Unit 06: Algebraic Representations and Applications (12 days)

Possible Lesson 01 (3 days)
Possible Lesson 02 (9 days)

POSSIBLE LESSON 01 (3 days)

This lesson is one approach to teaching the State Standards associated with this unit. Districts are encouraged to customize this lesson by supplementing with district-approved resources, materials, and activities to best meet the needs of learners. The duration for this lesson is only a recommendation, and districts may modify the time frame to meet students' needs. To better understand how your district is implementing CSCOPE lessons, please contact your child's teacher. (For your convenience, please find linked the TEA Commissioner's List of State Board of Education Approved Instructional Resources and Midcycle State Adopted Instructional Materials.)

Lesson Synopsis:
Students use concrete and pictorial models to solve equations. Students solve one- and two-step equations by connecting the concrete and pictorial models and recording each action verbally and with symbolic notation.

TEKS:
The Texas Essential Knowledge and Skills (TEKS) listed below are the standards adopted by the State Board of Education, which are required by Texas law. Any standard that has a strike-through (e.g. ~~sample phrase~~) indicates that portion of the standard is taught in a previous or subsequent unit.

The TEKS are available on the Texas Education Agency website at http://www.tea.state.tx.us/index2.aspx?id=6148

7.2	***Number, operation, and quantitative reasoning.. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:***
7.2F	**Select and use appropriate operations to solve problems and justify the selections.** ***Readiness Standard***
7.5	***Patterns, relationships, and algebraic thinking.. The student uses equations to solve problems. The student is expected to:***
7.5A	**Use concrete and pictorial models to solve equations and use symbols to record the actions.** ***Supporting Standard***
7.5B	**Formulate problem situations when given a simple equation and formulate an equation when given a problem situation.**

Readiness Standard

Underlying Processes and Mathematical Tools TEKS:

7.13	***Underlying processes and mathematical tools.. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:***
7.13A	**Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.**
7.13B	**Use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness.**
7.13C	**Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.**
7.13D	**Select tools such as real objects, manipulatives, paper/pencil, ~~and technology~~ or techniques such as mental math, estimation, and number sense to solve problems.**
7.14	***Underlying processes and mathematical tools.. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:***
7.14A	**Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.**
7.15	***Underlying processes and mathematical tools.. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:***
7.15B	**Validate his/her conclusions using mathematical properties and relationships.**

Performance Indicator(s):

Grade7 Mathematics Unit06 PI02

Use a given equation to develop a real-life scenario, and evaluate the solution with concrete or pictorial models. Create a presentation (e.g., poster, Prezi, etc.) of the scenario, and use symbols to record the actions of the models used to validate the solution.

Sample Performance Indicator:

- Create a presentation of a real-life scenario connected to the equation $y = 3x + 2$, and justify the solution with models and use symbols to record the actions.

Standard(s): 7.2F , 7.5A , 7.5B , 7.13A , 7.13B , 7.13C , 7.13D , 7.14A , 7.15B
ELPS ELPS.c.1C , ELPS.c.3J

Key Understanding(s):

- A solution to an equation from an everyday problem situation can be validated by using concrete models and pictorial representations to solve the equation and symbols to record the actions.
- The process of solving an equation involves using a plan or strategy to keep the values on both sides of the equation equally balanced and validating the solution for reasonableness.

Underdeveloped Concept(s):

- Students may think *equal* means *find the answer*, rather than *has the same value as.*
- Some students may think variables are letters representing an object as opposed to representing a number or quantity of objects.
- Some students may think that a variable is only a place holder as it is in an equation. A variable can actually have several purposes and can also denote a generalized arithmetic pattern, such as when writing an equation to represent the algebraic properties.

Vocabulary of Instruction:

- equal
- equation
- expression
- solution
- solving an equation
- variable

Materials List:

- algebra tiles (16 “1” tiles, 16 “-1” tiles, 6 “x” tiles, 6 “-x” tiles) (1 set per student, 1 set per teacher)
- Bag of Algebra Tiles (1 set per student, 1 set per teacher) (previously created)
- cardstock (3 sheets per 2 students)
- cardstock (optional) (2 sheets of red, 2 sheets of green) (1 set per 4 students, 1 set per teacher)
- math journal (1 per student)
- plastic zip bag (sandwich sized) (1 per 2 students)
- plastic zip bag (sandwich sized) (1 per student, 1 per teacher)
- scissors (1 per teacher)
- scissors (optional) (1 per teacher)

Attachments:

All attachments associated with this lesson are referenced in the body of the lesson. Due to considerations for grading or student assessment, attachments that are connected with Performance Indicators or serve as answer keys are available in the district site and are not accessible on the public website.

What Did You Get? KEY

What Did You Get?

Algebra Tiles

Maintain Your Balance KEY

Maintain Your Balance

Value Balance KEY

Value Balance

- Value Balance Blank Model
- Equation Models KEY
- Equation Models
- Family Reunion KEY
- Family Reunion
- Check It Out KEY
- Check It Out
- Balancing Practice KEY
- Balancing Practice

GETTING READY FOR INSTRUCTION

Teachers are encouraged to supplement and substitute resources, materials, and activities to meet the needs of learners. These lessons are one approach to teaching the TEKS/Specificity as well as addressing the Performance Indicators associated with each unit. District personnel may create original lessons using the Content Creator in the Tools Tab. All originally authored lessons can be saved in the “My CSCOPE” Tab within the “My Content” area.

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
1	**Topics:** • Solving equations **Engage 1** Students use logic and reasoning skills to follow a set of instructions that lead all students to the	**Spiraling Review** **ATTACHMENTS** • Teacher Resource: **What Did You Get?**

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	same result. Students use algebra tiles to represent each step in the problem pictorially and are introduced to solving equations symbolically. **Instructional Procedures:** 1. Prior to instruction, create a Bag of Algebra Tiles for each student and a Bag of Algebra Tiles for each teacher by placing 16 "1" tiles, 16 "-1" tiles, 6 "*x*" tiles, and 6 "*-x*" tiles in a plastic zip bag. If algebra tiles are not available, use class resource: **Algebra Tiles** to create a Bag of Algebra Tiles for each student and a Bag of Algebra Tiles for each teacher by copying pages 1 and 3 on green cardstock and pages 2 and 4 on red cardstock, laminating, cutting apart, and placing 16 "1" tiles, 16 "-1" tiles, 6 "*x*" tiles, and 6 "*-x*" tiles in a plastic zip bag. 2. Read each step aloud from teacher resource: **What Did You Get?**. Instruct students to record their solutions to each step in their math journal. Allow time for students to complete each step and record their solution. Facilitate a class discussion to debrief student solutions. Ask: • **What did you get for an answer?** *(7)* • **Did everyone get 7 for your answer?** *(yes)* • **Did everyone start with the same number?** *(no)* • **Why did everyone get 7 for an answer when you did not start with the same number?** *(When you double the number you selected, you have 2x. Adding 14 to 2x and then dividing this quantity by 2 takes you back to 7 more than the original number you selected, x + 7. Subtracting the original number you selected leaves a value of 7, x + 7 — x = 7.)* Select a Number | *x*	KEY (1 per teacher) • Teacher Resource: **What Did You Get?** (1 per teacher) • Class Resource (optional): **Algebra Tiles** (1 per 4 students, 1 per teacher) **MATERIALS** • algebra tiles (16 "1" tiles, 16 "-1" tiles, 6 "*x*" tiles, 6 "*-x*" tiles) (1 set per student, 1 set per teacher) • plastic zip bag (sandwich sized) (1 per student, 1 per teacher) • cardstock (optional) (2 sheets of red, 2 sheets of green) (1 set per 4 students, 1 set per teacher) • scissors (optional) (1 per teacher) • math journal (1 per student) **TEACHER NOTE** The solution to each problem from teacher resource: **What Did You Get?** if all calculations are performed correctly, is 7.

Suggested Day	Suggested Instructional Procedures	Notes for Teacher

Add the number to itself	$x + x = 2x$
Add fourteen	$2x + 14$
Divide by 2	$\frac{2x + 14}{2} = \frac{2x}{2} + \frac{14}{2}$
Subtract original number	$x + 7 - x$
Ending Value	7

3. Display teacher resource: **What Did You Get?**. Facilitate a class discussion about the various ways to make the same statement.
   Ask:

   - **What is the purpose of the variable in this expression?** *(The variable represents an unknown value.)*
   - **What does this instruction mean?** *Answers may vary. Choose a number and write it down; etc.*
   - **What did you do when I stated this instruction?** *Answers may vary. My number was 5, so 5 + 5 is 10; etc.*

4. Distribute a Bag of Algebra Tiles to each student.

TEACHER NOTE
Some students may fail to perform the same operation to both sides of the equal sign when solving an equation.

TEACHER NOTE
Algebra tiles help students make sense of the language of algebra. They are concrete models of abstract thought. Middle school students (and most high school students) need a firm foundation in the use of algebra before moving to purely abstract algebraic manipulations.

TEACHER NOTE
The algebra tiles are used as a visual model to represent an expression. When students use pictorial models to solve equations, the algebra tiles may be used as a visual model to show the difference between an expression and an equation.

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	5. Using algebra tiles, demonstrate each step from the displayed teacher resource: **What Did You Get?**. Model how to complete the pictorial models of the table. Instruct students to replicate the models with their algebra tiles and record a pictorial model of each step in their math journal. Ask: • **What can I use to represent the unknown number each one of you wrote down?** *(The "x" from the algebra tiles.)* • **What does your model look like when you double the unknown number you wrote down?** *(Two of the "x" variables from the algebra tiles.)* • **What can I use to represent the number 14?** *(Place fourteen of the positive "1" tiles from the algebra tiles with the model.)* • **What can you do to your model to show dividing by 2?** *(Create two equal groups of the algebra tiles with the same number and type of tiles in each of the two groups.)* • **What can you do to your model to show subtracting the unknown number you wrote down?** *(Pick up or remove the "x" from the model. There should be seven of the positive "1" tiles remaining.)* 6. Facilitate a class discussion to complete the symbolic model of the table the displayed teacher resource: **What Did You Get?**.	
	Topics: • Solving equations **Explore/Explain 1**	**ATTACHMENTS** • Teacher Resource: **Maintain Your Balance KEY** (1 per teacher) • Handout: **Maintain Your Balance** (1 per

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	Students are introduced to solving equations using algebra tiles to develop the conceptual idea of equivalence by balancing equations. **Instructional Procedures:** 1. Place students in pairs and distribute handout: **Maintain Your Balance** to each student. Instruct students to compare part A and part B by noting the changes that took place in part B and then determine what needs to be done to part A to create equivalence between the two scales. Allow time for students to complete the activity. Monitor and assess student pairs to check for understanding. Facilitate a class discussion about balancing equations. Ask: • **What was done to this side of the balance?** *(Problem 1a and 3a have a change on the left side of the balance. Problems 2a and 4a have a change on the right side of the balance.)* • **What must you do to this side of the balance to maintain equivalence?** *(the same thing that was done to the other side of the balance to maintain equivalence)* • **What mathematical symbol do you use to show when you remove the value of a number?** *(Use the subtraction symbol to show removing the value of a number.)* • **How can you verify the values on both sides of the balance scale are equivalent?** *(Simplify each expression on each side of the balance and check to see if the values are equivalent.)*	student) **TEACHER NOTE** Students may not want to write the subtraction sign when subtracting a negative number. They mistake the negative sign as the subtraction symbol. The negative numbers are placed within parentheses to avoid confusing a negative sign with the subtraction symbol. For example: (−5) — 2 = (−7) or 6 — (−3) = 9
2	**Topics:** • Solving equations	**Spiraling Review**

