

Exploring Data with Graphs and Numerical Summaries

Section 2.1: What Are the Types of Data?

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Learning Objectives

1. Know the definition of variable
2. Know the definition and key features of a categorical versus a quantitative variable
3. Know the definition of a discrete versus a continuous quantitative variable
4. Know the definition of frequency, proportion (relative frequencies), and percentages
5. Create Frequency Tables

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Learning Objective 1: Variable

- A **variable** is any characteristic that is recorded for the subjects in a study
- Examples: Marital status, Height, Weight, IQ
- A variable can be classified as either
 - **Categorical**, or
 - **Quantitative (Discrete, Continuous)**

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Learning Objective 2: Categorical Variable

- A variable can be classified as categorical if each observation belongs to one of a set of (non-numerical) categories.
- Examples:
 - Gender (Male or Female)
 - Religious Affiliation (Catholic, Jewish, ...)
 - Type of residence (Apt, Condo, ...)
 - Belief in Life After Death (Yes or No)

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Learning Objective 2: Quantitative Variable

- A variable is called quantitative if observations on it take numerical values that represent different magnitudes of the variable
- Examples:
 - Age
 - Number of siblings
 - Annual Income

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Learning Objective 2: Main Features of Quantitative and Categorical Variables

- **For Quantitative variables: key features are the center and spread (variability)**
- **For Categorical variables: a key feature is the percentage of observations in each of the categories**

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Learning Objective 3: Discrete Quantitative Variable

- A quantitative variable is discrete if its possible values form a set of separate numbers, such as 0,1,2,3,....
- Discrete variables have gaps between their possible values – usually they have a finite collection of values.
- Examples:
 - Number of pets in a household
 - Number of children in a family
 - Number of foreign languages spoken by an individual

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Learning Objective 3: Continuous Quantitative Variable

- A quantitative variable is continuous if its possible values form an interval
- Continuous variables have an infinite number of possible values (in principle)
- Examples:
 - Height/Weight
 - Age
 - Blood pressure

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Learning Objective 4:
Proportion & Percentage (Relative Frequencies)

- The number of observations that fall in a certain category is the **frequency** (count) of observations in that category divided by the total number of observations
 - Frequency of that class
Sum of all frequencies
- The Percentage is the proportion multiplied by 100. Proportions and percentages are also called **relative frequencies**.

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Learning Objective 4:
Frequency, Proportion, & Percentage Example

- If 4 students received an “A” out of 40 students, then,
 - 4 is the frequency
 - $0.10 = 4/40$ is the proportion and relative frequency
 - 10% is the percentage $.1 * 100 = 10\%$

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Learning Objective 5:
Frequency Table

- A **frequency table** is a listing of possible values for a variable, together with the number of observations and/ or relative frequencies for each value

No. hours	Frequency	Percent
0-1	232	25.6
2-3	403	44.5
4-5	181	20.0
6-7	45	5.0
8 or more	44	4.9
Total	905	100.0

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Class Problem #3

- A stock broker has been following different stocks over the last month and has recorded whether a stock is up, the same, or down in value. The results were

Performance of stock	Up	Same	Down
Count	21	7	12

1. What is the variable of interest
2. What type of variable is it?
3. Add proportions to this frequency table

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Exploring Data with Graphs and Numerical Summaries

Section: How Can We Describe Data Using Graphical Summaries?

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Learning Objectives

1. Distribution
2. Graphs for categorical data: bar graphs and pie charts
3. Graphs for quantitative data: dot plot, stem-leaf, and histogram
4. Constructing a histogram
5. Interpreting a histogram
6. Displaying Data over Time: time plots

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Learning Objective 1: Distribution

- A **distribution** is an association of the possible values a variable takes with the occurrence of those values (frequency or relative frequency)
- A **graph** or **frequency table** describes a distribution in pictorial or in tabular form.

