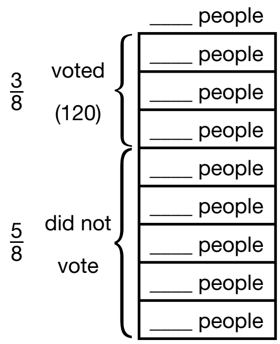
Name ________________________________

• Finding the Whole Group When a Fraction Is Known

Example: $\frac{3}{8}$ of the town voted. If 120 of the people voted, how many people lived in the town?

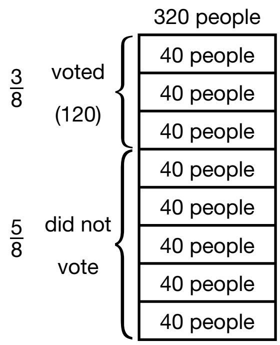


1. Draw a diagram divided into the same number of parts as the denominator. (For $\frac{3}{8}$, draw 8 parts.)

2. Bracket the same number of parts as the numerator. (For $\frac{3}{8}$, bracket 3 parts.) Then bracket the remaining parts.

3. Divide the given whole number by the numerator. $120 \div 3 = 40$

4. Write that answer in each part.



5. Add all the parts to find the whole group. There were 320 people in the town.

Practice:

1. Kareem said that four fifths of his age is 16 years.

How old is Kareem? __________

2. Two fifths of the audience are senior citizens. If 60 people in the audience are seniors, how many people are in the audience in all? __________

3. Charlie bought a book for \$25. This was $\frac{5}{6}$ of the regular price.

What was the regular price of the book? __________

• Implied Ratios

There are two ways to solve ratio problems: *multiplying by a rate* and *completing a proportion.*

Example: If 12 books weigh 20 pounds, how much would 30 books weigh?

1. Solve by *multiplying by a rate***:**

$$30\ \cancel{\text{books}} \times \frac{20 \text{ pounds}}{12\ \cancel{\text{books}}} = 50 \text{ pounds}$$

2. Solve by *completing a proportion:*

Make and complete a ratio box.

	Case 1	Case 2
Books	12	30
Pounds	20	*P*

Use the numbers in the ratio box to write a proportion.

$$\frac{\text{books}}{\text{pounds}} = \frac{12}{20} = \frac{30}{P} \qquad 12 \cdot P = 20 \cdot 30$$

$$12P = 600$$

$$P = \frac{600}{12}$$

$$P = 50 \text{ pounds}$$

Practice:

1. If Jim feeds his dog 7 pounds of dog food in 14 days, how much does he feed his dog in 30 days? ________

2. In 35 minutes, 49 customers entered the store. At this rate, how many customers will enter the store in 2 hours? ________

3. Vincent drove 75 miles in 90 minutes. How far could he drive at that rate in 1 hour? ________

• Multiplying and Dividing Positive and Negative Numbers

- To multiply and divide two signed numbers:
  1. Multiply or divide as with whole numbers.
  2. Place a sign on the answer.
     If the signs are the *same,* the answer is *positive.*
     If the signs are *different,* the answer is *negative.*

Examples: Multiplication / Division

Multiplication	Division
$(+6)(+2) = +12$	$\frac{+6}{+2} = +3$
$(-6)(-2) = +12$	$\frac{-6}{-2} = +3$
$(-6)(+2) = -12$	$\frac{-6}{+2} = -3$
$(+6)(-2) = -12$	$\frac{+6}{-2} = -3$

Practice:

Simplify 1–9.

1. $-7\ (-6)$ ____________

2. $-4\ (+6)$ ____________

3. $\frac{-5}{-15}$ ____________

4. $\frac{7.5}{-1.5}$ ____________

5. $\frac{1}{4}\left(\frac{-8}{10}\right)$ ____________

6. $\frac{-4.8}{-8}$ ____________

7. $\frac{900}{-3}$ ____________

8. $12\ (-10)$ ____________

9. $\left(\frac{-2}{5}\right)\left(\frac{-15}{30}\right)$ ____________

• Fractional Part of a Number, Part 2

- To find a fractional part of a number:
  1. Translate the question into an equation.
     Replace “is” with $=$.
     Replace “of” with $\times$.
  2. Solve.