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	Explore/Explain 2 Students continue to work with algebra tiles and the balance model, connecting the concrete to the pictorial and symbolic models. Students record and justify each action verbally and with symbolic notation. **Instructional Procedures:** 1. Distribute handout: **Value Balance** and a Bag of Algebra Tiles each student. 2. Display problem 1 from teacher resource: **Value Balance**. Using algebra tiles, demonstrate how to create concrete and pictorial models for problem 1. Instruct students to verbalize the concrete steps and record a pictorial model representing the concrete steps for problem 1 on their handout: **Value Balance**. Remind students to model the problem with algebra tiles, and then to summarize the process used to solve the equation, and to record the symbolic notation of the model. Instruct students to use the symbols from the handout: ø represents (−1), o represents 1 and ____ represents *x*. Ask: • **What models will represent "2*x* + 3"?** *(two x's and 3 positives)* • **Where would you place the models for "2*x* + 3" on the value balance?** *(On the left side of the value balance.)* • **What models will be placed on the right side of the value balance?** *(9 positives)* • **What pictorial model will we record to model the equation: 2*x* + 3 = 9?** Pictorial Model	**ATTACHMENTS** • Teacher Resource: **Value Balance KEY** (1 per teacher) • Handout: **Value Balance** (1 per student) • Teacher Resource: **Value Balance** (1 per teacher) • Handout (optional): **Value Balance Blank Model** (1 per student) • Teacher Resource: **Equation Models KEY** (1 per teacher) • Handout: **Equation Models** (1 per student) • Teacher Resource: **Equation Models** (1 per teacher) **MATERIALS** • Bag of Algebra Tiles (1 set per student, 1 set per teacher) (previously created) **TEACHER NOTE** Review with students how 3 — 4 is equivalent to 3 + (−4) or 2 — (−3) is equivalent to 2 + 3. Remind students about creating zero pairs. For

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	___ ___ o o o | o o o o o o o o o o	example: 2 + (−2) = 0.
	• **What should be done first to both sides of the value balance? Explain.** *(Add 3 negatives to each side of the equation to create 3 zero pairs.)* • **What pictorial model will record this action?**	**TEACHER NOTE** The teacher will model two problems with the students. Since some students become accustomed to solving equations in the form: 2*x* + 3 = 9, Problem 2 on the handout: **Value Balance** is used as an example to model equations in the form: 4 = 3x — 2.
	Pictorial Model ___ ___ o o o ø ø ø | o o o o o o o o o o o ø ø ø	
	• **What should be done next to both sides of the value balance? Explain.** *(Remove the zero pairs from each side of the equation.)* • **What pictorial model will record this action?**	**TEACHER NOTE** Some students may want to move an item from one side of the equation to the other side. Reinforce the importance of equivalence and how it is maintained by doing "like actions" on both sides of the equation. For example: "Remove 4 negatives from **both** sides of the equation. The only exception is when "zero pairs" are removed from the model to simplify the equation.
	Pictorial Model ___ ___ ~~o o o ø ø ø~~ | o o o o o o ~~o o o ø ø ø~~ ___ ___ | o o o o o o	
	• **What action should be done next?** *(Divide both sets of models on the value balance into two equal groups because we want to find the value of one x.)* • **What pictorial model will record this action?**	

Suggested Day	Suggested Instructional Procedures	Notes for Teacher

Pictorial Model	
___ ___	o o o o o o

- **What is the value of *x*? Explain.** *(x = 3. There is one x in one of the two groups on the left side of the value balance and 3 positives in one of the two groups on the right side of the value balance.)*
- **What pictorial model will record this action?**

Pictorial Model	
___	o o o

- **How can you record the actions we performed with the models using symbols?**

Symbolic Model
2x + 3 = 9
2x + 3 + (–3) = 9 + (–3)
2x + 0= 6
2x = 6

TEACHER NOTE
Remind students that to solve an equation they will be finding the value for the variable that makes an equation true.

TEACHER NOTE
7.2(C) During the first six weeks, students used models to investigate operations involving integers. Some review may be needed for students to relate "3 — 4" to "3 + (−4)."

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	$\frac{2x}{2} = \frac{6}{2}$ x = 3	

- **How do you check the solution? Explain.** *(Write the original equation. Substitute the solution for x in place of x in the equation: 2x + 3 = 9. Simplify the equation by doing the correct order of operations on each side of the equation. After simplifying each side of the equation, the values on both sides of the = sign should be equivalent.)*

3. Display problem 2 from teacher resource: **Value Balance.** Repeat the same process of demonstrating the solution process for problem 2 by creating concrete models using algebra tiles and recording pictorial models for solving equations. Instruct students to replicate the model on their handout: **Value Balance**. Facilitate a class discussion to debrief the solution process for problem 2.
4. Place students in pairs and distribute handout: **Equation Models** to each student. Instruct student pairs to take turns modeling the equation using algebra tiles and describing the actions as the other partner records the pictorial model on their handout. Explain to students that the following pictorial models for may be used when solving equations: ø represents (– 1), o represents 1, ____ represents x and | represents –x. Students will then switch roles for the same problem and repeat the process until the handout has been completed. Allow time for students to complete the activity. Monitor and assess student pairs to check for understanding. Facilitate individual discussions about balancing equations, as needed.
   Ask:

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	• **What is the model for this equation?** *(Example for #1: There are 2 negative tiles on the left side of the balance and an x and 4 positive tiles on the right side of the balance.)* • **What do you think will be a reasonable solution for *x*? Explain.** *(Example for #1: The value for x must be a negative number because the problem states x + 4 = (−2), which means x has to be a negative number less than (– 4) because 4 + (−4) = 0.)* • **What action can you do to both sides of your model to maintain equivalence?** *(Add the same tiles to both sides of the equation to create zero pairs.)* • **What must you do to create "zero pairs" with this value?** *(Add the opposite of the number. For example: Add 4 to (−4) because 4 + (−4) = 0.)* • **Did you perform the same action on both sides of your balance scale to maintain equivalence? Explain.** *(Yes. Example for #1: 4 negatives were added to both sides of the equation in order to create zero pairs, with 4 on the right side of the equation.)* • **How do you decide what operation you want to do on both sides of the equation?** *(I look at the number that is on the same side of the equation with the x's and do whatever operation will zero the number.)* • **How are the pictorial model and the action you performed with the algebra tiles related?** *(The pictorial model is a visual recording of the action performed.)* 5. Display teacher resource: **Equation Models**. Facilitate a class discussion to complete the symbolic column and formalize the concept of balancing equations. Ask: • **How did you decide what operation to begin with? Explain.** *Answers may vary. Added 4 negatives to both sides of the balance to create zero pairs with positive 4; etc.* • **How are the pictorial model and symbolic model related?** *(The pictorial model is a*	

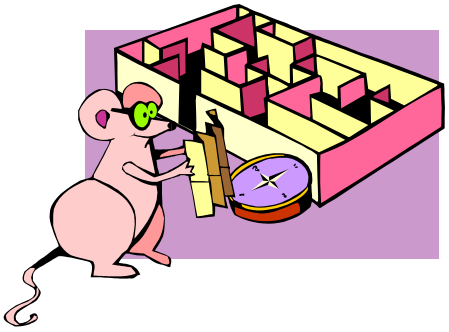
Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	visual recording of the symbolic model which uses symbols. Each picture in the pictorial model is representative of the numbers and variables used in the symbolic model.) • **What would be the next steps to solve this equation?** *Answers may vary. Remove the zero pairs created on each side of the equation, divide the quantities on both sides of the balance into two equal groups to model dividing by 2; etc.* • **Why don't you add 4 positives in the equation: (−2) = x + 4 to the left side of the equation to make zero pairs with the (−2)?** *(Then the equation would not be balanced. Adding 4 positives to the left side of the equation means that 4 positives would need to be added to the right side of the equation to maintain balance.)*	
3	**Topics:** • Solving equations **Elaborate 1** Students connect an equation's symbolic model, pictorial model, and solutions. **Instructional Procedures:** 1. Prior to instructions, create a card set: **Family Reunion** for every 2 students by copying on cardstock, cutting apart, and placing in a plastic zip bag. 2. Place students in pairs and distribute a card set: **Family Reunion** to each pair. Instruct students to match the symbolic model, a pictorial model, and solution cards to create a family, and then record the families in their math journal with the symbolic notation and pictorial steps to solve each equation. Allow time for students to complete the activity. Monitor and assess students to	**Spiraling Review** **ATTACHMENTS** • Teacher Resource: **Family Reunion KEY** (1 per teacher) • Card Set: **Family Reunion** (1 set per 2 students) • Teacher Resource: **Check It Out KEY** (1 per teacher) • Handout: **Check It Out** (1 per student) • Teacher Resource (optional): **Balancing Practice KEY** (1 per teacher) • Handout (optional): **Balancing Practice** (1

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	check for understanding. Facilitate a class discussion to debrief student solutions, as needed. 3. Distribute handout: **Check It Out** to each student as independent practice or homework.	per student) **MATERIALS** • cardstock (3 sheets per 2 students) • scissors (1 per teacher) • plastic zip bag (sandwich sized) (1 per 2 students) • math journal (1 per student) **ADDITIONAL PRACTICE** The handout (optional): **Balancing Practice** may be used as additional practice if needed. **State Resources** **TEXTEAMS: MS Algebraic Reasoning** – Cover Up
	Evaluate 1 **Instructional Procedures:**	

Suggested Day	Suggested Instructional Procedures	Notes for Teacher
	1. Assess student understanding of related concepts and processes by using the Performance Indicator(s) aligned to this lesson. **Performance Indicator(s):** **Grade7 Mathematics Unit06 PI02** Use a given equation to develop a real-life scenario, and evaluate the solution with concrete or pictorial models. Create a presentation (e.g., poster, Prezi, etc.) of the scenario, and use symbols to record the actions of the models used to validate the solution. Sample Performance Indicator: • Create a presentation of a real-life scenario connected to the equation $y = 3x + 2$, and justify the solution with models and use symbols to record the actions. Standard(s): 7.2F , 7.5A , 7.5B , 7.13A , 7.13B , 7.13C , 7.13D , 7.14A , 7.15B ELPS ELPS.c.1C , ELPS.c.3J	

05/02/13

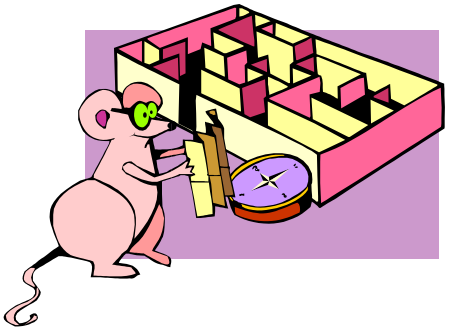
What Did You Get? KEY



Instructions	What This Means	Say It a Different Way
1. Write down any number.	Choose a number and write it.	Pick any number.
2. Add the number you wrote down to itself.	Find the sum of the number you selected when it is added twice.	Double the number you selected.
3. Add fourteen.	Add 14 to your sum from step 2.	Find the sum of the number you doubled and fourteen.
4. Divide by two.	Create 2 equal sized groups.	Take half of the sum.
5. Subtract your original number.	Subtract the original number from the quotient in step 4.	Take away the original number.

Instruction	Pictorial Model	Symbolic Model
1. Write down any number.	▭	n
2. Add the number you wrote down to itself.	▭ ▭	n + n or 2n
3. Add fourteen.	▭ ▭ • • • • • • • • • • • • •	2n + 14
4. Divide by two.	• • • • • • • ▭ • • • • • • • ▭	n + 7
5. Subtract your original number.	• • • • • • •	7

What Did You Get?



Instructions	What This Means	Say It a Different Way
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3. Add fourteen.		
4. Divide by two.		
5. Subtract your original number.		

