

EECS40**Spring 2004
Professor Sanders****Midterm Exam # 2
April 15, 2004
Time Allowed: 80 minutes**Name: SOLUTIONS, _____
Last First

Student ID #: _____, Signature: _____

Discussion Section: _____

This is a closed-book exam, except for use of two 8.5 x 11 inch sheet of your notes.
Show all your work to receive full or partial credit. Write your answers clearly in the
spaces provided.

Problem #:	Points:
1	/10
2	/20
3	/20
Total	/50

1.

a) (5 points)

A silicon sample is uniformly doped with Boron to a concentration of $10^{16} \text{ atoms/cm}^3$. Determine the resistivity of the sample at room temperature.

Use electron mobility $\mu_n = 1000 \text{ cm}^2/\text{V}\cdot\text{s}$, hole mobility $\mu_p = 400 \text{ cm}^2/\text{V}\cdot\text{s}$, $Q = 1.6 \cdot 10^{-19} \text{ C}$ and $n_i = 10^{10}$ at room temperature.

$$N_a = 10^{16} \text{ cm}^{-3} \quad p = 10^{16} \text{ cm}^{-3} \gg n_i$$

p-type :

$$\rho = \frac{1}{q p \mu_p} = \frac{1}{1.6 \times 10^{-19} \text{ C} \cdot 10^{16} \text{ cm}^{-3} \cdot 400 \text{ cm}^2/\text{V}\cdot\text{s}} = \frac{1}{0.64} \Omega\cdot\text{cm} = 1.56 \Omega\cdot\text{cm}$$

b) (5 points)

The same sample is then to be counter doped to a depth of $5 \mu\text{m}$ with Arsenic atoms to create a resistor technology with resistance of $100 \Omega/\square$.

Determine the required Arsenic doping density.

$$R_s = \frac{\rho'}{t}$$

$$\rho' = R_s t = 100 \Omega \cdot 5 \mu\text{m} = 0.05 \Omega\cdot\text{cm}$$

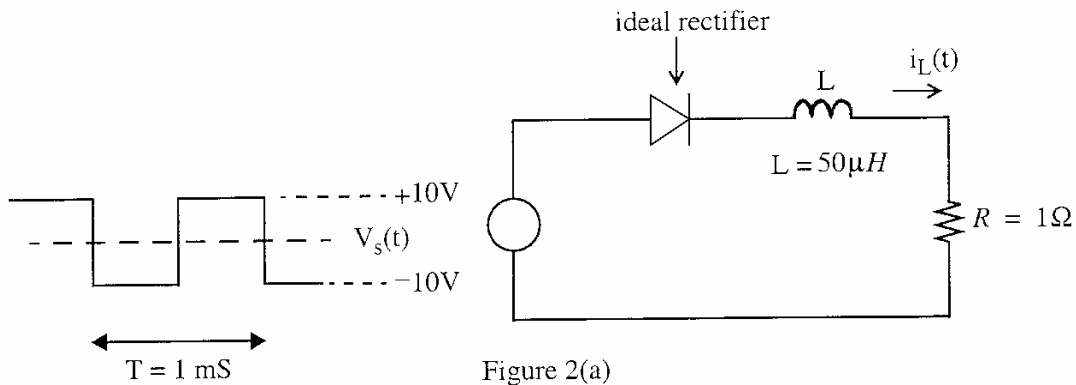
$\rho' \ll \rho \Rightarrow$ the sample becomes n-type

$$\rho' = \frac{1}{q n \mu_n + q p \mu_p} \approx \frac{1}{q n \mu_n} = \frac{1}{q (N_d - N_a) \mu_n}$$

$$\therefore N_d - N_a = n = \frac{1}{q \mu_n \rho'} = \frac{1}{1.6 \times 10^{-19} \text{ C} \cdot 1000 \text{ cm}^2/\text{V}\cdot\text{s} \cdot 0.05 \Omega\cdot\text{cm}} = 1.25 \times 10^{17} \text{ cm}^{-3}$$

$$N_d = n + N_a = 1.25 \times 10^{17} \text{ cm}^{-3} + 10^{16} \text{ cm}^{-3} = 1.35 \times 10^{17} \text{ cm}^{-3}$$

