Implementing the Eight Recommendations from the What Works Clearinghouse IES Practice Guide on Struggling Students in Math

Participant Resource Guide

for Secondary Teachers

Version 1.0

CORE Mission

CORE serves as a trusted advisor at all levels of preK–12 education, working collaboratively with educators to support literacy and math achievement growth for all students.

Our implementation support services and products help our customers build their own capacity for effective instruction by laying a foundation of research-based knowledge, supporting the use of proven tools, and developing leadership.

As an organization committed to integrity, excellence, and service, we believe that with informed school and district administrators, expert teaching, and well-implemented programs, all students can become proficient academically.

Objectives: You will be able to . . .

- Understand the practical application of the eight recommendations in the Institute of Education Sciences' (IES) Practice Guide for struggling learners in math
- Recognize the relationship between tier 1 instruction and tiers 2 and 3 instruction in math, and the role played by assessments
- Explain the connections between several crucial strategies for teaching and learning math
- Describe key stages and understandings in developing proficiency with number sense, algorithms, fractions, equality, proportionality, and beginning algebra

Agenda

I. Overview
   - RtI in Math
   - The eight recommendations

II. Assessments – uses not abuses

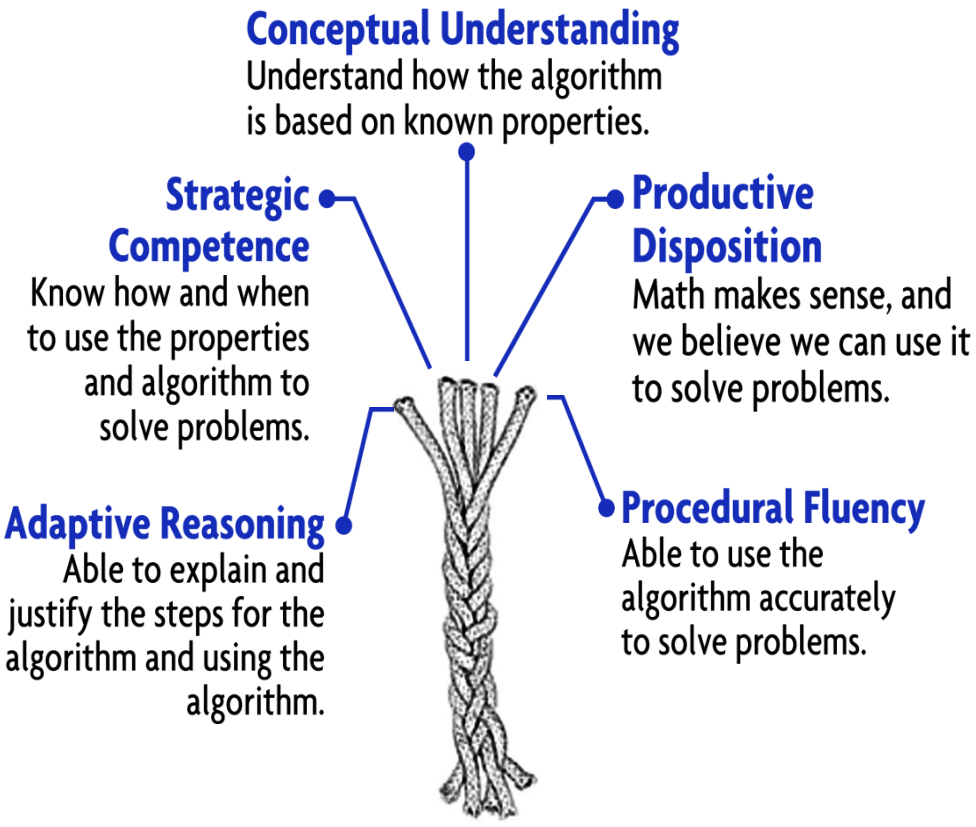
III. Number sense
   - Whole numbers
   - Equality
   - Fractions