Instructions	Pictorial Model	Symbolic Model
1. Write down any number.		
2. Add the number you wrote down to itself.		
3. Add fourteen.		
4. Divide by two.		
5. Subtract your original number.		

Algebra Tiles

Copy on green cardstock. Cut apart. One page for every 4 students. Sixteen tiles per student.

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

Algebra Tiles

Copy on red cardstock. Cut apart. One page for every 4 students. Sixteen tiles per student.

−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1	−1
−1	−1	−1	−1	−1	−1	−1	−1

Algebra Tiles

Copy on green cardstock. Cut apart. One page for every 4 students. Six tiles per student.

X	*X*	*X*	*X*	*X*	*X*	*X*	*X*
X	*X*	*X*	*X*	*X*	*X*	*X*	*X*
X	*X*	*X*	*X*	*X*	*X*	*X*	*X*

Algebra Tiles

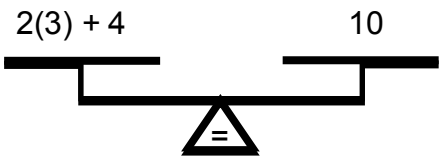
Copy on red cardstock. Cut apart. One page for every 4 students. Six tiles per student.

$-x$	$-x$	$-x$	$-x$	$-x$	$-x$	$-x$	$-x$
$-x$	$-x$	$-x$	$-x$	$-x$	$-x$	$-x$	$-x$
$-x$	$-x$	$-x$	$-x$	$-x$	$-x$	$-x$	$-x$

Maintain Your Balance KEY

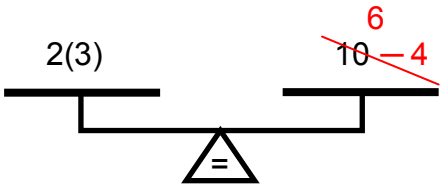
Study each balance scale in Part A. In Part B the scale has been changed. Indicate what needs to be done to maintain the same equivalence as in Part A.

(1a)



Value Balance

(1b)

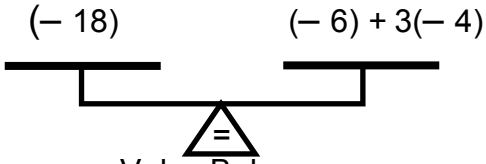


Value Balance

What changed? (Subtract 4)

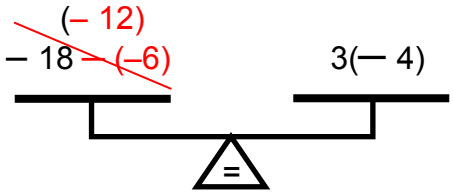
What needs to be done to maintain equivalence?

(2a)



Value Balance

(2b)

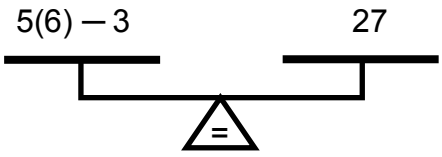


Value Balance

What changed? (Subtract (–6))

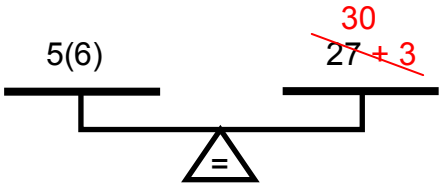
What needs to be done to maintain equivalence?

(3a)



Value Balance

(3b)

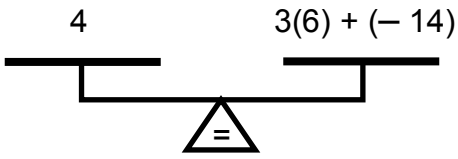


Value Balance

What changed? (Add 3)

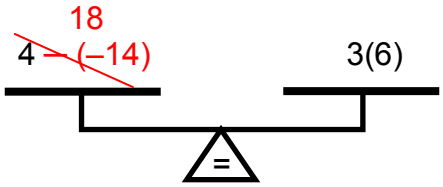
What needs to be done to maintain equivalence?

(4a)



Value Balance

(4b)



Value Balance

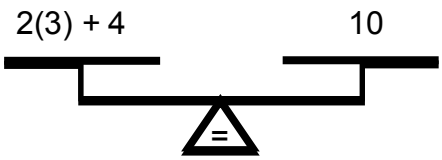
What changed?(Subtract (–14))

What needs to be done to maintain equivalence?

Maintain Your Balance

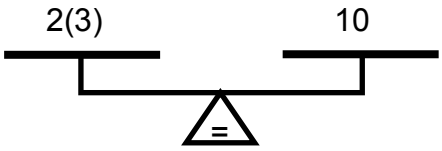
Study each balance scale in Part A. In Part B the scale has been changed. Indicate what needs to be done to maintain the same equivalence as in Part A.

(1a)



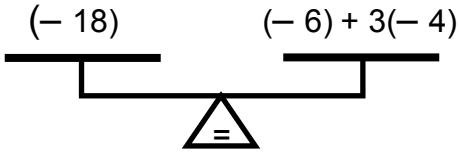
Value Balance

(1b)



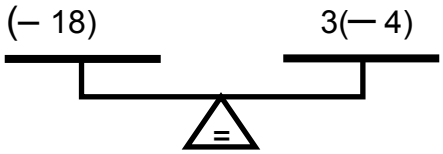
Value Balance
What changed?
What needs to be done to maintain equivalence?

(2a)



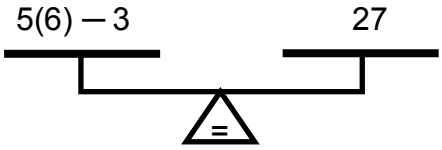
Value Balance

(2b)



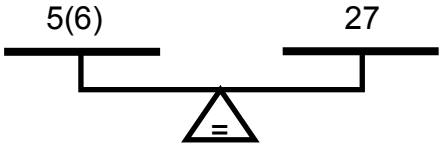
Value Balance
What changed?
What needs to be done to maintain equivalence?

(3a)



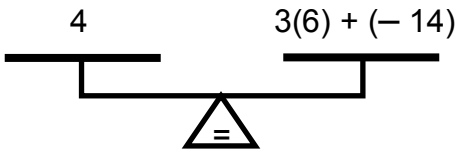
Value Balance

(3b)



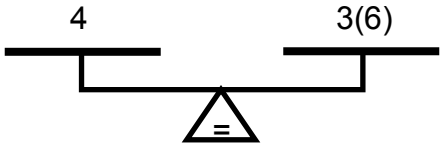
Value Balance
What changed?
What needs to be done to maintain equivalence?

(4a)



Value Balance

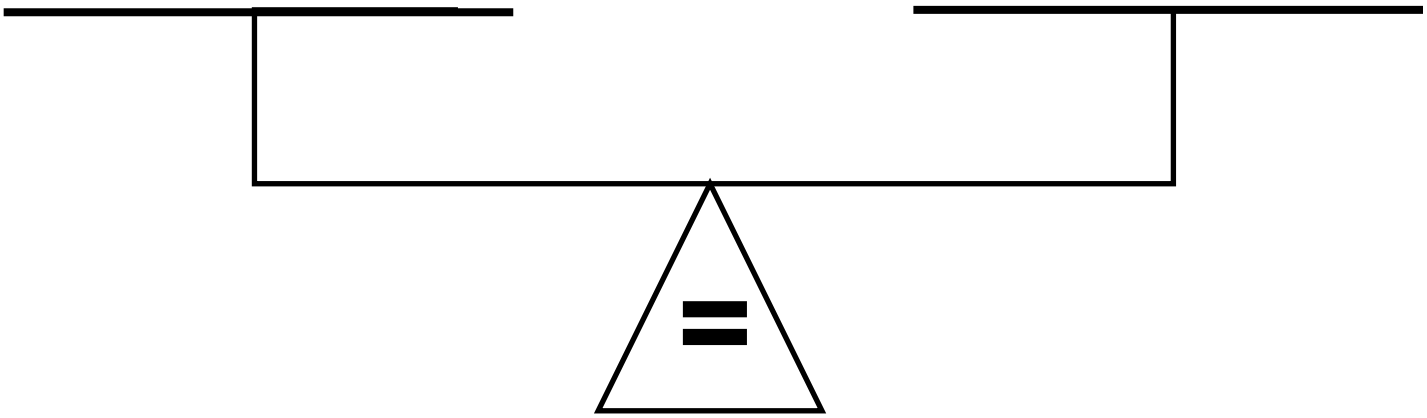
(4b)



Value Balance
What changed?
What needs to be done to maintain equivalence?

Value Balance KEY

Problem 1: 2x + 3 = 9



Value Balance

Let ø represent (– 1) and o represent 1 and ____ represent x.

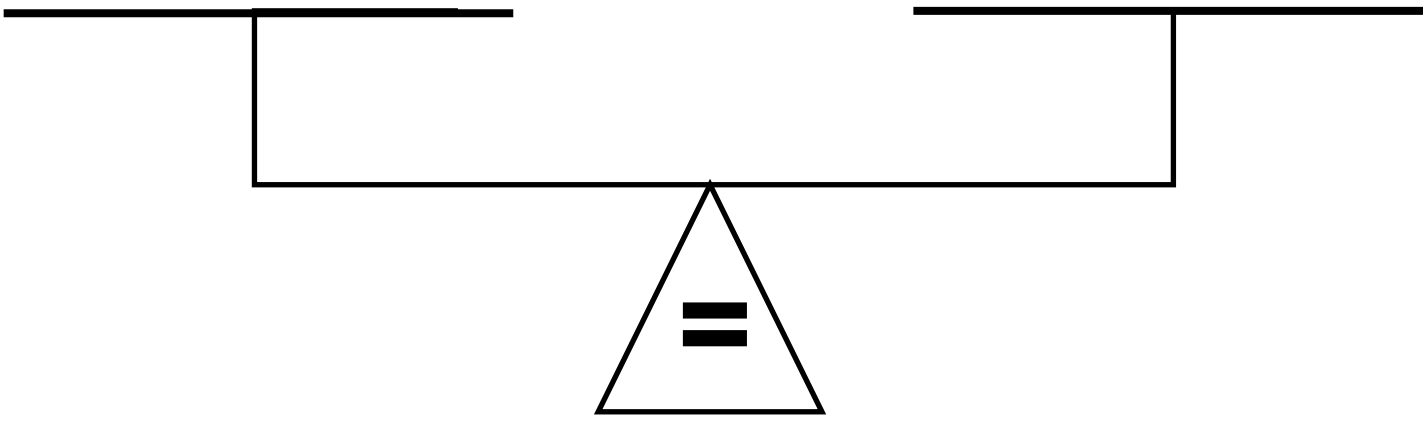
Pictorial Model		Symbolic Model
____ ____ o o o	o o o o o o o o o	2x + 3 = 9
____ ____ o o o ø ø ø	o o o o o o o o o ø ø ø	2x + 3 +(–3) = 9+(–3)
____ ____ ~~o o o ø ø ø~~	o o o o o o ~~o o o ø ø ø~~	2x = 6
____ ____	o o o o o o	2x = 6
____ ____	o o o o o o	$\frac{2x}{2} = \frac{6}{2}$
____	o o o	x = 3

Describe the Action:	Check the Solution:
Record the original equation	2x + 3 = 9
Substitute the solution for x in place of x in the equation	2(3) + 3 = 9
Simplify both sides of the equation	9 = 9
	The solution x = 3 is correct

Value Balance KEY

Problem 2: 4 = 3x – 2

4 = 3x – 2 is equivalent to 4 = 3x + (– 2)



Value Balance

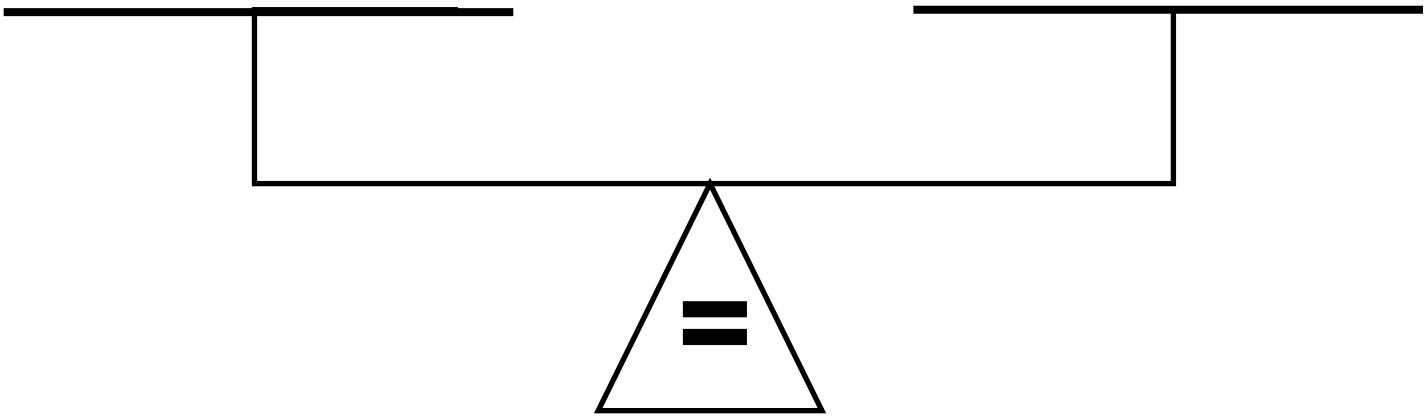
Let ø represent (– 1) and o represent 1 and ____ represent x.