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Learning Objective 2: Graphs for Categorical Variables

Using pie charts and bar graphs to summarize categorical variables

- **Pie Chart:** A circle having a “slice of pie” for each category
- **Bar Graph:** A graph that displays a vertical bar for each category

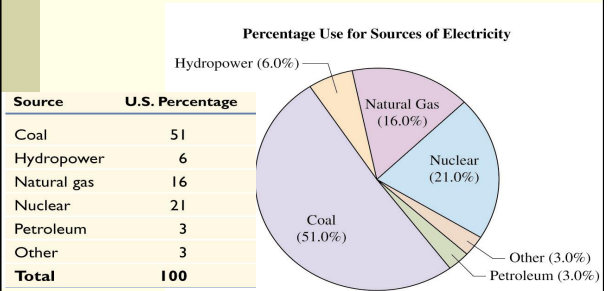
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Learning Objective 2:
Pie Charts

- Pie charts:
 - used for summarizing a categorical variable
 - Drawn as a circle where each category is represented as a “slice of the pie”
 - The size of each pie slice is proportional to the percentage of observations falling in that category

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Learning Objective 2:
Pie Chart Example



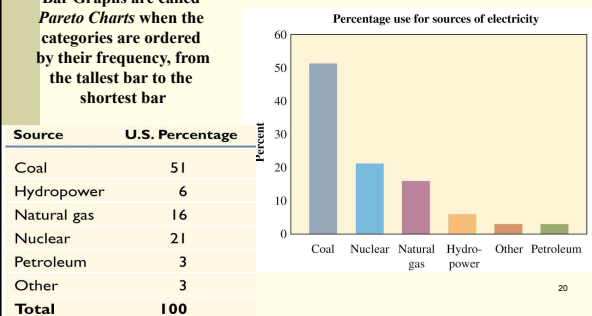
Learning Objective 2:
Bar Graphs

- Bar graphs are used for summarizing a categorical variable
- Bar Graphs display a vertical bar for each category
- The height of each bar represents either counts (“frequencies”) or percentages (“relative frequencies”) for that category
- Usually easier to compare categories with a bar graph than with a pie chart
- Bars for different categories don’t touch

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Learning Objective 2:
Bar Graph Example

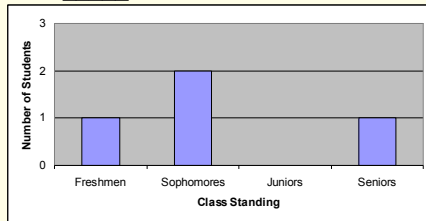
Bar Graphs are called *Pareto Charts* when the categories are ordered by their frequency, from the tallest bar to the shortest bar



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Learning Objective 2:
Class Exercise

There are 7 students in a class who are either freshman, sophomores, juniors, or seniors. The number of students in this class who are juniors is _____.



Learning Objective 3:
Graphs for Quantitative Data

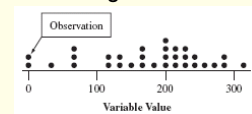
- **Dot Plot:** shows a dot for each observation placed above its value on a number line
- **Stem-and-Leaf Plot:** portrays the individual observations
- **Histogram:** uses bars to portray the data

Learning Objective 3:
Which Graph?

- **Dot-plot and stem-and-leaf plot:**
 - More useful for small data sets
 - Data values are retained
- **Histogram**
 - More useful for large data sets
 - Most compact display
 - More flexibility in defining intervals

Learning Objective 3:
Dot Plots

- Dot Plots are used for summarizing a quantitative variable

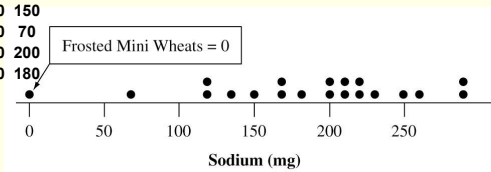


- To construct a dot plot
 1. Draw a horizontal line
 2. Label it with the name of the variable
 3. Mark regular values of the variable on it
 4. For each observation, place a dot above its value on the number line

Learning Objective 3:
Dot plot for Sodium in Cereals

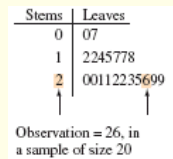
Sodium Data:

0 210
260 125
220 290
210 140
220 200
125 170
250 150
170 70
230 200
290 180



Learning Objective 3:
Stem-and-leaf plots

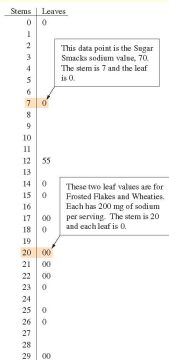
- Stem-and-leaf plots are used for summarizing quantitative variables
- Separate each observation into a **stem** (first part of the number) and a **leaf** (typically the last digit of the number)
- Write the stems in a vertical column ordered from smallest to largest, including empty stems; draw a vertical line to the right of the stems
- Write each leaf in the row to the right of its stem; order leaves if desired



Learning Objective 3:
Stem-and-Leaf Plot for Sodium in Cereal

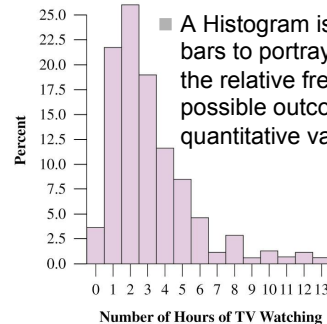
Sodium Data:

0 210
260 125
220 290
210 140
220 200
125 170
250 150
170 70
230 200
290 180



Learning Objective 4:
Histograms

- A Histogram is a graph that uses bars to portray the frequencies or the relative frequencies of the possible outcomes for a quantitative variable



Learning Objective 4:
Steps for Constructing a Histogram

1. Divide the range of the data into intervals of equal width
2. Count the number of observations in each interval, creating a frequency table
3. On the horizontal axis, label the values or the endpoints of the intervals.
4. Draw a bar over each value or interval with height equal to its frequency (or percentage), values of which are marked on the vertical axis.
5. Width of all bars the same and all touch
6. Label and title appropriately

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Learning Objective 4:
Histogram for Sodium in Cereals

Sodium

Data:

0 210
260 125
220 290
210 140
220 200
125 170
250 150
170 70
230 200
290 180

TABLE 2.4: Frequency Table for Sodium in 20 Breakfast Cereals.

The table summarizes the sodium values using eight intervals and lists the number of observations in each, as well as the proportions and percentages.

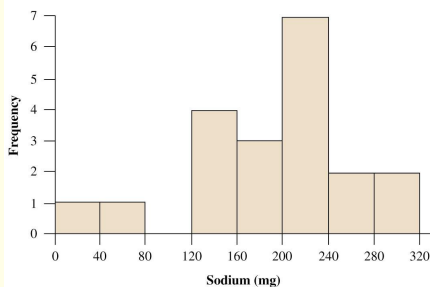
Interval	Frequency	Proportion	Percentage
0 to 39	1	0.05	5%
40 to 79	1	0.05	5%
80 to 119	0	0.00	0%
120 to 159	4	0.20	20%
160 to 199	3	0.15	15%
200 to 239	7	0.35	35%
240 to 279	2	0.10	10%
280 to 319	2	0.10	10%

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Learning Objective 4:
Histogram for Sodium in Cereals

Sodium Data:

0 210
260 125
220 290
210 140
220 200
125 170
250 150
170 70
230 200
290 180



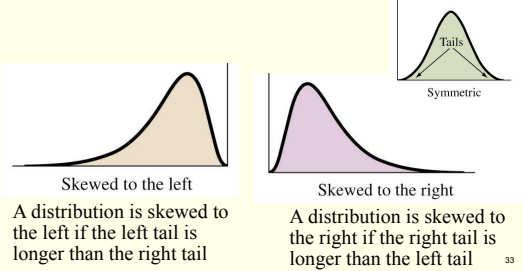
Learning Objective 5:
Interpreting Histograms

- Overall pattern consists of center, spread, and shape
 - Assess where a distribution is **centered** by finding the **median** (50% of data below median 50% of data above).
 - Assess the **spread** of a distribution.
 - **Shape** of a distribution: roughly symmetric, skewed to the right, or skewed to the left