IV. Proportionality

V. Beginning algebra

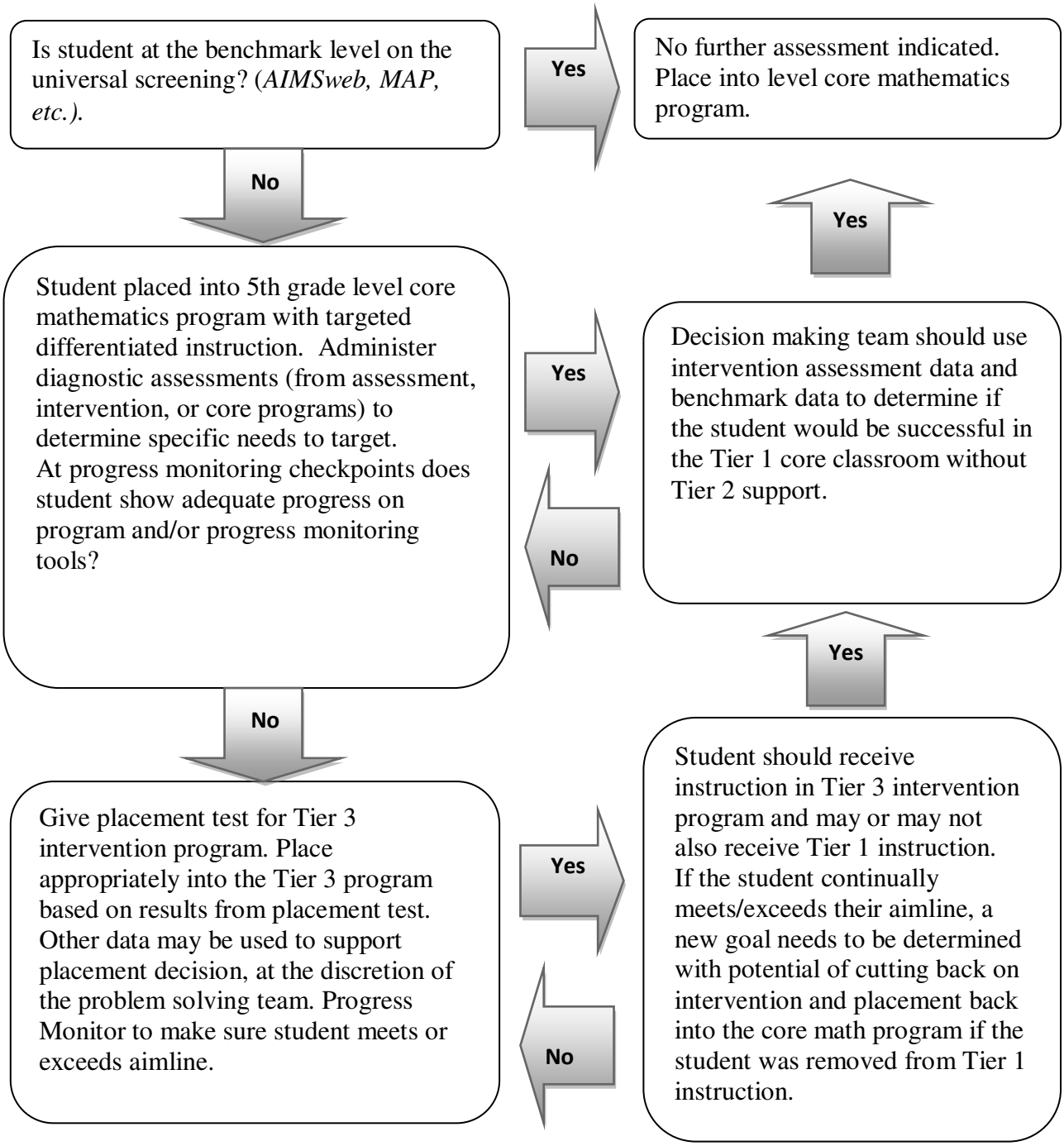
Five Proficiency Strands in Mathematics

(National Research Council 2001)



Sample RtI Placement and Movement Plan
(Tier 2 – Tier 3 Interventions)
Draft Document

Start: UNIVERSAL SCREENING



Definitions

Adequate Progress: Maintaining grade level performance or making adequate gains towards reaching grade level performance.

Adequate Yearly Progress (AYP): Progress made towards maintaining or gaining appropriate grade level performance. School wide AYP is determined by state guidelines.

Individual Planning Form (IPF): A document which specifies individualized prescriptive intervention designed for a student by the Problem Solving Team. This form spells out the intervention(s) in detail, and can be used during observations to determine if the student is receiving the intervention as prescribed.

Problem Solving Team (PST): A multidisciplinary team assembled to determine the next course of action for students who are not making adequate progress even though receiving interventions designed to increase performance. This team should include the teacher and principal, as well as any others who can provide input into the individual student's needs. If making a decision regarding a possible special education placement, this team must meet federal guidelines for a multidisciplinary team.

Profiting from Instruction: A student is said to be profiting from instruction if he/she is making adequate progress in the classroom or intervention (see definition for Adequate Progress), and/or performing successfully as measured by program assessments with or without extra support.

Program Assessments: Weekly, unit and End of Year assessments provided by the publisher of a given core or intervention program. These assessments measure whether or not the student is mastering key skills covered in the program.

Progress Monitoring: Frequent, short assessments that provide information on the progress an individual student is making as a result of instruction. Internal progress monitoring is monitored by program assessments, which determine if a student is receiving sufficiently robust instruction to master key program skills, and may also indicate if a program or intervention is sufficiently robust for a given group of students. External progress monitoring is an assessment such as AIMSweb or MAP which provides information on the progress an individual is making based on normative data.

Creating Equations with the Digits 1–9

Create as many equations as you can with the following conditions:

- Use the digits 1–9 to create many different equations.
- Use some or all of the digits in each equation.
- Do not use any digit more than once within any single equation.
- Do not use the digit zero.
- You may use any math operation, including exponents.
- Create equations representative of the grade level you teach or above.

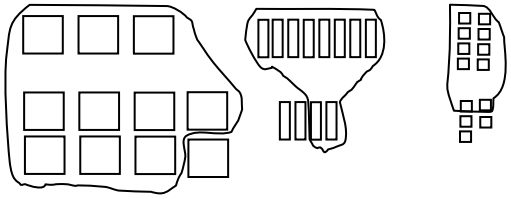
How Many Number Facts? – Multiplication

Learning multiplication number facts to the point of instant recall may at first appear to require learning 121 or more facts for all combinations of factors from 0 to 10. However, use of number properties, counting strategies, and number relationships, such as the commutative property, etc., can significantly reduce this number as well as make the process of memorizing the facts easier and more meaningful.

Make a list of the strategies and properties that will reduce the number of facts to be memorized. Explain how each strategy or property can be applied, and how many facts on the chart below it can be applied to.