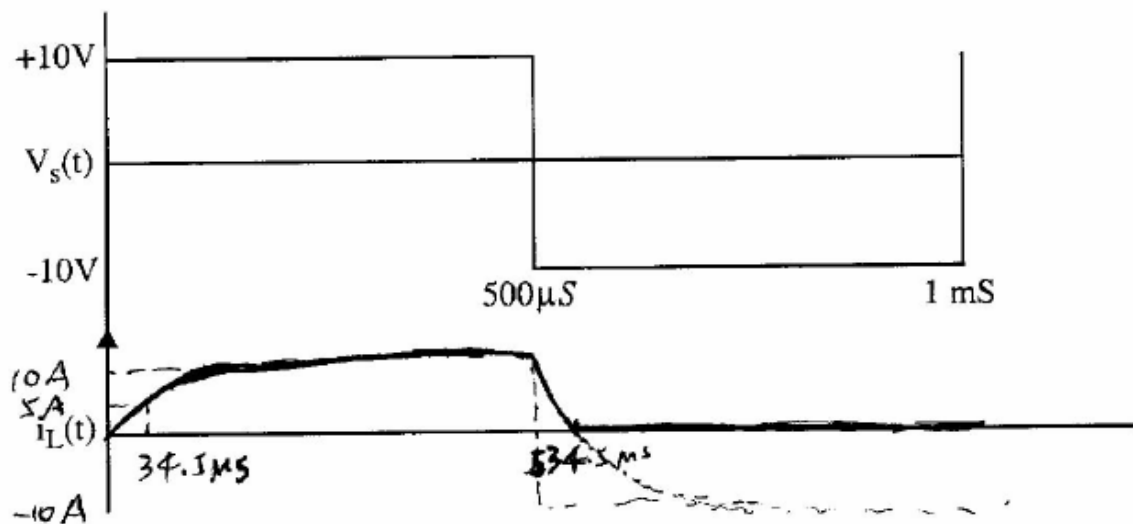
2.



a) (10 points)

The diode in Figure 2(a) is ideal. The waveform $V_s(t)$ is a balanced square wave with amplitude of 10 V and period 1 mS. Take $L = 50 \mu H$ and $R = 1 \Omega$.

The circuit operates in a periodic steady state. Sketch and carefully dimension one period of the $i_L(t)$ waveform on the axes below. Make reasonable approximations.



$$\tau = \frac{L}{R} = 0.05 \text{ ms}$$

$$0.69 \tau = 0.0345 \text{ ms} = 34.5 \mu\text{s}$$

b) (10 points)

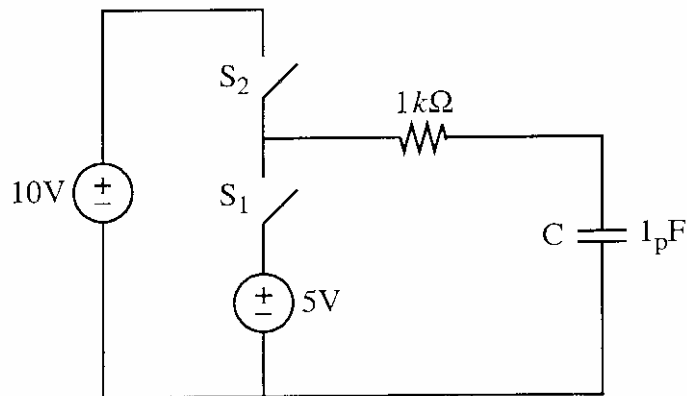


Figure 2(b)

In the circuit of Figure 2(b), switch S_1 is initially closed and switch S_2 is initially open and the circuit is in equilibrium. Switch S_1 is then opened and switch S_2 is closed for a sufficiently long time so that the circuit can be considered to be in equilibrium. How much energy is dissipated in the $1\text{ k}\Omega$ resistor during the transient?

Hint: Think in terms of net charge and energy flow. Detailed transient analysis is **NOT** needed.

Energy changed in capacitor

$$\begin{aligned}
 \Delta W_c &= \frac{1}{2} C V_2^2 - \frac{1}{2} C V_1^2 \\
 &= \frac{1}{2} C (100\text{ V}^2 - 25\text{ V}^2) \\
 &= \frac{1}{2} \times 10^{-12}\text{ F} \times 75\text{ V}^2 \\
 &= 3.75 \times 10^{-11}\text{ J}
 \end{aligned}$$

Total energy delivered by voltage source V_2 :

$$\begin{aligned}
 W_{\text{source}} &= \int V_2 i\, dt = V_2 \int i\, dt = V_2 \Delta Q \\
 &= V_2 (C V_2 - C V_1) \\
 &= 5 \times 10^{-11}\text{ J}
 \end{aligned}$$

Energy dissipated in resistor

$$W_R = W_S - \Delta W_c = 1.25 \times 10^{-11}\text{ J}$$

3.

