\alpha = \frac{\left(20t - 5t^2\right)}{5} = \frac{dw}{dt}$$

$$\int_{t=0}^{t^2} \left(4t - t^2\right) dt = \int_0^{\omega} dw$$

$$w = \frac{4t^2}{2} - \frac{t^3}{3}$$

When it comes to rest

w = 0

$$\Rightarrow t = 0 \,\&\, t = 6s$$

$$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \text{ s}$$

$$\theta = 2\frac{t^3}{3} - \frac{1}{12}t^4$$

$$= 2 \times 6^2 \times 2 - \frac{1}{12} \times 6^2 \times 6 \times 6^3$$

$$= 6^2\left(4 - 3\right) = 6^2 = 36$$

$$= \frac{36}{2 \times 3.14} revolutions = \frac{11.46}{2} revolution = 5.73$$

Ans: 4