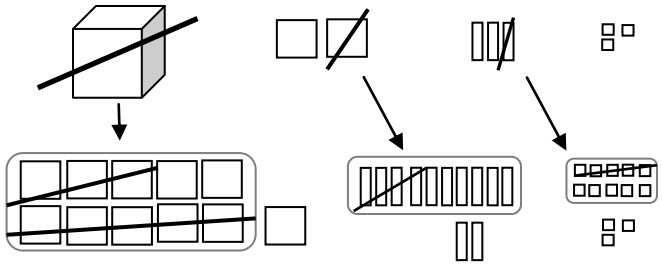
+	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Progression of Learning for Multidigit Addition
Concrete and Visual Representations → Accessible Algorithms → Standard U.S. Algorithm

Problem	Concrete Representation	Accessible Algorithm 1	Accessible Algorithm 2	Standard U.S. Algorithm
388 + 845 = 388 + 845	Regrouping 11 hundreds = 1 thousand & 1 hundred 12 tens = 1 hundred & 2 tens 13 ones = 1 ten & 3 ones 	Record partial sums adding like place values from left to right or right to left. 388 + 845 = 1100 + 120 + 13 = 1233 388 + 845 = 13 + 120 + 1100 = 1233	Add from right to left. Write or mark the regrouping below the corresponding place value in the addends. Then add straight down. 388 + 845 = 3 388 + 845 = 33 388 + 845 = 1233	Add from right to left. Mark regroupings above the corresponding place values in the addends. 1 1 388 +845 1233
Students should see and write problems horizontally and vertically.	• Connect the concrete or visual representations to numerical representations of the values. 1000 + 100 + 100 + 20 + 10 + 3 = 1,233 • Connect the expanded form of the numerical representation (shown below) with the concrete or visual representation. 3 (hundreds) + 8 (tens) + 8 (ones) + 8 (hundreds) + 4 (tens) + 5 (ones) 11 (hundreds + 12 (tens) + 13 (ones) = 1(thousand) + 2(hundreds) + 3(tens) + 3(ones) • Connect back to the problem as originally written: 388 + 845 = 1233	Connect the 1100 to the 11 hundreds, the 120 to the 12 tens, and the 13 to the 13 ones of the concrete representation.	Connect to the actual values, such as 5 + 8 = 13. We can see the 1 ten and 3 ones close to each other. It looks like 13, except the 1 is raised to show it is another of the tens to be added together.	Connect regroupings to the • same regroupings shown in the concrete representation • partial sums in accessible algorithm 1 • regroupings marked below the addends in accessible algorithm 2

Progression of Learning for Multidigit Subtraction
Concrete and Visual Representations → Accessible Algorithms → Standard U.S. Algorithm

<table>
<tr><th>Problem</th><th>Concrete Representation</th><th>Accessible Algorithm</th><th>Standard U.S. Algorithm</th></tr>
<tr>
<td>1233 – 845 =

1233
– 845</td>
<td>Decomposing/Ungrouping
1 thousand = 10 hundreds
1 hundred = 10 tens
1 ten = 10 ones
 
3 hundreds 8 tens 8 ones</td>
<td>Decompose (ungroup) everything first, as needed (from left to right, or right to left), until each top number is greater than the corresponding bottom number.
Subtract like place values from left to right or right to left
11 12
~~12~~ ~~13~~ 13
~~1~~ ~~2~~ ~~3~~ ~~3~~
– 8 4 5
3 8 8

11 12
~~1~~ ~~2~~ 13
1 ~~2~~ ~~3~~ ~~3~~
– 8 4 5
3 8 8</td>
<td>Decompose or ungroup from right to left while subtracting like place values. Alternate between subtraction and ungrouping.
11 12
~~12~~ ~~13~~ 13
~~1~~ ~~2~~ ~~3~~ ~~3~~
– 8 4 5
3 8 8</td>
</tr>
<tr>
<td>Students should see and write problems horizontally and vertically.</td>
<td>• Discuss the need to take away 8 hundreds, 4 tens, and 5 ones with the original representation (shown on the top), where there are not enough hundreds, tens, or ones to accomplish the task.
• Discuss decomposing (ungrouping) as the inverse of regrouping.
• Connect the expanded form of the numerical representation (shown below) with the concrete or visual representation.
11 (hundreds) + 12 (tens) + 13 (ones)
– 8 (hundreds) + 4 (tens) + 5 (ones)
3 (hundreds + 8 (tens) + 8 (ones)
• Connect back to the problem as originally written:
1233 – 845 = 388</td>
<td>Connect decomposing or ungrouping of numbers to the concrete representation and to the expanded form of the numerical representation.
• 1 thousand = 10 hundreds; plus the 2 hundreds already there, makes 12 hundreds
• 1 hundred = 10 tens; plus the 3 tens already there, makes 13 tens
• 1 ten = 10 ones; plus the 3 ones already there, makes 13 ones.</td>
<td>Connect decomposing or ungrouping to the same decomposing/ungrouping shown in the concrete representation, the expanded numerical representation, and the accessible algorithm.</td>
</tr>
</table>

Maximum Difference

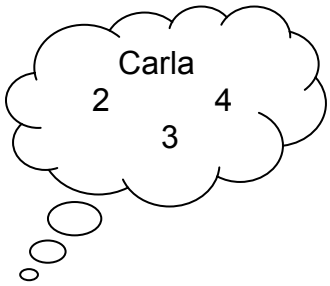
In a game, Carla and Maria are making subtraction problems using tiles numbered 1 to 5. The player whose subtraction problem gives the largest answer wins the game. Look at where each girl placed two of her tiles.

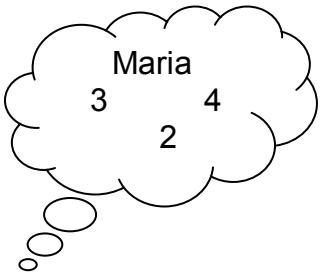
Carla

1		
-	5	

Maria

		5
-		1





Who will win the game? _____

Explain how you know this person will win.

U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1996 Math Assessment

Defining a Fraction

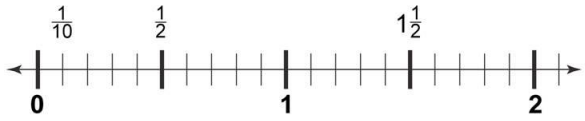
A fraction is a quotient of one number (or expression) by another, indicated by **a**/**b**. The dividend **a** is the numerator and the nonzero divisor **b** is the denominator.