Pictorial Model		Symbolic Model
o o o o	____ ____ ____ ø ø	4 = 3x + (– 2)
o o o o o o	____ ____ ____ ø ø o o	4 + 2 = 3x + (– 2) + 2
o o o o o o	____ ____ ____ ~~ø ø o o~~	6 = 3x + 0
o o o o o o	____ ____ ____	6 = 3x
o o o o o o	____ ____ ____	$\frac{6}{3} = \frac{3x}{3}$
o o	____	2 = x

Describe the Action:	Check the Solution:
Record the original equation	4 = 3x + (– 2)
Substitute the solution for x in place of x in the equation	4 = 3(2) + (– 2)
Simplify both sides of the equation	4 = 4
	The solution x = 2 is correct

Value Balance

Problem 1: 2x + 3 = 9



Value Balance

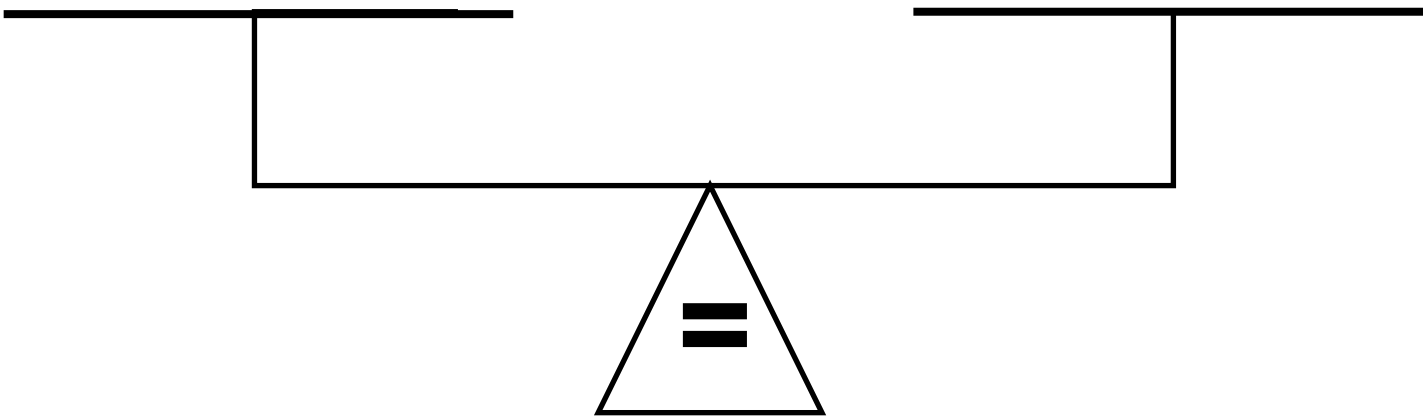
Let ø represent (– 1) and o represent 1 and ____ represent x.

Pictorial Model		Symbolic Model
		=
		=
		=
		=
		=
		=

Describe the Action:	Check the Solution:

Value Balance

Problem 2: 4 = 3x — 2



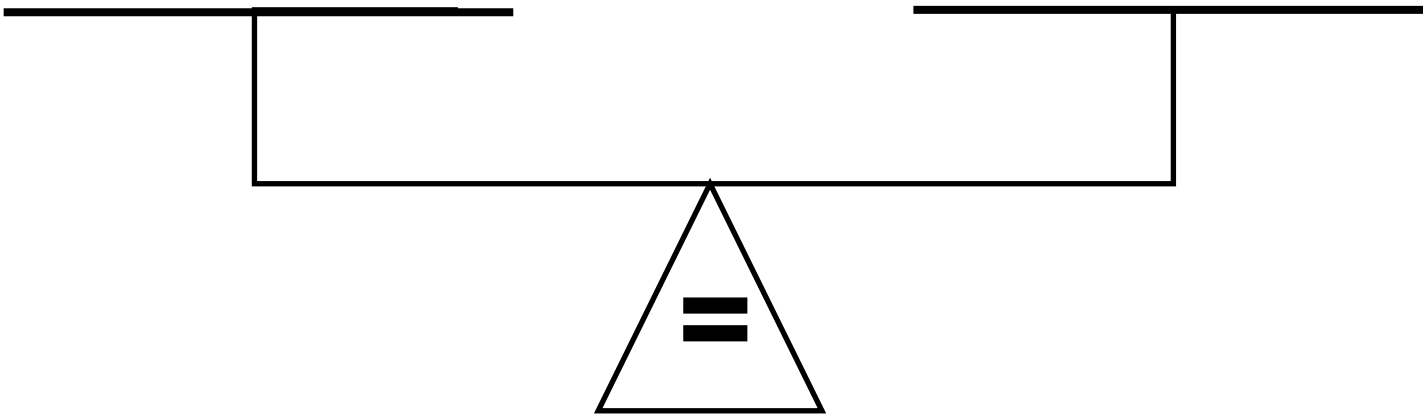
Value Balance

Let ø represent (– 1) and o represent 1 and ____ represent x.

Pictorial Model		Symbolic Model
		=
		=
		=
		=
		=
		=

Describe the Action:	Check the Solution:

Value Balance Blank Model



Value Balance

Let ø represent (–1) and o represent 1 and ____ represent x.

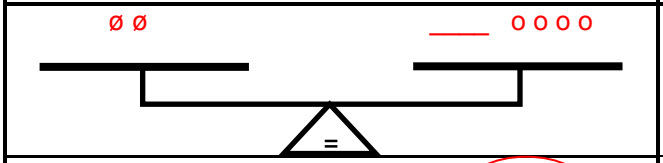
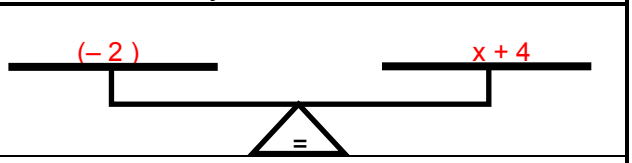
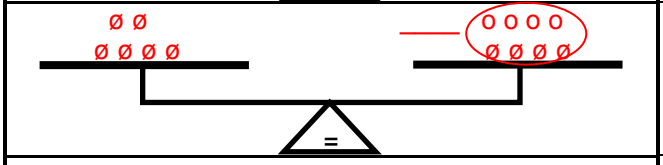
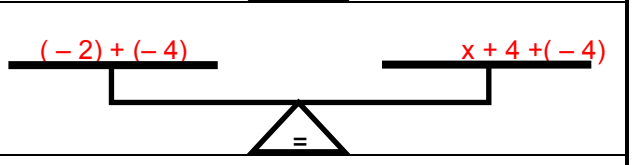
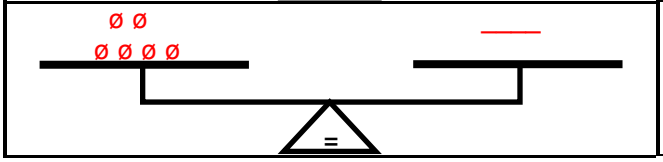
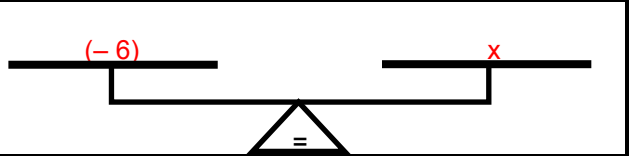
Pictorial Model		Symbolic Model
		=
		=
		=
		=
		=
		=

Describe the Action:	Check the Solution:

Equation Models KEY

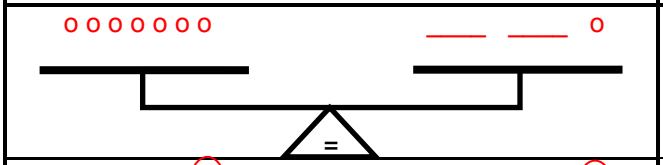
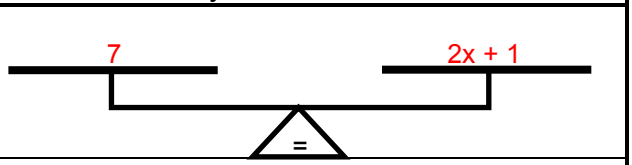
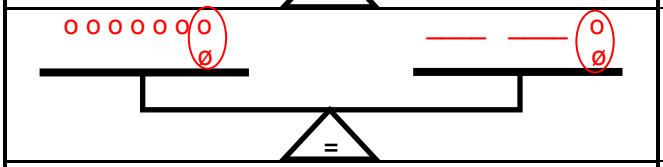
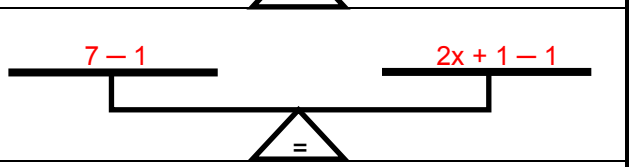
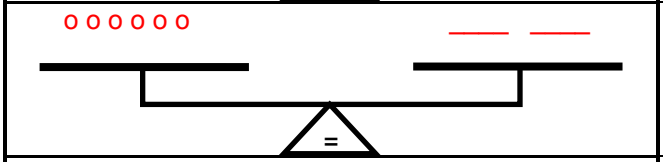
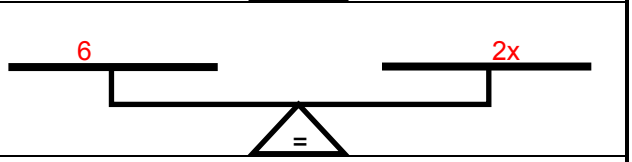
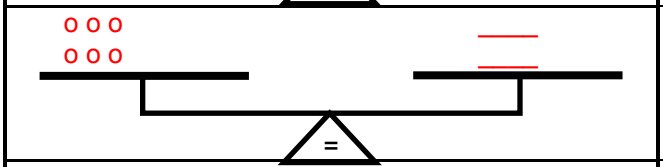
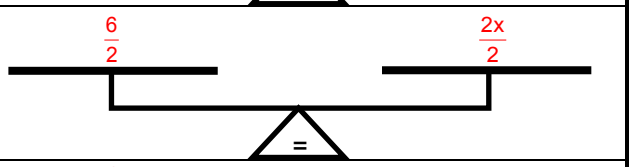
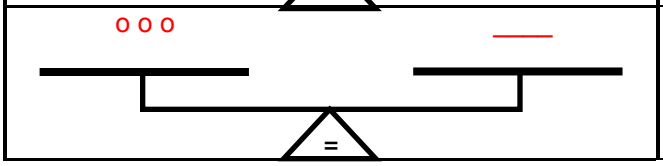
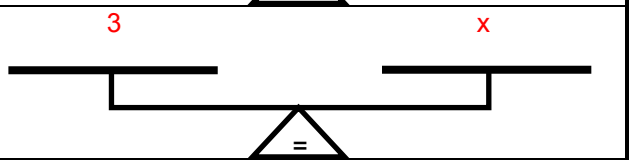
Note: ø represents (– 1), o represents 1, ____ represents x and | represents -x