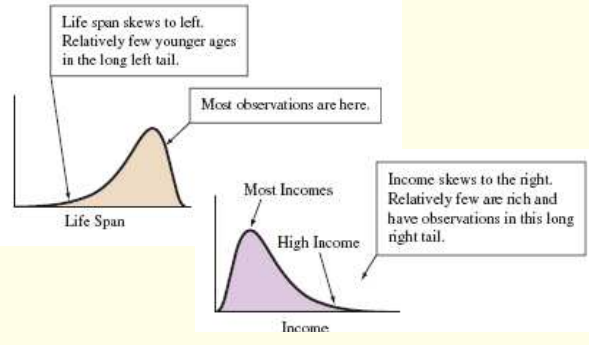
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Learning Objective 5:
Shape

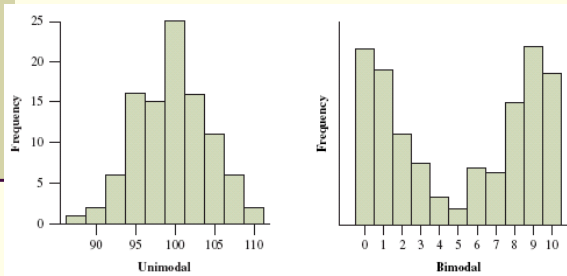
- Symmetric Distributions: if both left and right sides of the histogram are mirror images of each other



Learning Objective 5:
Examples of Skewness



Learning Objective 5:
Shape: Type of Mound



Learning Objective 5:
Shape and Skewness

- Consider a data set containing IQ scores for the general public:
- What shape would you expect a histogram of this data set to have?
 - a. **Symmetric**
 - b. **Skewed to the left**
 - c. **Skewed to the right**
 - d. **Bimodal**

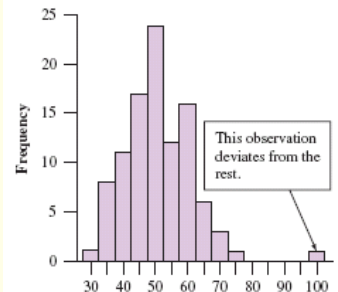
Learning Objective 5:
Shape and Skewness

- Consider a data set of the scores of students on a very easy exam in which most score very well but a few score very poorly:
- What shape would you expect a histogram of this data set to have?
 - a. **Symmetric**
 - b. **Skewed to the left**
 - c. **Skewed to the right**
 - d. **Bimodal**

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Learning Objective 5:
Outlier

- An Outlier falls far from the rest of the data



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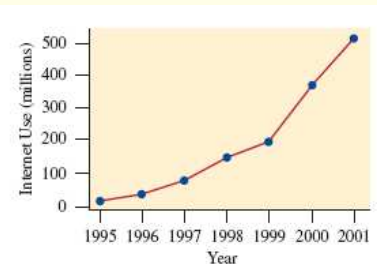
Learning Objective 6:
Time Plots

- Used for displaying a time series, a data set collected over time.
- Plots each observation on the vertical scale against the time it was measured on the horizontal scale. Points are usually connected.
- Common patterns in the data over time, known as trends, should be noted.

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Learning Objective 6:
Time Plots Example

- A Time Plot from 1995 – 2001 of the number of people worldwide who use the Internet



Exploring Data with Graphs and Numerical Summaries

Section: How Can We Describe the Center of Quantitative Data?

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Learning Objectives

1. Calculating the mean
2. Calculating the median
3. Comparing the Mean & Median
4. Definition of Resistant
5. Know how to identify the mode of a distribution

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Learning Objective 1: Mean

- The mean is the sum of the observations divided by the number of observations
- It is the center of mass

$$\bar{x} = \sum \frac{x}{n}$$

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Learning Objective 1: Calculate Mean

Cereal	Sodium
Frosted Mini Wheats	0
Raisin Bran	210
All Bran	260
Apple Jacks	125
Capt Crunch	220
Cheerios	290
Cinnamon Toast	210
Crackling Oat Bran	140
Crispix	220
Frosted Flakes	200
Fruit Loops	125
Grape Nuts	170
Honey Nut Cheerios	250
Life	150
Oatmeal Raisin Crisp	170
Sugar Smacks	70
Special K	230
Wheaties	200
Corn Flakes	290
Honeycomb	180