The Penguin Dictionary of Mathematics 1998

CORE MATH

Fractions as Numbers

A fraction is a representation of a number. As such it can be placed on the number line.



CORE MATH

Fractions in Algebra

- Fractions are a way to write numbers or expressions.
- All of the following are ways in which fractions will appear throughout K–12 math:

$$\frac{6}{1},\ \frac{1}{3},\ \frac{\sqrt{2}}{2},\ \frac{\pi}{2},\ \frac{x-5}{2x-4},$$

$$\frac{\text{favorable outcomes}}{\text{possible outcomes}},\ \frac{\text{opposite side}}{\text{hypotenuse}}$$

CORE MATH

Fractions as Part-Whole

1. Part-Whole

   3 out of 4 equal-size shares

Adding It Up 2001

CORE MATH

Fractions as Division

2. Quotient (as division)

$\frac{3}{4}$ = 3 divided by 4

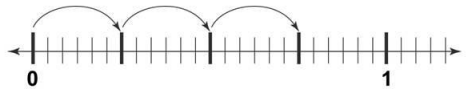
Adding It Up 2001

CORE MATH

Fractions as Measurement

3. Measure

$\frac{3}{4}$ of the way from the beginning of a unit to the end



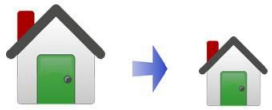
CORE MATH

Fractions as Operator or Scale Factor

4. Operator or scale factor

$\frac{3}{4}$ of a quantity; or

dilation – resizing something by a scale factor of $\frac{3}{4}$



CORE MATH

Fractions as Ratio

5. Ratio

Comparison: 3 hearts for every 4 stars

$\frac{3}{4}$

CORE MATH

Paper Folding Fractions

Directions for subgroups with **square** papers:

- Fold one sheet in half diagonally three times, and unfold.
- Fold one sheet in half horizontally three times, and unfold.
- Fold one sheet in half mixed vertically and horizontally three times total, and unfold.

Directions for subgroups with **letter-size** papers:

- Fold one sheet in half vertically (long way) three times, and unfold.
- Fold one sheet in half horizontally (short length) three times and unfold.
- Fold one sheet in half mixed vertically and horizontally three times total, and unfold.

Questions for all subgroups:

1. What are the units on each of your three papers?

2. What fraction do these units represent?

3. Shade three sections on each paper. How much of your paper is this?
   Write this in two ways.

4. Are the shaded regions on each of your three papers equal? Why or why not?
   Prove this in two ways – visually and with mathematical reasoning.

5. Are any of your shaded regions equal to the shaded regions of the papers held by the other subgroup at your table group? Why or why not?

Sarah Paints Her Fence

1. Show and name two other fractions equivalent to 1/3.

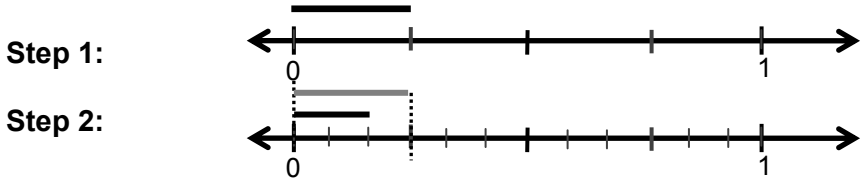
2. Sarah painted 3/8 of the fence around her house. Her friend Johnny painted 1/3 of the fence around her house. Who painted the most fence?

3. Mark came over to Sarah's house and painted 1/4 of the fence. What fraction of the fence did Johnny and Mark paint altogether?

4. How much more of the fence did Johnny and Mark paint than Sarah?

Modeling Fraction Multiplication

Number Line: All of the following steps would usually be done on one number line.

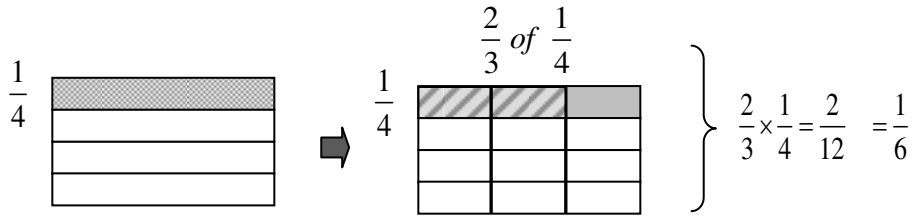


- Split the number line from 0 to 1 into four equal parts to identify ¼ or one part out of four parts.
- Split each ¼ into three equal parts = 1/3 × ¼.
- Identify 2 of these three parts within the one-fourth = 2/3 × ¼.
- Count up the total parts taken, (2).
- Count up the total number of parts, (12).

$$\frac{2}{3}\times\frac{1}{4}=\frac{2}{12}\quad=\frac{1}{6}$$

Area Model: All of the following steps would usually be done on one area model.

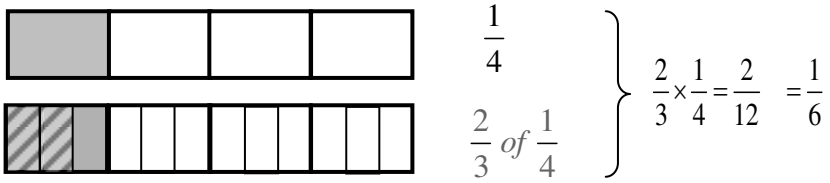
- Divide the whole into four equal parts and identify ¼ of the whole.
- Divide each fourth into three equal parts.
- Identify 2 of the three parts within the ¼, (2 parts).
  - Recognize that the product of the numerators is the number of parts taken.
- Count the total number of parts, (12 parts).
  - Recognize that the product of the denominators equals the total number of parts.



$$\frac{2}{3}\times\frac{1}{4}=\frac{2}{12}\quad=\frac{1}{6}$$

Tape Diagram: All of the following steps would usually be done on one tape diagram.