1. (– 2) = *x* + 4

Pictorial Model	Symbolic Model
	
	
	

Describe the actions:	Check the solution:
Record the original equation	(–2) = x + 4
Substitute the solution for x in place of x in the equation	(–2) = (– 6) + 4
Simplify both sides of the equation	(–2) = (–2)

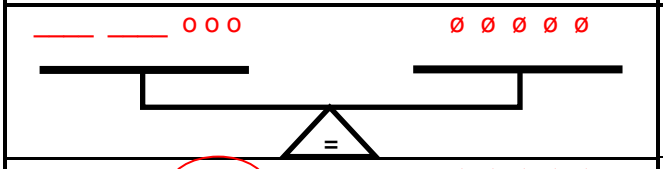
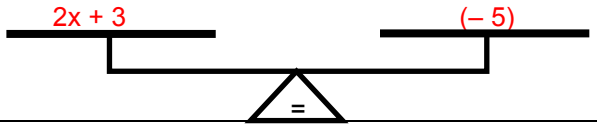
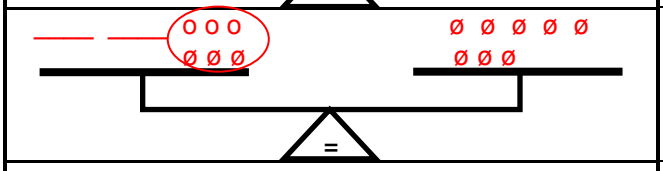
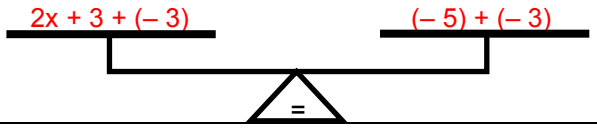
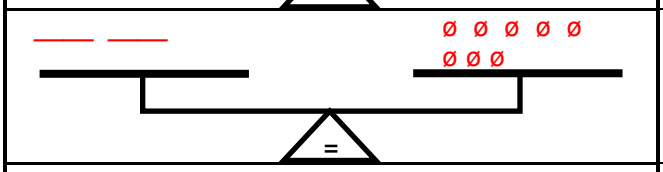
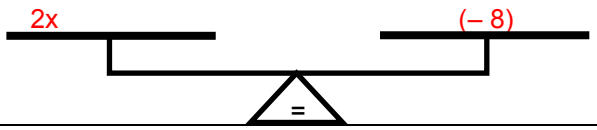
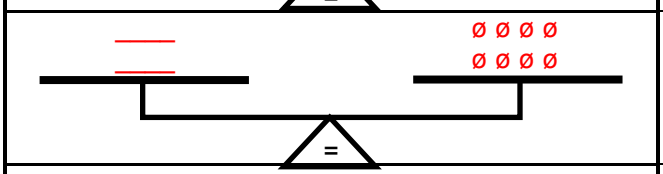
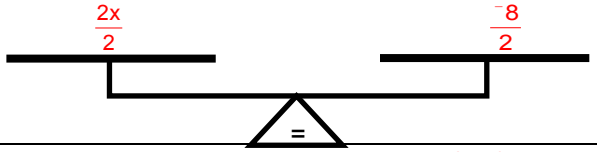
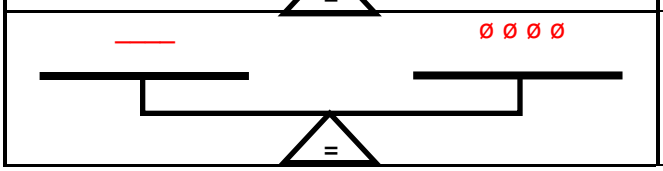
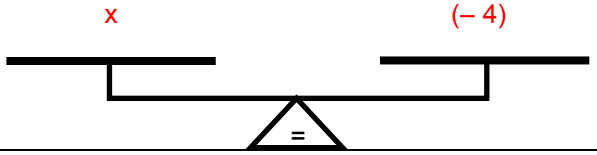
2. 7 = 2*x* + 1

Pictorial Model	Symbolic Model
	
	
	
	
	

Describe the actions:	Check the solution:
Record the original equation	7 = 2x + 1
Substitute the solution for x in place of x in the equation	7 = 2(3) + 1
Simplify both sides of the equation	7 = 7

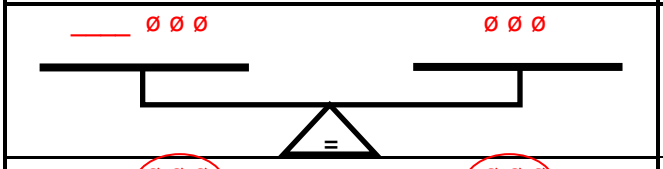
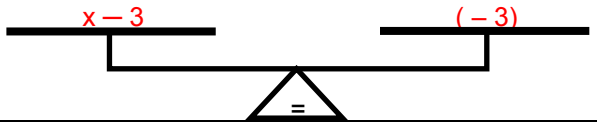
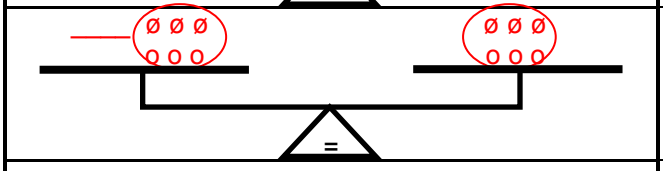
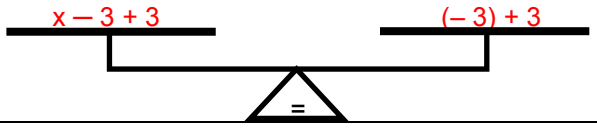
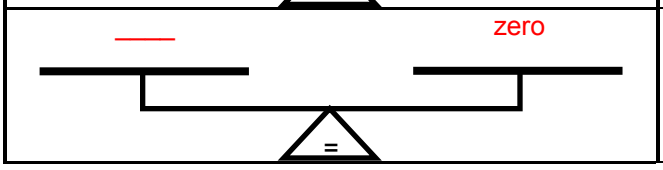
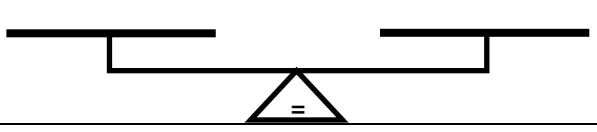
Equation Models KEY

3. $2x + 3 = (-5)$

Pictorial Model	Symbolic Model
	
	
	
	
	

Describe the actions:	Check the solution:
Record the original equation	2x + 3 = (–5)
Substitute the solution for x in place of x in the equation	2(–4) + 3= (–5)
Simplify both sides of the equation	(–5) = (–5)

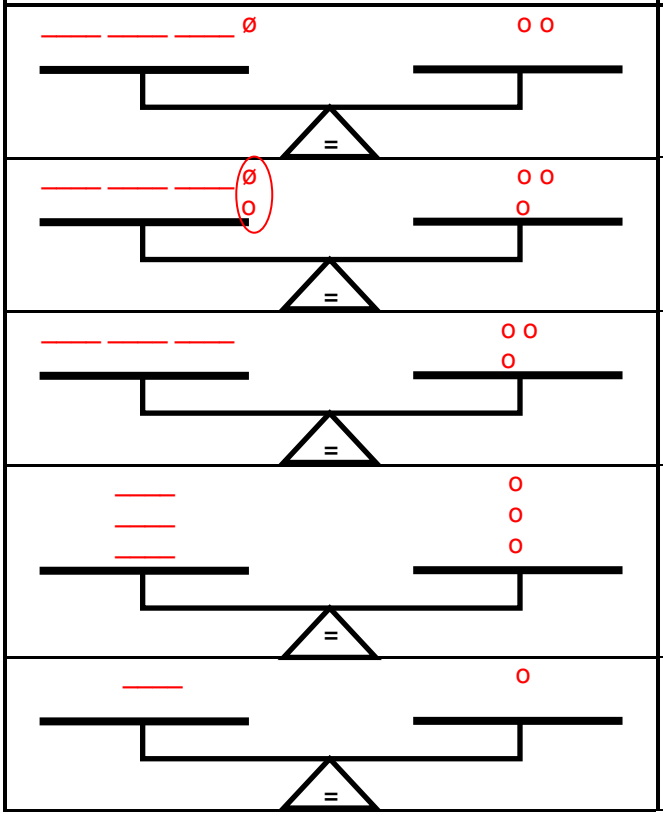
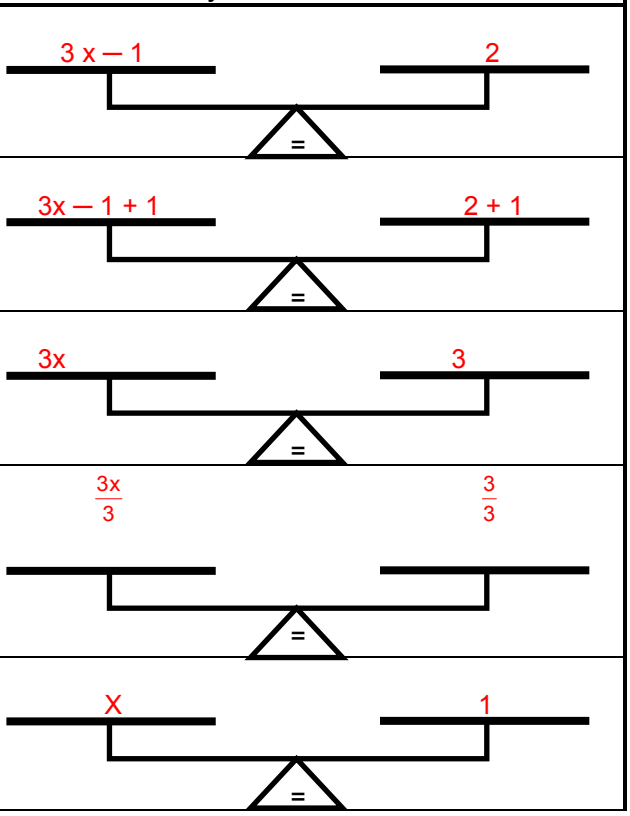
4. $x - 3 = (-3)$

Pictorial Model	Symbolic Model
	
	
	

Describe the actions:	Check the solution:
Record the original equation	x – 3 = (–3)
Substitute the solution for x in place of x in the equation	0 – 3 = (–3)
Simplify both sides of the equation	(–3) = (–3)