Example: What fraction of 56 is 42? question

$$W_F \times 56 = 42 \quad \text{equation}$$

$$\frac{W_F \times 56}{56} = \frac{42}{56} \quad \text{divided by 56}$$

$$W_F = \frac{3}{4} \quad \text{simplified}$$

Example: Seventy-five is what decimal part of 20? question

$$75 = W_D \times 20 \quad \text{equation}$$

$$\frac{75}{20} = \frac{W_D \times 20}{20} \quad \text{divided by 20}$$

$$W_D = 3.75 \quad \text{simplified}$$

Example: Three fourths of what number is 60? question

$$\frac{3}{4} \times W_N = 60 \quad \text{equation}$$

$$\frac{4}{3} \times \frac{3}{4} \times W_N = 60 \times \frac{4}{3} \quad \text{multiplied by } \frac{4}{3}$$

$$W_N = 80 \quad \text{simplified}$$

Practice:

Write and solve an equation for each problem.

1. Sixty-four is four tenths of what number? ______________________

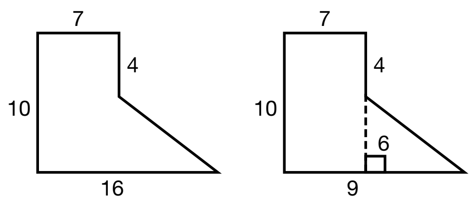
2. One fifth of what number is 345? ______________________

3. Three hundred is $\frac{3}{4}$ of what number? ______________________

• Area of a Complex Figure
• Area of a Trapezoid

- To find the area of a **complex figure,** divide it into rectangles and triangles.
  1. Draw lines to divide the figure into rectangles and triangles.
  2. Find the area of each part and add them together.

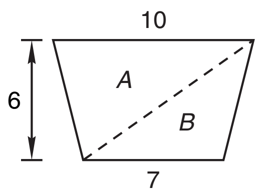
Example: Find the area of this figure. Corners that look square are square. Measurements are in millimeters.



$$
\begin{aligned}
&\text{Area of rectangle} &&= 7 \times 10 &&= 70 \text{ mm}^2\\
+\ &\text{Area of triangle} &&= \frac{6 \times 9}{2} &&= 27 \text{ mm}^2\\
\hline
&\text{Total area} && &&= 97 \text{ mm}^2
\end{aligned}
$$

- There are two ways to find the area of a **trapezoid:**
  1. Draw a *diagonal* line segment to divide the trapezoid into triangles. Then find the area of each triangle and add them together.

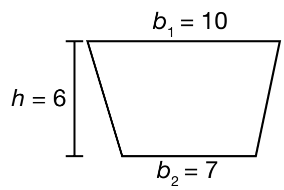
Example: Find the area of this trapezoid. Measurements are in centimeters.



$$
\begin{aligned}
&\text{Area of triangle } A &&= \frac{10 \times 6}{2} &&= 30 \text{ cm}^2\\
+\ &\text{Area of triangle } B &&= \frac{7 \times 6}{2} &&= 21 \text{ cm}^2\\
\hline
&\text{Total area} && &&= 51 \text{ cm}^2
\end{aligned}
$$

2. Use the formula $A = \frac{1}{2}(b_1 + b_2)h$

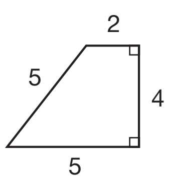
$$
\begin{aligned}
A &= \frac{1}{2}(10 + 7)(6)\\
&= \frac{17}{2}(6)\\
&= 51 \text{ cm}^2
\end{aligned}
$$



Practice:

Find the area of each figure. Dimensions are in meters.