- Divide the whole into four equal parts and identify ¼ of the whole.
- Divide each fourth into three equal parts.
- Identify 2/3 of the ¼, (2 parts).
- Count the number of parts, (12 parts).



$$\frac{2}{3}\times\frac{1}{4}=\frac{2}{12}\quad=\frac{1}{6}$$

Three Fraction Multiplication Problems

- **Teacher vs. Heater**
  A teacher spent 5/6 of her day teaching class. For ¾ of this time, the heater was on and was very noisy. During what fraction of the day did the teacher teach with the noisy heater on?

- **Chocolate Chips**
  I have 2 2/3 cups of chocolate chips. I am going to use ¾ of the chocolate chips for baking cookies. How many cups of chocolate chips will I use for my cookies?

- **Field Trip Lunches**
  Twenty-four students are on a field trip. 5/6 of the students brought lunches. In ¾ of the lunches there is a fruit or vegetable. How many students brought a lunch with a fruit or vegetable?

1. Use a visual model to illustrate solving your problem.

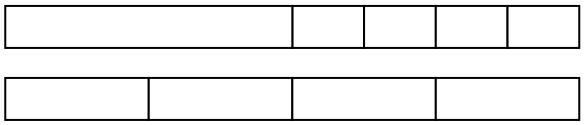
2. Show a numerical method to solve the problem.

3. Connect the numerical method to your visual model.

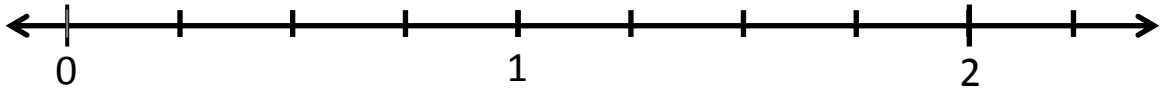
Two Fraction Division Problems

- Use the given visual tool to model solving each problem.
- Solve each problem numerically and be ready to explain the connection between the numerical method and the visual method.

Painting Randy's House
Randy has 1¾ rooms left to paint in his house. He can paint a ½ room each day. How many days will it take Randy to paint the rest of his house?



Dividing My Flour
I have 1¾ cups of flour. I want to divide this in half and put each half in a plastic bag. How much flour will be in each bag?



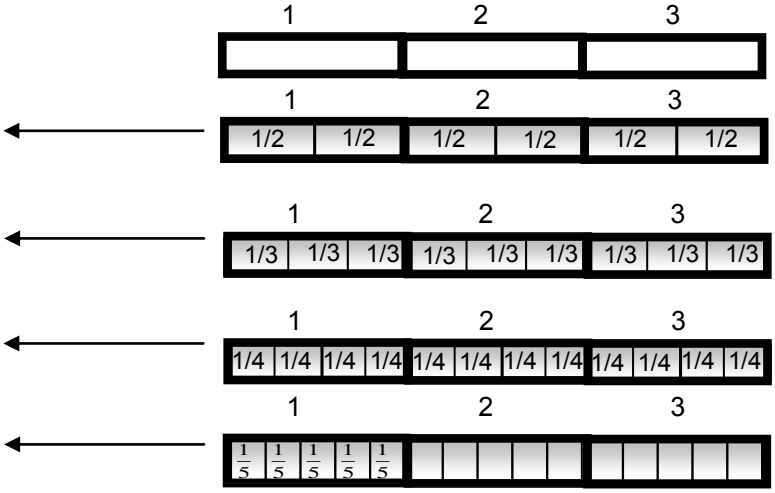
Fraction Division Through Patterns and Tape Diagrams

Fill in the quotients for each table. Utilize the tape diagrams to help determine the quotients. Write a conjecture for a shortcut or fast way to determine the quotients.

Table 1

Expression	Quotient
$3 \div 3$	
$3 \div 1$	
$3 \div \frac{1}{2}$	
$3 \div \frac{1}{3}$	
$3 \div \frac{1}{4}$	
$3 \div \frac{1}{5}$	

Samples of how students can use tape diagrams to determine the quotients

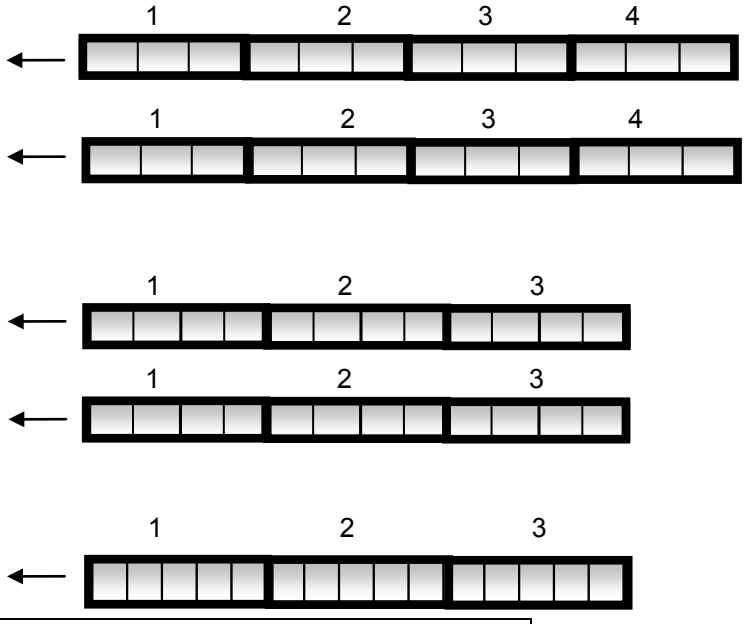


Conjecture: A fast way to divide a whole number by a unit fraction is to:

_____.

Table 2

Expression	Quotient
$4 \div \frac{1}{3}$	
$4 \div \frac{2}{3}$	
$3 \div \frac{1}{4}$	
$3 \div \frac{2}{4}$	
$3 \div \frac{3}{4}$	
$3 \div \frac{1}{5}$	
$3 \div \frac{3}{5}$	



Conjecture: A fast way to divide a whole number by a proper fraction is to:

_____.

Standard Fraction Division Algorithm

"Invert and Multiply"

Why?

What Can You Tell Me?

The ratio of male teachers to female teachers at Sheppard Elementary School is 1 to 5. There are 24 teachers at the school.

Write down five true statements based on this information.

1.

2.

3.

4.

5.

Proportional Reasoning vs. Proportions

# People	Gallons of Water
8	2
10	?

Solve for the unknown quantity in the table above using:

- the cross-multiplication algorithm
- another method

David's Dogs

Carlos is working at ***David's Dogs*** after school to earn his spending money. He takes the orders and delivers them to the tables. He has to take the orders quickly, so he uses his own note system. When the guests are finished eating, he calculates the check (bill). Use his notes for the orders listed below to determine the check for each table.

Adapted from Lombard and Fulton 2000

1. Determine the checks (bills) for each of the tables listed below.

   Table 4 **Table 5**

2. What are the important concepts related to solving the bill for these two tables?

What Is My Number?

1.

2.

3.

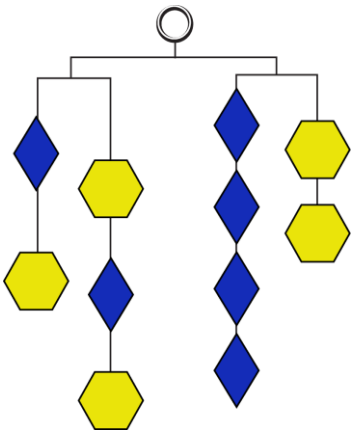
4.

5.

6.

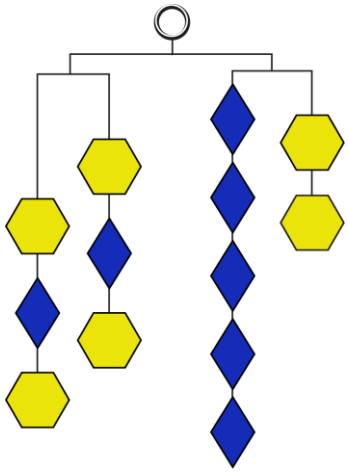
Solving Equations Using a Mobile

A. Cheryl and Darrell are making mobiles to sell as a fundraiser. The diamonds (rhombi) weigh one ounce each. The string that is used will only support two pounds.



1. Determine the weight of the hexagons if the mobile shown here balances.
2. Will the string support this weight?

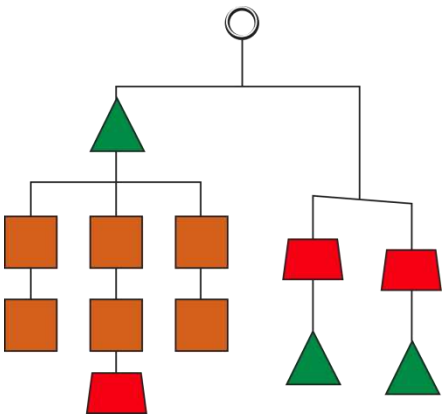
B. John and Marlene are making mobiles to sell as a fundraiser. The string that is used will only support two pounds. The rhombi are one ounce each.