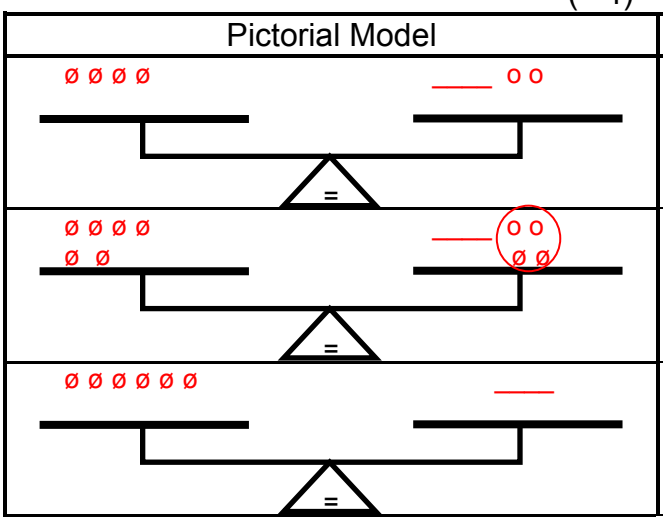
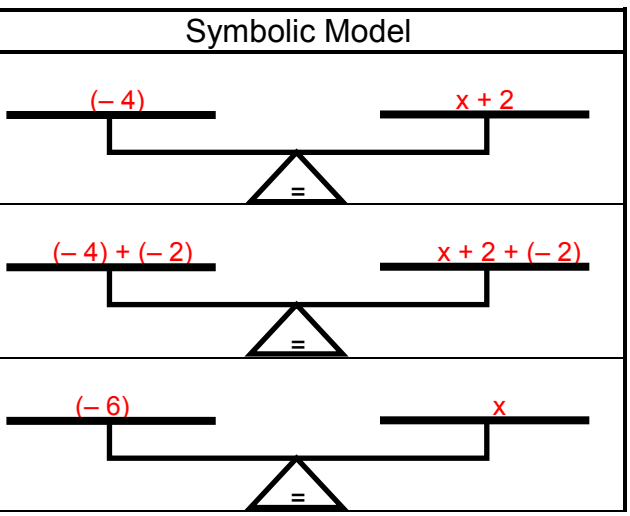
Equation Models KEY

5. $3x - 1 = 2$

Pictorial Model	Symbolic Model
	

Describe the actions:	Check the solution:
Record the original equation	3x – 1 = 2
Substitute the solution for x in place of x in the equation	3(1) – 1 = 2
Simplify both sides of the equation	3 = 3

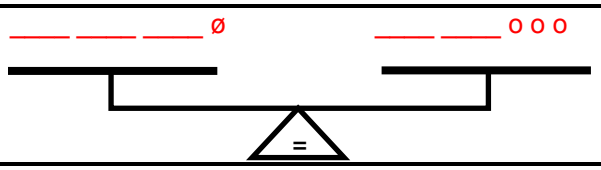
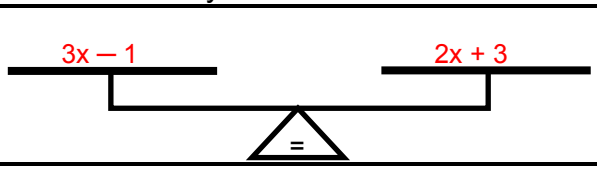
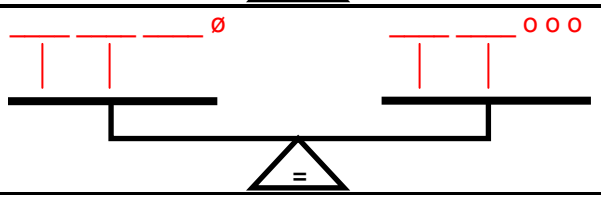
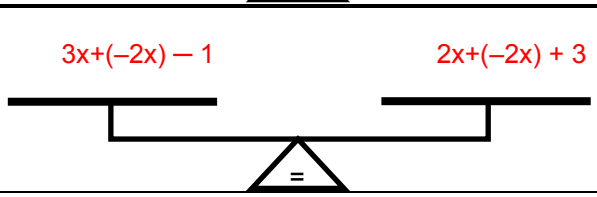
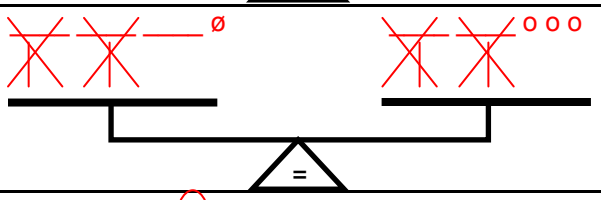
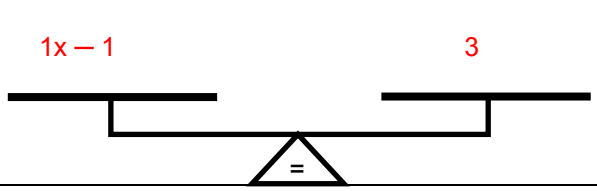
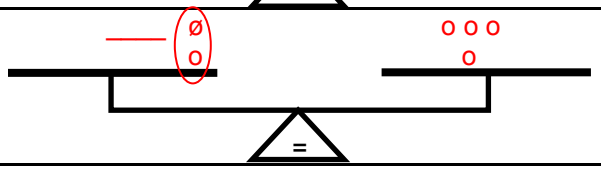
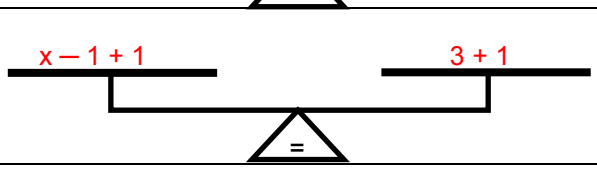
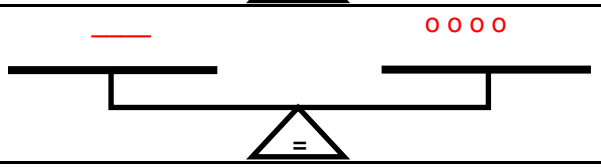
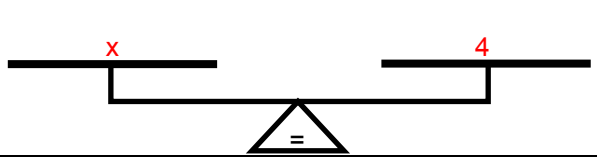
6. $(-4) = x + 2$

Pictorial Model	Symbolic Model
	

Describe the actions:	Check the solution:
Record the original equation	(–4) = x + 2
Substitute the solution for x in place of x in the equation	(–4) = (–6) + 2
Simplify both sides of the equation	(–4) = (–4)

Equation Models KEY

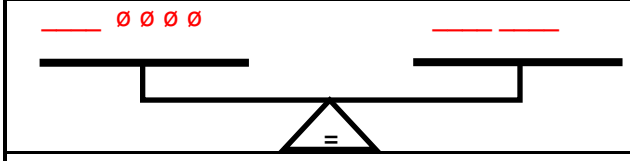
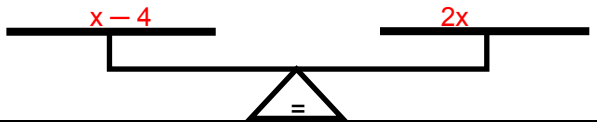
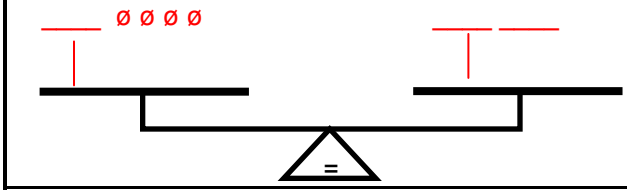
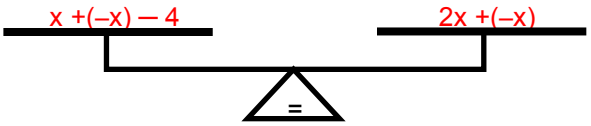
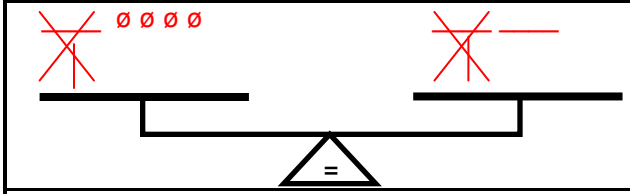
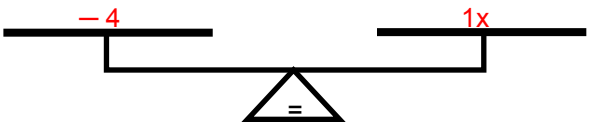
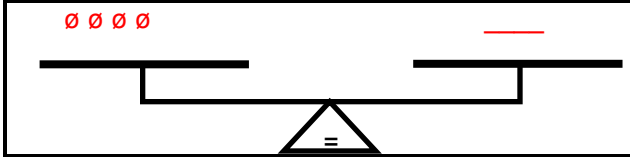
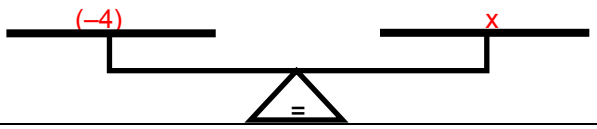
7. $3x - 1 = 2x + 3$

Pictorial Model	Symbolic Model
	
	
	
	
	

Describe the actions:	Check the solution:
Record the original equation	3x – 1 = 2x + 3
Substitute the solution for x in place of x in the equation	3(4) – 1 = 2(4) + 3
Simplify both sides of the equation	11 = 11

Equation Models KEY

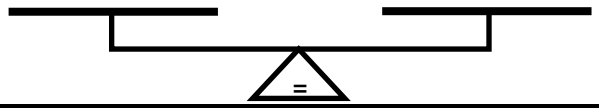
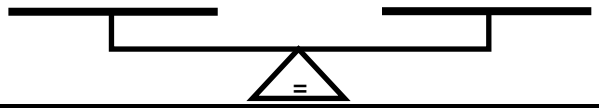
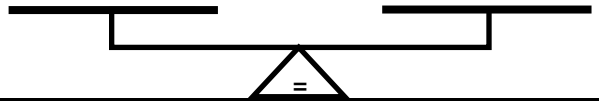
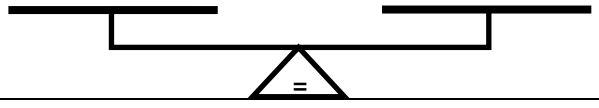
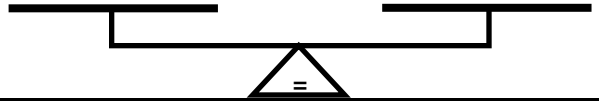
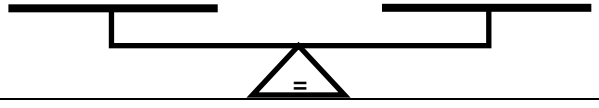
8. $x - 4 = 2x$

Pictorial Model	Symbolic Model
	
	
	
	

Describe the actions:	Check the solution:
Record the original equation	x – 4 = 2x
Substitute the solution for x in place of x in the equation	(–4) – 4 = 2(–4)
Simplify both sides of the equation	(–8) = (–8)