The Follower Shows the Mean of the Cereal Sodium Data

EDIT	TESTS
1:1-Var Stats	
2:2-Var Stats	
3:Med-Med	
4:LinReg(ax+b)	
5:QuadReg	
6:CubicReg	
7:QuartReg	

1-Var Stats
x̄=185.5
sx=71.8
Σx=734650
Sx=71.24642189
σx=69.44242219
n=20

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Learning Objective 2: Median

- The median is the midpoint of the observations when they are ordered from the smallest to the largest (or from the largest to smallest)
- Order observations
- If the number of observations is:
 - Odd, then the median is the middle observation
 - Even, then the median is the average of the two middle observations

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Learning Objective 2: Median

1) Sort observations by size.
 n = number of observations

Order	Data	Order	Data
1	78	1	78
2	91	2	91
3	94	3	94
4	98	4	98
5	99	5	99
6	101	6	101
7	103	7	103
8	105	8	105
9	114	9	114
		10	121

2.a) If n is **odd**, the median is observation $(n+1)/2$ down the list

← $n = 9$
 $(n+1)/2 = 10/2 = 5$
Median = 99

2.b) If n is **even**, the median is the mean of the two middle observations

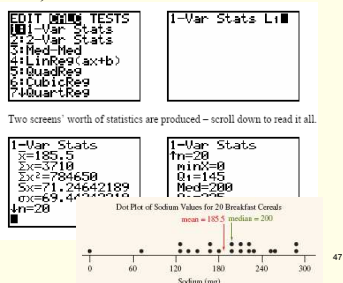
$n = 10$ →
 $(n+1)/2 = 5.5$
Median = $(99+101)/2 = 100$

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Learning Objective 1 & 2: Calculate Mean and Median

Cereal	Sodium
Frosted Mini Wheats	0
Raisin Bran	210
All Bran	260
Apple Jacks	125
Capt. Crunch	220
Cheerios	290
Cinnamon Toast	210
Crackling Oat Bran	140
Crispix	220
Frosted Flakes	200
Fruit Loops	125
Grape Nuts	170
Honey Nut Cheerios	250
Life	150
Oatmeal Raisin Crisp	170
Sugar Snacks	70
Special K	230
Wheaties	200
Corn Flakes	290
Honeycomb	180

Enter data into L1
STAT; CALC; 1:1-Var Stats; Enter
L1;Enter



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Leaning Objectives 1 & 2: Find the mean and median

CO₂ Pollution levels in 8 largest nations measured in metric tons per person:

2.3 1.1 19.7 9.8 1.8 1.2 0.7 0.2

- a. Mean = 4.6 Median = 1.5
- b. Mean = 4.6 Median = 5.8
- c. Mean = 1.5 Median = 4.6

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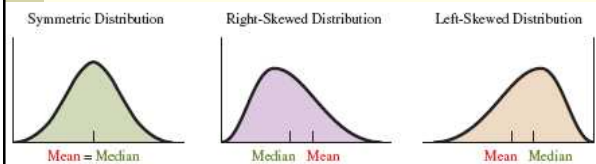
Learning Objective 3:
Comparing the Mean and Median

- The mean and median of a symmetric distribution are close together.
 - For symmetric distributions, the mean is typically preferred because it takes the values of all observations into account

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Learning Objective 3:
Comparing the Mean and Median

- In a skewed distribution, the mean is farther out in the long tail than is the median
 - For skewed distributions the median is preferred because it is better representative of a typical observation



Learning Objective 4:
Resistant Measures

- A numerical summary measure is **resistant** if extreme observations (outliers) have little, if any, influence on its value
 - The Median is resistant to outliers
 - The Mean is not resistant to outliers

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Learning Objective 5:
Mode

- Mode
 - Value that occurs most often
 - Highest bar in the histogram
 - The mode is most often used with categorical data

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Exploring Data with Graphs and Numerical Summaries

Section: How Can We Describe the Spread of Quantitative Data?