1. Determine the weight of the new hexagons if the mobile shown here balances.
2. Will the string support this weight?

C. On the following balanced mobile the total weight is 48 ounces and  +  = 6 ounces.

1. What is the weight of each shape?

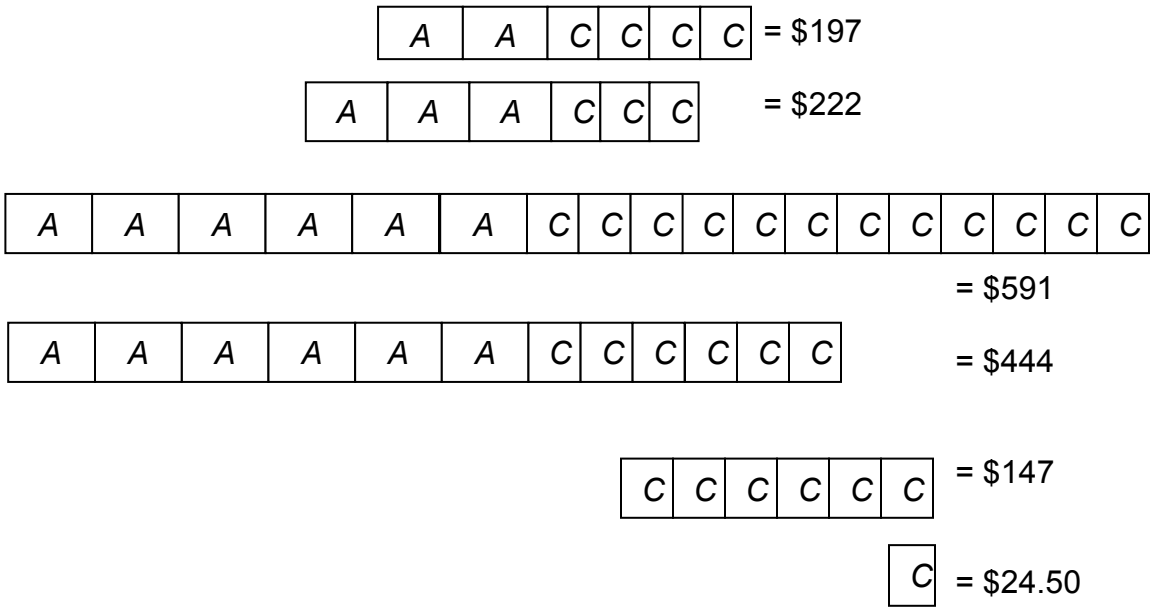


Francis and Anita at the Park Again

A theme park charges different prices for adult and child passes.

- Francis bought 2 adult and 4 child passes for $197.
- Anita bought 3 adult and 3 child passes for $222.

How much does each type of pass cost?



$3(2A + 4C = 197) \rightarrow 6A + 12C = 394$

$2(3A + 3C = 222) \rightarrow \underline{6A + 6C = 444}$

$6C = 147 \rightarrow C = 24.5$

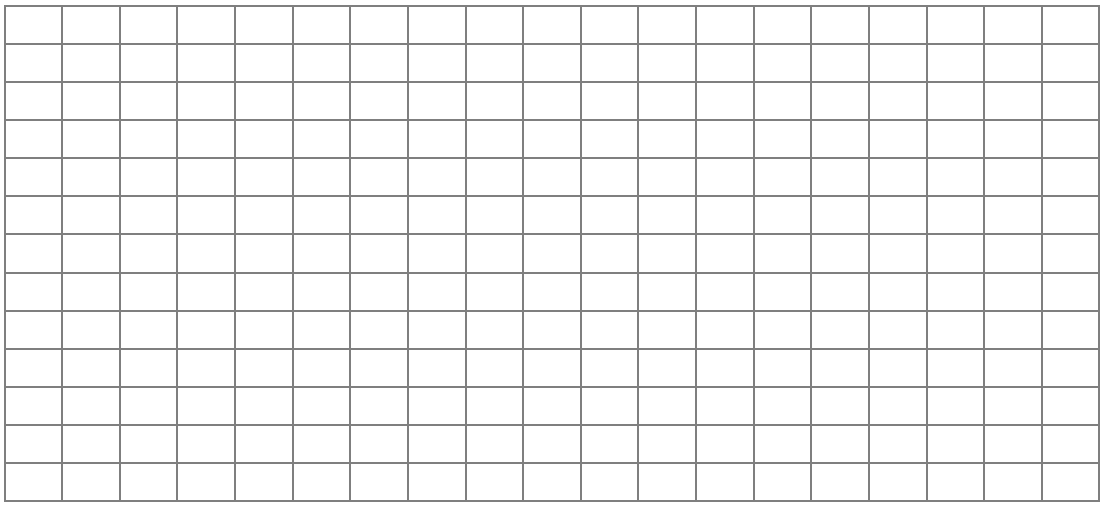
Frank and Jim at the Corner Store

Use tape diagrams to solve the following problem and connect your diagrams to numerical methods.

At a corner store,

- Frank bought two sodas and three hot dogs for $9.03, before tax.
- Jim bought three sodas and one hot dog for $6.02, before tax.

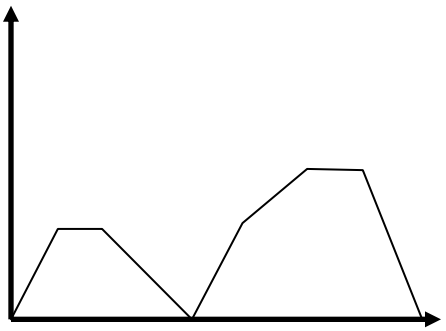
How much do sodas and hot dogs cost?



Every Graph Tells a Story

With a partner, sitting back-to-back, tell each other a story for one of the graphs without telling the other person which graph you are using. Have the other person determine which graph matches the story he/she is hearing. Discuss why the story matches the graph.

A.



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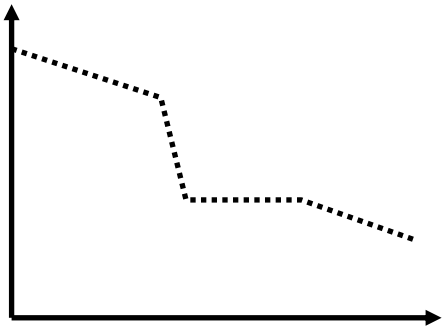

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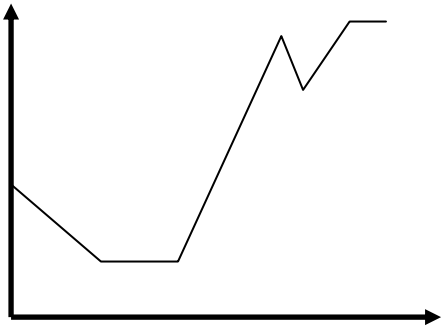

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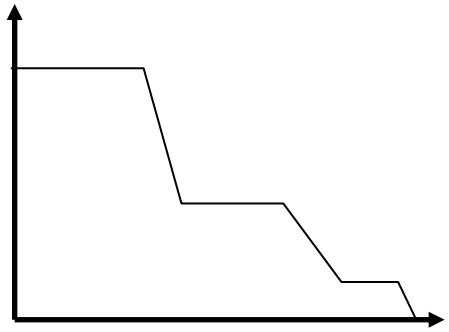