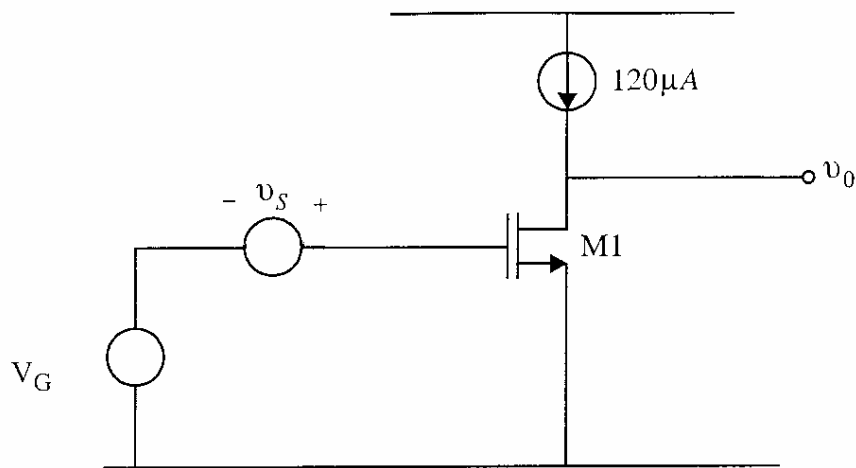


Figure 3

$$\lambda = 0.1 \text{ V}^{-1}$$

$$V_T = 0.5 \text{ V}$$

$$k' = 100 \mu\text{A}/\text{V}^2$$

$$\frac{W}{L} = 2$$

Mosfet M1 in Figure 3 is modeled by $i_D = \frac{1}{2} k' \frac{W}{L} (v_{GS} - V_T)^2 (1 + \lambda v_{DS})$ in saturation with parameters listed in Figure 3.

a) (5 points)

Determine the required bias voltage V_G so that M1 is biased in saturation with $V_{DS} = 2 \text{ V}$. Take $v_S = 0$

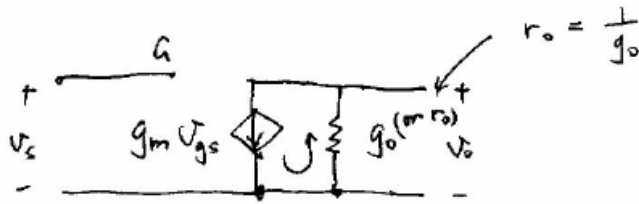
$$i_D = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$\Rightarrow 120 \mu\text{A} = \frac{1}{2} \times 100 \mu\text{A}/\text{V}^2 \times 2 \times (V_{GS} - 0.5 \text{ V})^2 (1 + 0.1 \text{ V}^{-1} \times 2 \text{ V})$$

$$\Rightarrow V_{GS} = 1.5 \text{ V}$$

b) (10 points)

Draw the small signal model for this circuit. Compute the parameters of this small signal model.



$$g_m = \frac{\partial i_D}{\partial v_{gs}} = k' \frac{W}{L} (v_{gs} - V_T) (1 + \lambda v_{ds})$$

$$= 100 \mu A/V^2 \times 2 \times (1.5V - 1V) (1 + 0.1V^{-1} \times 2V)$$

$$= 2.4 \times 10^{-4} S$$

$$g_o = \frac{\partial i_D}{\partial v_{ds}} = \frac{1}{2} k' \frac{W}{L} (v_{gs} - V_T)^2 \cdot \lambda$$

$$= \frac{1}{2} \times 100 \mu A/V^2 \times 2 \times (1.5V - 1V)^2 \times 0.1V^{-1}$$

$$= 10^{-5} S$$

c) (5 points)

Determine the small signal gain $A_v = \frac{v_o}{v_s}$.

$$v_o = -g_m v_{gs} \cdot r_o$$

$$= -g_m v_s \cdot \frac{1}{g_o}$$

$$A_v = \frac{v_o}{v_s} = - \frac{g_m}{g_o} = - \frac{2.4 \times 10^{-4} S}{10^{-5} S} = -24$$