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Learning Objectives

1. Calculate the Range
2. Calculate the standard deviation
3. Know the properties of the standard deviation
4. Know how to interpret the magnitude of s :
The Empirical Rule

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Learning Objective 1: Range

- One way to measure the spread is to calculate the range. The **range** is the difference between the largest and smallest values in the data set;

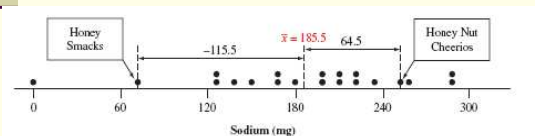
$$\text{Range} = \text{max} - \text{min}$$

- The range is strongly affected by outliers

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Learning Objective 2: Standard Deviation

- Each data value has an associated deviation from the mean, $x - \bar{x}$
- A deviation is positive if it falls above the mean and negative if it falls below the mean
- The sum of the deviations is always zero



Learning Objective 2:
Standard Deviation

- Gives a measure of variation by summarizing the *deviations* of each observation from the mean and calculating an *adjusted average* of these deviations

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

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Learning Objective 2:
Standard Deviation

$$s = \sqrt{\frac{1}{(n - 1)} \sum_{i=1}^n (x_i - \bar{x})^2}$$

- Find the mean
- Find the deviation of each value from the mean
- Square the deviations
- Sum the squared deviations
- Divide the sum by $n-1$ (for samples only)

(gives *variance*, or *squared deviation* from mean)

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Learning Objective 2:
Standard Deviation

Metabolic rates of 7 men (cal./24hr.):

1792 1666 1362 1614 1460 1867 1439

$$\begin{aligned} \bar{x} &= \frac{1792 + 1666 + 1362 + 1614 + 1460 + 1867 + 1439}{7} \\ &= \frac{11,200}{7} \\ &= 1600 \end{aligned}$$

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Learning Objective 2:
Standard Deviation

Observations x_i	Deviations $x_i - \bar{x}$	Squared deviations $(x_i - \bar{x})^2$
1792	1792 - 1600 = 192	(192) ² = 36,864
1666	1666 - 1600 = 66	(66) ² = 4,356
1362	1362 - 1600 = -238	(-238) ² = 56,644
1614	1614 - 1600 = 14	(14) ² = 196
1460	1460 - 1600 = -140	(-140) ² = 19,600
1867	1867 - 1600 = 267	(267) ² = 71,289
1439	1439 - 1600 = -161	(-161) ² = 25,921
	sum = 0	sum = 214,870

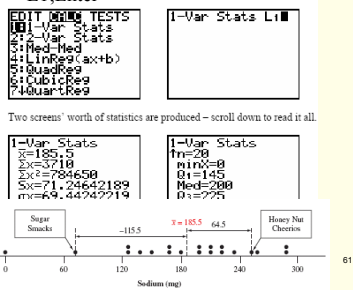
$$s^2 = \frac{214,870}{7 - 1} = 35,811.67$$

$$s = \sqrt{35,811.67} = 189.24 \text{ calories}^{\circ}$$

Learning Objective 2: Calculate Standard Deviation

Cereal	Sodium
Frosted Mini Wheats	0
Raisin Bran	210
All Bran	260
Apple Jacks	125
Capt Crunch	220
Cheerios	290
Cinnamon Toast	210
Crackling Oat Bran	140
Crispix	220
Frosted Flakes	200
Fruit Loops	125
Grape Nuts	170
Honey Nut Cheerios	250
Life	150
Oatmeal Raisin Crisp	170
Sugar Smacks	70
Special K	230
Wheaties	200
Corn Flakes	290
Honeycomb	180

Enter data into L1
STAT; CALC; 1:1-Var Stats; Enter
L1;Enter



Learning Objective 3: Properties of the Standard Deviation

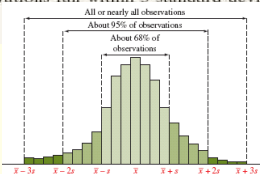
- s measures the spread of the data
- s = 0 only when all observations have the same value, otherwise $s > 0$. As the spread of the data increases, s gets larger.
- s has the same units of measurement as the original observations. The **variance** = s^2 has units that are squared
- s is not resistant. Strong skewness or a few outliers can greatly increase s.