1.



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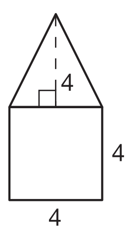

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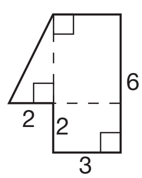

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Name ______________________________

• Complex Fractions

- A complex fraction is a fraction that contains a fraction.
- The fraction bar means *"divide by."*

Example: Simplify $\dfrac{15}{7\frac{1}{3}}$.

1. Write the numerator and denominator as *fractions.*

$$\frac{15}{7\frac{1}{3}} \longrightarrow \frac{\frac{15}{1}}{\frac{22}{3}}$$

2. Rewrite as a division problem.

$$\frac{15}{1} \div \frac{22}{3}$$

3. Multiply by the reciprocal of the second fraction.

$$\frac{15}{1} \times \frac{3}{22} = \frac{45}{22} = 2\frac{1}{22}$$

- Changing a percent to a fraction uses the same process. A percent is a fraction with a denominator of 100.

Example: Change $83\frac{1}{3}\%$ to a fraction.

1. Write the percent as a fraction with a denominator of 100. $\qquad \dfrac{83\frac{1}{3}}{100} \longrightarrow \dfrac{\frac{250}{3}}{\frac{100}{1}}$

2. Rewrite as a division problem. $\qquad \dfrac{\frac{250}{3}}{\frac{100}{1}} \longrightarrow \dfrac{250}{3} \div \dfrac{100}{1}$

3. Multiply by the reciprocal of the divisor. $\qquad \dfrac{250}{3} \times \dfrac{1}{100} = \dfrac{250}{300} = \dfrac{5}{6}$

Practice:

Simplify 1–3.

1. $\dfrac{8\frac{1}{3}}{2\frac{1}{2}}$ __________ **2.** $\dfrac{12\frac{1}{2}}{100}$ __________ **3.** $\dfrac{26}{3\frac{1}{4}}$ __________

Write 4–6 as a fraction.

4. $3\frac{1}{3}\%$ __________ **5.** $16\frac{2}{3}\%$ __________ **6.** $91\frac{2}{3}\%$ __________

• Percent of a Number, Part 2

- To find a **percent of a number:**
  1. Translate the question into an equation.
  2. Solve.

Example: What percent of 40 is 25? question

$$W_P \times 40 = 25 \quad \text{equation}$$

$$\frac{W_P \times 40}{40} = \frac{25}{40} \quad \text{divided by 40}$$

$$W_P = \frac{5}{8} \quad \text{simplified}$$

$$\frac{5}{8} \times 100\% = 62\frac{1}{2}\% \quad \text{converted to a percent}$$

Shortcut: Use the set-up $\frac{\text{is}}{\text{of}} = \frac{\text{percent}}{100}$ and substitute.

1. Write 100 on the lower right side.
2. Write the known numbers in the places of *is, of,* or *percent*.
3. Write a "?" in the place of the unknown that you are solving for.
4. Make a loop. Multiply. Divide by the number outside the loop.

Example: Fifty *is* what percent *of* 40?

$$\frac{\text{is}}{\text{of}} = \frac{50}{40} = \frac{?}{100} \longrightarrow (100 \times 50) \div 40 = 125 \longrightarrow \text{50 is 125\% of 40.}$$

Example: 75% *of* what number *is* 600?

$$\frac{\text{is}}{\text{of}} = \frac{600}{?} = \frac{75}{100} \longrightarrow (100 \times 600) \div 75 = 800 \longrightarrow \text{75\% of 800 is 600.}$$

When the problem says "translate" or "write an equation," use the equation method. At other times the shortcut method may be used to solve a percent problem.