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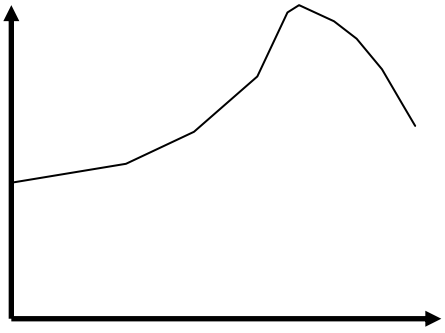
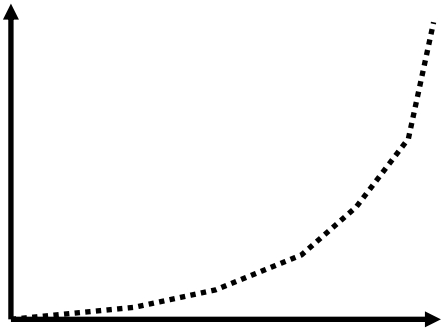


F.



Analyzing Patterns

- Complete each table of values. Analyze the patterns in the table.
- To the right of each table describe in words or symbols the pattern you see.
- Be prepared to discuss how some tables promote recursive thinking and some promote functional thinking.

TABLE 1	
x	***y***
1	5
2	10
3	15
4	
5	
8	
90	

TABLE 2	
x	***y***
1	6
2	11
3	16
4	
5	
8	
90	

TABLE 3	
x	***y***
0	3
1	8
2	13
3	18
4	
5	

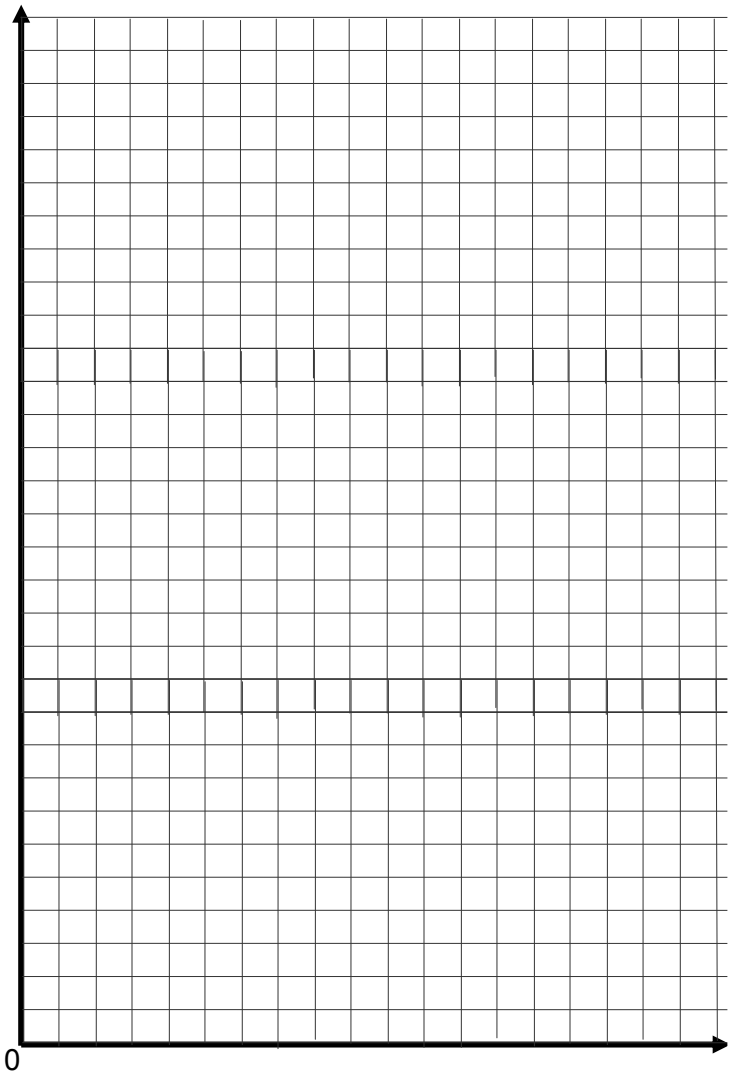
TABLE 4	
x	***y***
1	3
3	9
7	21
10	
11	
15	

TABLE 5	
x	***y***
1	1
2	4
3	9
4	
5	
6	

TABLE 6	
x	***y***
2	6
4	12
6	18
8	
10	
12	

Tying Knots in a Rope

1. Collect data on your rope length as a function of the number of knots in the rope.
   - Use the ruler to measure the length of your rope.
   - Record the length of rope at 0 knots on the t-chart.
   - Tie one knot at the one end of your rope.
   - Measure the new length of rope, and enter this length on the table.
2. Tie 4 more knots, measuring after each knot, and entering the data on the table.
3. Graph your data on the coordinated axis below.
4. Draw a line of best fit for your data.
5. Answer the questions on this and the next page about your rope, table, and graph.
6. Create a group graph with all ropes in the group represented on one set of coordinate axes. Scale the horizontal axis by ones and the vertical axis by fives.



Number of Knots	Length of Rope
0	
1	
2	
3	
4	
5	

7. If we had not seen your rope, what does your graph tell us about your rope? How does the graph tell us these things?

8. Write an equation for the length (L) of your rope as a function of the number of knots that have been tied.

9. What would be the length of your rope if it had eight knots in it? Explain your reasoning.

10. How does the equation created above relate to the table of values, the graph, and the actual ropes?

11. What is one other question that you would ask students about the ropes?

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12. **Group Chart Analysis**: When signaled to do so by the instructor, you will analyze the ropes on the charts of two other groups. For each chart, write down what you know about the ropes based on the graphs.

    Group Chart # ___

    Group Chart # ___