Learning Objective 4: Magnitude of s: Empirical Rule

EMPIRICAL RULE

If a distribution of data is bell-shaped, then approximately

- 68% of the observations fall within 1 standard deviation of the mean, that is, between $\bar{x} - s$ and $\bar{x} + s$ (denoted $\bar{x} \pm s$).
- 95% of the observations fall within 2 standard deviations of the mean ($\bar{x} \pm 2s$).
- All or nearly all observations fall within 3 standard deviations of the mean ($\bar{x} \pm 3s$).



Exploring Data with Graphs and Numerical Summaries

Section: How Can Measures of Position
Describe Spread?

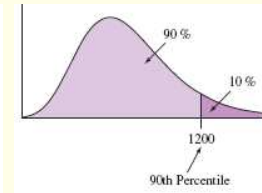
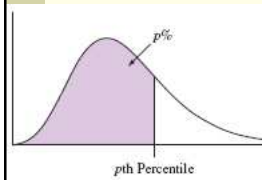
Learning Objectives

1. Obtaining quartiles and the 5 number summary
2. Calculating interquartile range and detecting potential outliers
3. Drawing boxplots
4. Comparing Distributions
5. Calculating a z-score

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Learning Objective 1: Percentile

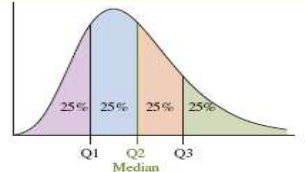
- The **p^{th} percentile** is a value such that p percent of the observations fall below or at that value.



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Learning Objective 1: Finding Quartiles

- Splits the data into four parts
 - Arrange the data in order
 - The **median** is the **second quartile**, Q_2
 - The **first quartile**, Q_1 , is the median of the lower half of the observations
 - The **third quartile**, Q_3 , is the median of the upper half of the observations



Learning Objective 1: Measure of spread: **quartiles**

Quartiles divide a ranked data set into four equal parts.

The **first quartile**, Q_1 , is the value in the sample that has 25% of the data at or below it and 75% above

The **second quartile** is the same as the **median** of a data set. 50% of the obs are above the median and 50% are below

The **third quartile**, Q_3 , is the value in the sample that has 75% of the data at or below it and 25% above

1	0.6
2	1.2
3	1.6
4	1.9
5	1.5
6	2.1
7	2.3
8	2.3
9	2.5
10	2.8
11	2.9
12	3.3
13	3.4
14	3.6
15	3.7
16	3.8
17	3.9
18	4.1
19	4.2
20	4.5
21	4.7
22	4.9
23	5.3
24	5.6
25	6.1

$Q_1 = \text{first quartile} = 2.2$

$M = \text{median} = 3.4$

$Q_3 = \text{third quartile} = 4.35$

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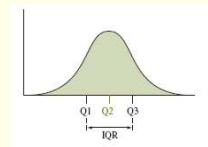
Learning Objective 1 Quartile Example

- Find the first and third quartiles
Prices per share of 10 most actively traded stocks on NYSE (rounded to nearest \$)
2 4 11 13 14 15 31 32 34 47
- a. $Q_1 = 2$ $Q_3 = 47$
 - b. $Q_1 = 12$ $Q_3 = 31$
 - c. $Q_1 = 11$ $Q_3 = 32$
 - d. $Q_1 = 12$ $Q_3 = 33$

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Learning Objective 2: Calculating Interquartile range

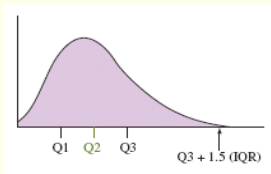
- The interquartile range is the distance between the third quartile and first quartile:
- $IQR = Q_3 - Q_1$
- IQR gives spread of middle 50% of the data



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Learning Objective 2: Criteria for identifying an outlier