Practice:

1. What percent of 75 is 25? __________

2. Write an equation to solve this problem: Fifty-six is what percent of 200?

____

3. Thirty percent of what number is 90? __________

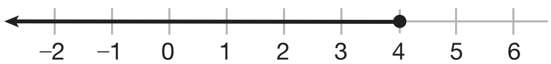
4. Twenty-four is 40% of what number? __________

Name ________________________________

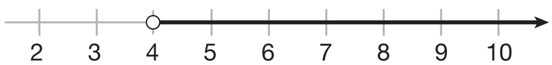
• Graphing Inequalities

- To graph an inequality on a number line:
  1. Use a dot or an empty circle to represent the given number.
     Draw a dot if the number is included in the graph.
     Draw an empty circle if the number is not included in the graph.
  2. Draw a shaded line to represent other numbers included in the graph.
  3. Draw an arrowhead to show that there are more numbers included that cannot be seen on the given number line.

Example: Graph $x \le 4$ on a number line.
The comparison $x \le 4$ means "x is *less than or equal to 4*."
On a number line:
1. Start at the answer "*equal to 4*." Draw a dot at 4 to show that 4 is included.
2. Draw a line on all the numbers *less than 4*.
3. Draw an arrowhead to show that there are more numbers less than 4.

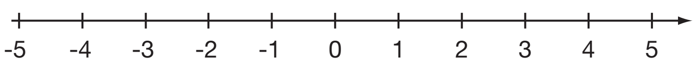


Example: Graph $x > 4$ on a number line.
The comparison $x > 4$ means "x is *greater than (but does not include) 4*."
On a number line:
1. Start at the given number 4. Draw an empty circle at 4 to show that 4 is not included (x is not equal to 4).
2. Draw a line on all the numbers *greater than 4*.
3. Draw an arrow to show that there are more numbers greater than 4.

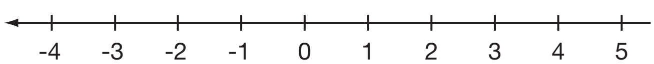


Practice:

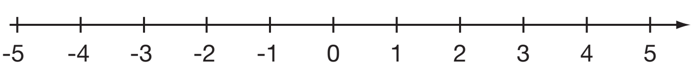
1. Graph $x > 4$.



2. Graph $y \le 3$.



3. Graph $x \ge -2$.

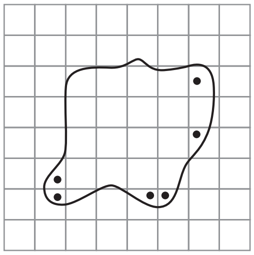


Name ______________________________

• Estimating Areas

- Area is measured in square units.
- To estimate the area of an irregular shape, use a grid and count the squares contained inside the shape.

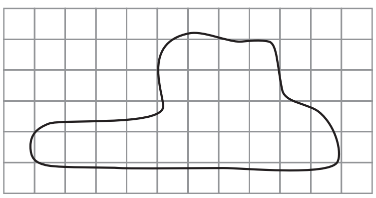
Example: Estimate the area of the shape on the grid. Each square represents 1 square inch.



1. Count the number of whole or nearly whole squares.
2. Mark each "half square" with a dot.
3. Find the total.
   17 whole squares + 6 "half squares" = 20 squares
   The area of the shape is about 20 square inches.

Practice:

Estimate the area of each shape on the grid. Each square represents 1 cm^2.

1. 

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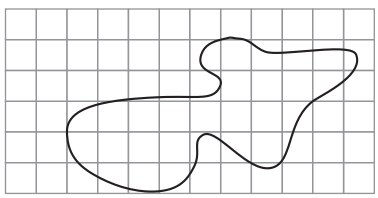
2. 

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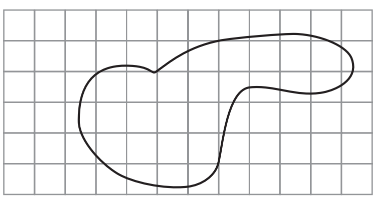
3. 