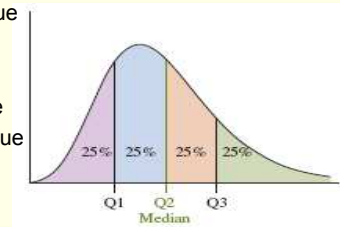
- An observation is a potential outlier if it falls more than $1.5 \times IQR$ below the first quartile or more than $1.5 \times IQR$ above the third quartile



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Learning Objective 3: 5 Number Summary

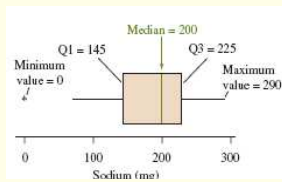
- The five-number summary of a dataset consists of the
 - Minimum value
 - First Quartile
 - Median
 - Third Quartile
 - Maximum value



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Learning Objective 3: Boxplot

- A box goes from the Q1 to Q3
- A line is drawn inside the box at the median
- A line goes from the lower end of the box to the smallest observation that is not a potential outlier and from the upper end of the box to the largest observation that is not a potential outlier
- The potential outliers are shown separately



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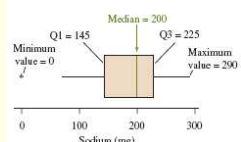
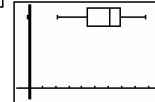
Learning Objective 3: Boxplot for Sodium Data

Cereal	Sodium
Frosted Mini Wheats	0
Raisin Bran	210
All Bran	260
Apple Jacks	125
Capt Crunch	220
Cheerios	290
Cinnamon Toast	210
Crackling Oat Bran	140
Crispix	220
Frosted Flakes	200
Fruit Loops	125
Grape Nuts	170
Honey Nut Cheerios	250
Life	150
Oatmeal Raisin Crisp	170
Sugar Snacks	70
Special K	230
Wheaties	200
Corn Flakes	290
Honeycomb	180

```

1-Var Stats
n=20
min=0
Q1=145
Med=200
Q3=225
max=290

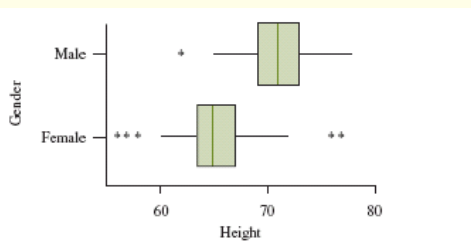
```



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Learning Objective 4: Comparing Distributions

Box Plots do not display the shape of the distribution as clearly as histograms, but are useful for making graphical comparisons of two or more distributions



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Learning Objective 5: Z-Score

- The **z-score** for an observation is the number of standard deviations away from the mean:

$$z = \frac{\text{observation} - \text{mean}}{\text{standard deviation}}$$

- An observation from a bell-shaped distribution is a potential outlier if its z-score < -3 or > +3

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Exploring Data with Graphs and Numerical Summaries

section: How Can Graphical Summaries Be Misused?

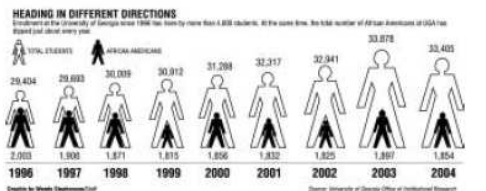
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Learning Objective 1: Guidelines for Constructing Effective Graphs

- Label both axes and provide proper headings
- To better compare relative size, the vertical axis should start at 0.
- Be cautious in using anything other than bars, lines, or points
- It can be difficult to portray more than one group on a single graph when the variable values differ greatly

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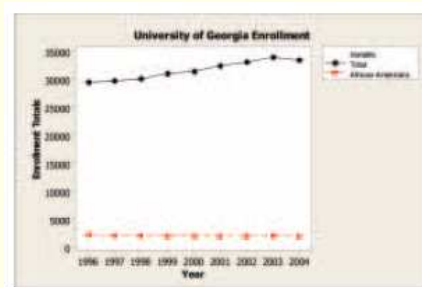
Learning Objective 1: Example



▲ FIGURE 2.18: An Example of a Poor Graph. QUESTION: What's misleading about the way the data are presented?

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Learning Objective 1: Example



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