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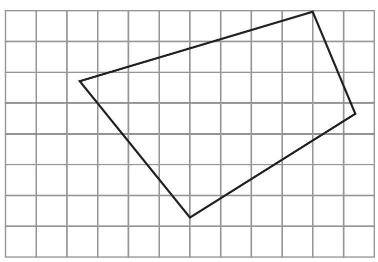
4. 

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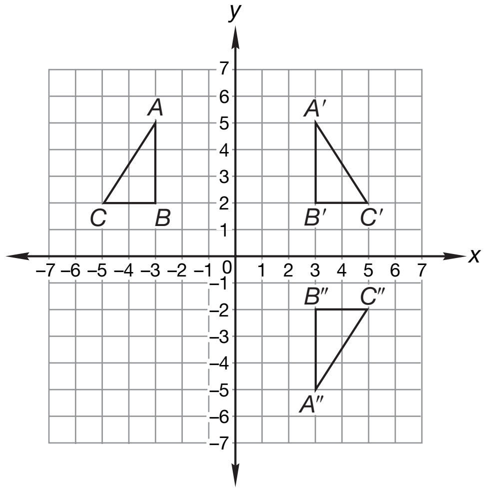
Name ______________________________

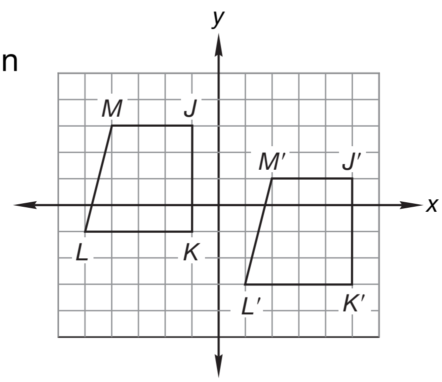
• Transformations

These **transformations** allow a figure to change position without changing size or shape.

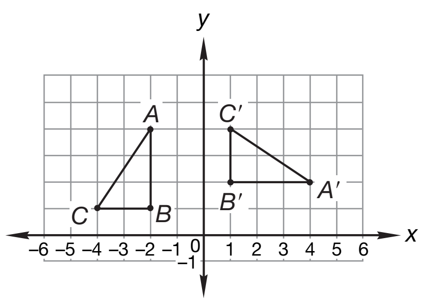
- **Flip:** A figure can flip like a coin. This is called **reflection** and makes a mirror image of the figure.
  - If a figure reflects (flips) in the *y*-axis, the reflection appears on the opposite side of the *y*-axis the same distance from the *y*-axis. $\triangle A'B'C'$ is a reflection in the *y*-axis of $\triangle ABC$.
  - If a figure reflects (flips) in the *x*-axis, the reflection appears on the opposite side of the *x*-axis the same distance from the *x*-axis. $\triangle A''B''C''$ is a reflection in the *x*-axis of $\triangle A'B'C'$.



- **Slide:** A figure can move or slide to a new position without a flip or turn. This is called **translation** and moves a figure right, left, up, or down. Quadrilateral $J'K'L'M'$ is a translation of quadrilateral *JKLM* 6 units to the right and 2 units down.

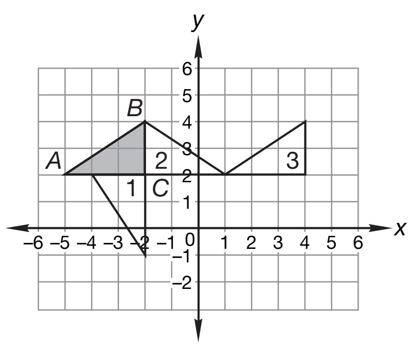


- **Turn:** A figure can turn or rotate about a specified point. This is called **rotation** and turns a figure around its *center* of rotation. The origin is the center of rotation for $\triangle ABC$ and it's image $\triangle A'B'C'$.



Practice:

Identify the transformation of $\triangle ABC$ that each figure represents.



1. Figure 1 ______________

2. Figure 2 ______________

3. Figure 3 ______________