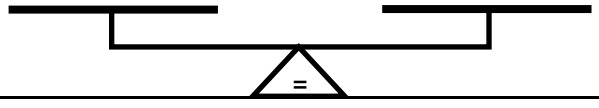
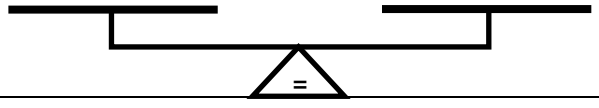
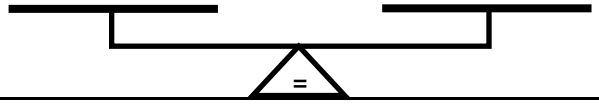
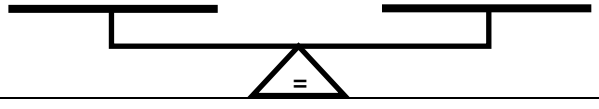
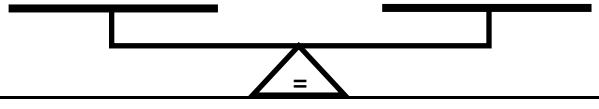
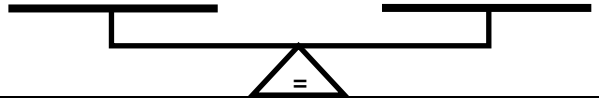
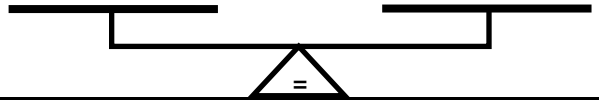
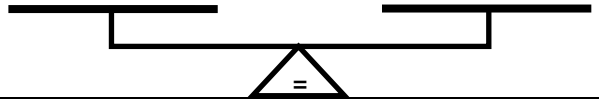
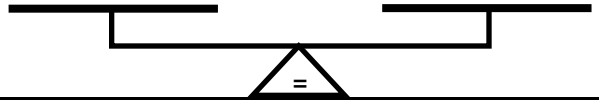
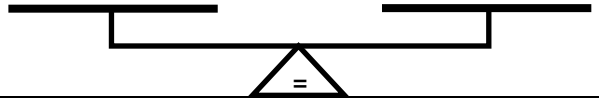
Equation Models

1. $(-2) = x + 4$

Pictorial Model	Symbolic Model
	
	
	

Describe the actions:	Check the solution:

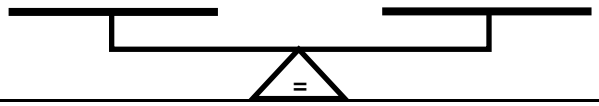
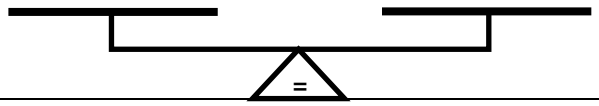
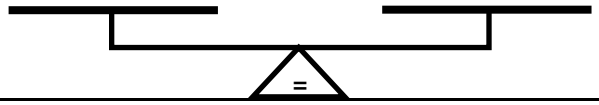
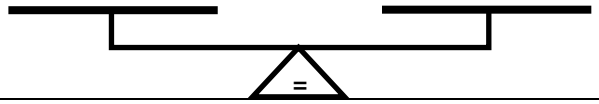
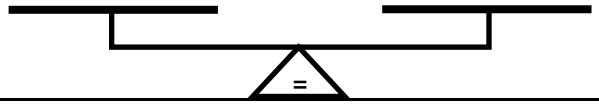
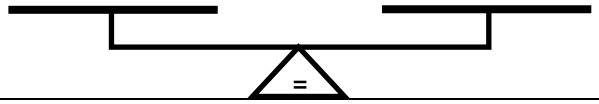
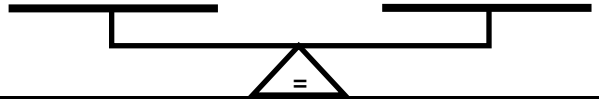
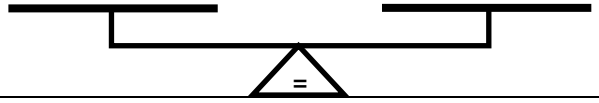
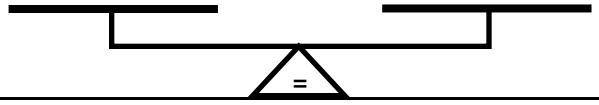
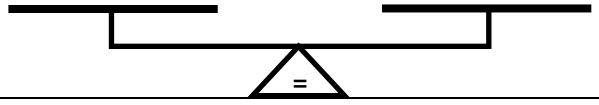
2. $7 = 2x + 1$

Pictorial Model	Symbolic Model
	
	
	
	
	

Describe the actions:	Check the solution:

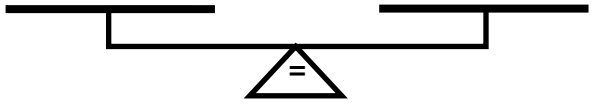
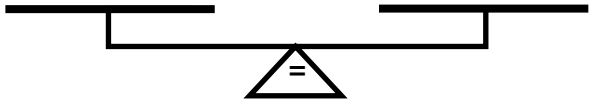
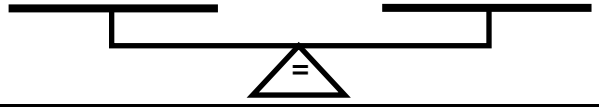
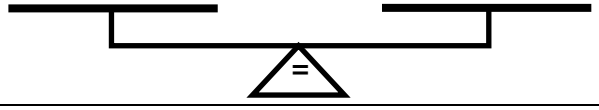
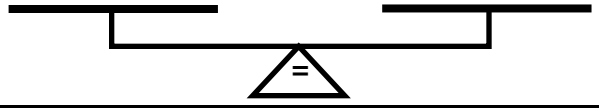
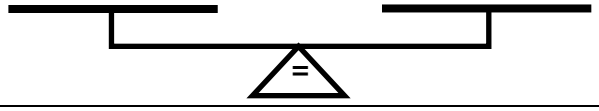
Equation Models

3. $2x + 3 = (-5)$

Pictorial Model	Symbolic Model
	
	
	
	
	

Describe the actions:	Check the solution:

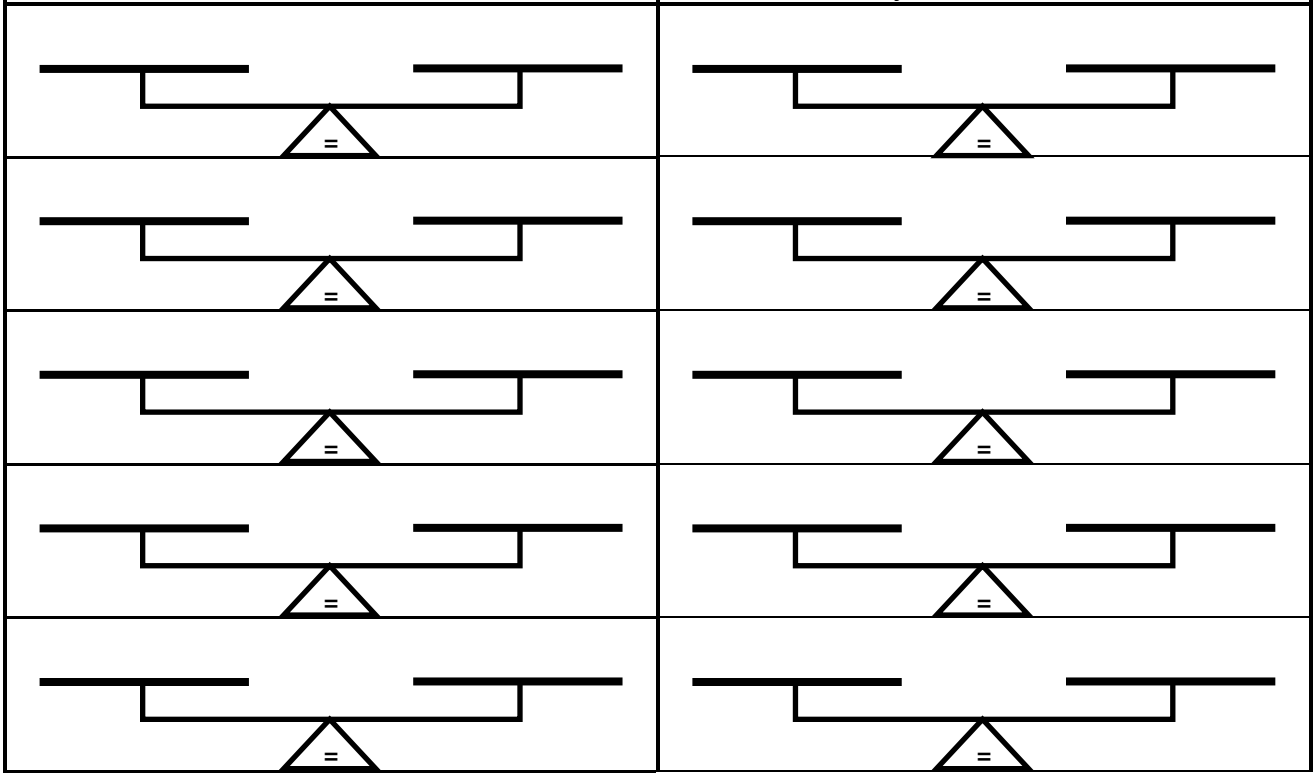
4. $x - 3 = (-3)$

Pictorial Model	Symbolic Model
	
	
	

Describe the actions:	Check the solution:

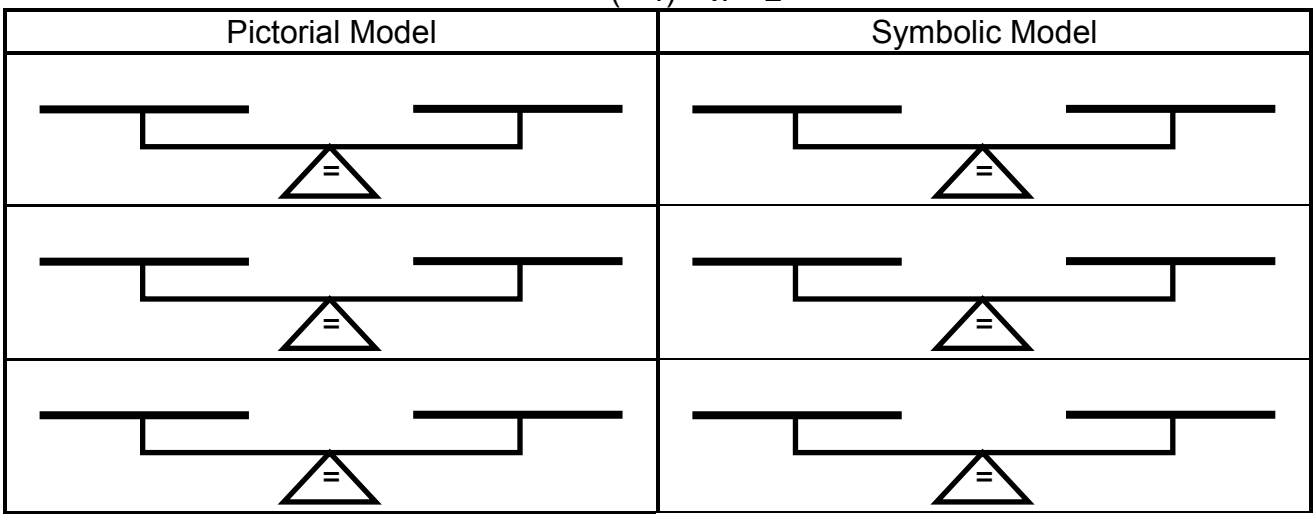
Equation Models

5. $3x - 1 = 2$

Pictorial Model	Symbolic Model
	

Describe the actions:	Check the solution:

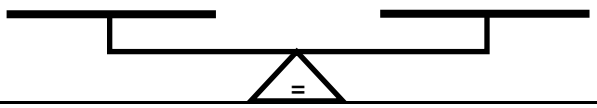
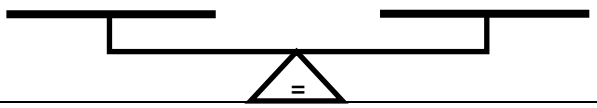
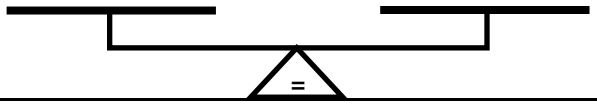
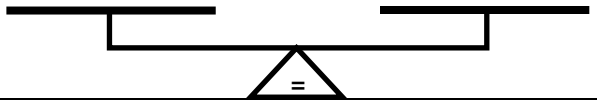
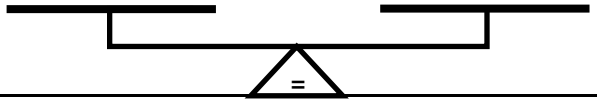
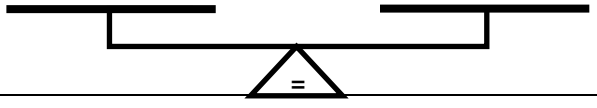
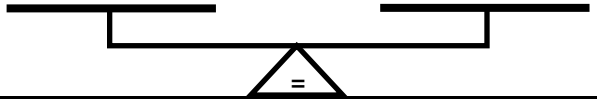
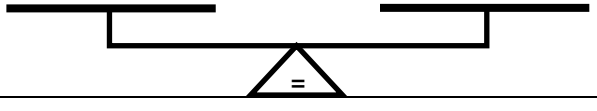
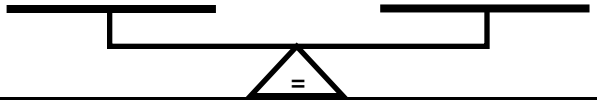
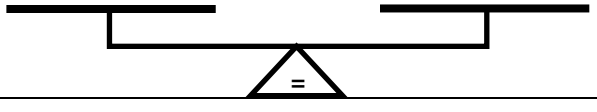
6. $(-4) = x + 2$

Pictorial Model	Symbolic Model
	

Describe the actions:	Check the solution:

Equation Models

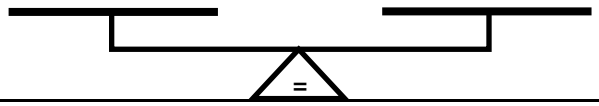
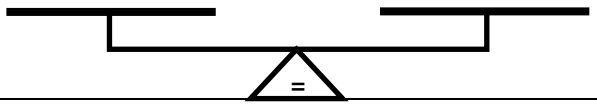
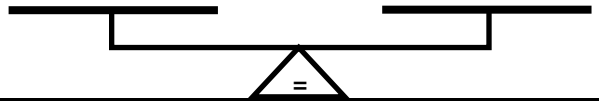
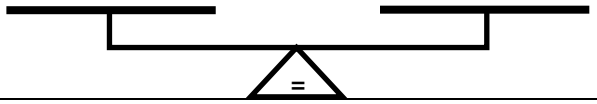
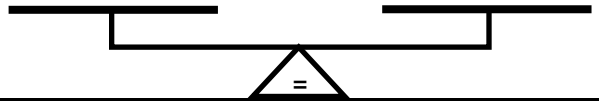
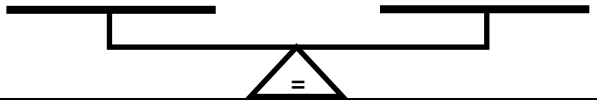
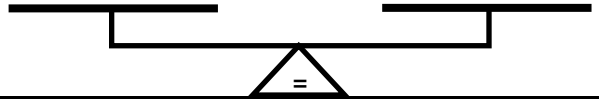
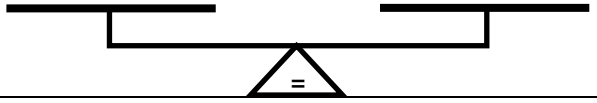
7. $3x - 1 = 2x + 3$

Pictorial Model	Symbolic Model
	
	
	
	
	

Describe the actions:	Check the solution:

Equation Models

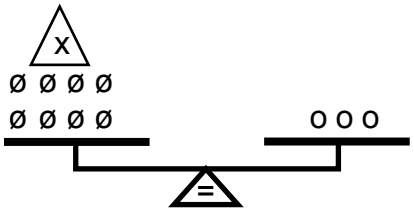
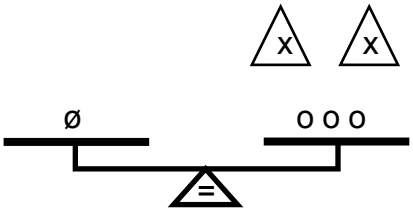
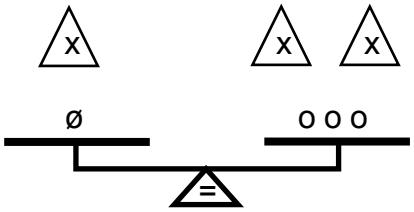
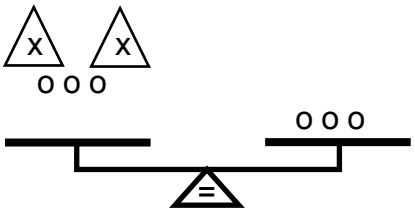
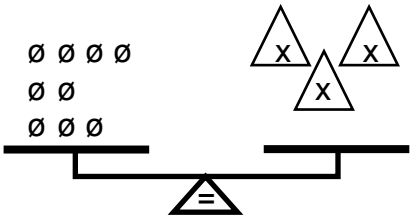
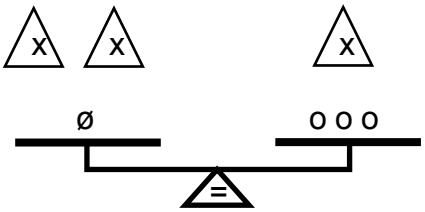
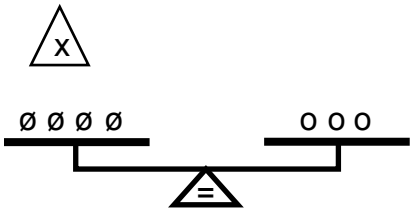
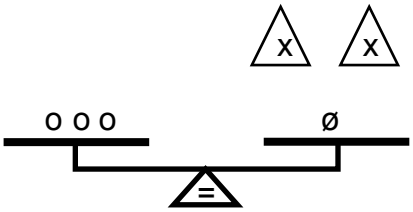
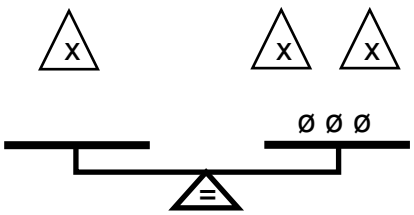
8. $x - 4 = 2x$

Pictorial Model	Symbolic Model
	
	
	
	

Describe the actions:	Check the solution:

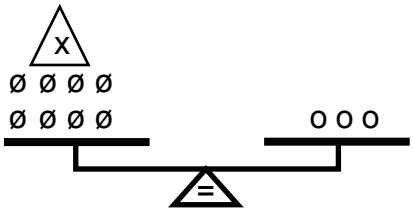
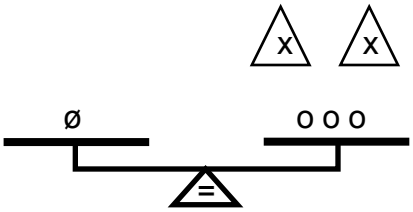
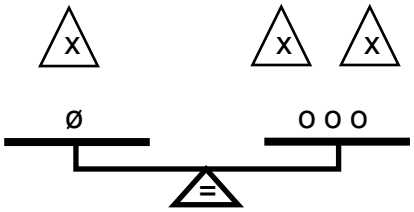
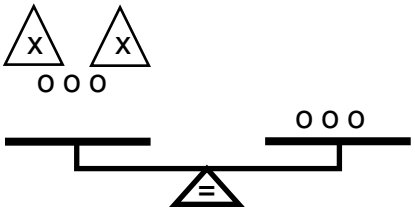
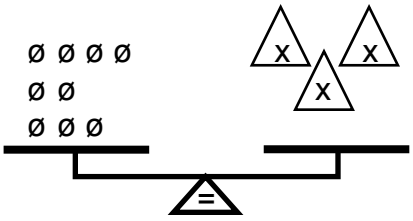
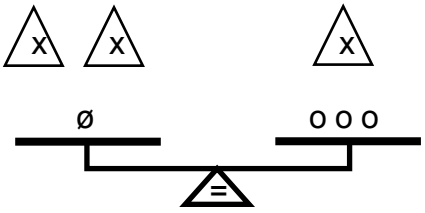
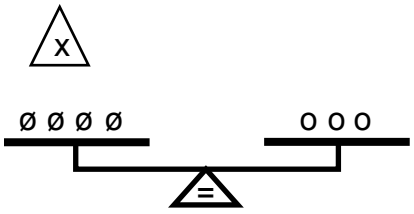
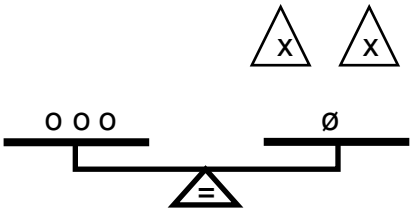
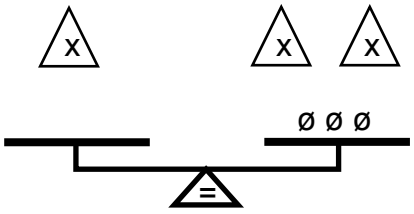
Family Reunion KEY

Copy the cards (3 sheets) on cardstock and cut apart one deck of cards for each pair of students. There are 27 cards to a deck. Three cards (symbolic, pictorial, solution) make a family. There are 9 families in each deck. o = 1 and ø = –1 and  = x

1 Family Reunion x – 8 = 3; x = 11 	2 Family Reunion (– 1) = 2x + 3; x = (– 2) 	3 Family Reunion x – 1 = 2x + 3; x = (– 4) 
4 Family Reunion 2x + 3 = 3; x = 0 	5 Family Reunion (– 9) = 3x; x = (– 3) 	6 Family Reunion 2x – 1 = x + 3; x = 4 
7 Family Reunion x – 4 = 3; x = 7 	8 Family Reunion 3 = 2x – 1; x = 2 	9 Family Reunion x = 2x – 3; x = 3 

Family Reunion

Copy the cards (3 sheets) on cardstock and cut apart one deck of cards for each pair of students. There are 27 cards to a deck. Three cards (symbolic, pictorial, solution) make a family. There are 9 families in each deck. Key: o = 1 and ø = –1 and  = x

1 Family Reunion 	2 Family Reunion 	3 Family Reunion 
4 Family Reunion 	5 Family Reunion 	6 Family Reunion 
7 Family Reunion 	8 Family Reunion 	9 Family Reunion 

Family Reunion

Copy the cards (3 sheets) on cardstock and cut apart one deck of cards for each pair of students. There are 27 cards to a deck. Three cards (symbolic, pictorial, solution) make a family. There are 9 families in each deck. o = 1 and ø = (–1) and  = x

Family Reunion $x - 8 = 3$	Family Reunion $(-1) = 2x + 3$	Family Reunion $x - 1 = 2x + 3$
Family Reunion $2x + 3 = 3$	Family Reunion $(-9) = 3x$	Family Reunion $2x - 1 = x + 3$
Family Reunion $x - 4 = 3$	Family Reunion $3 = 2x - 1$	Family Reunion $x = 2x - 3$

Family Reunion

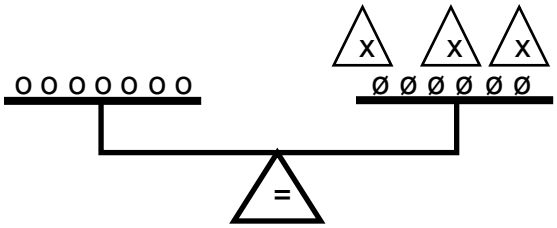
Copy the cards (3 sheets) on cardstock and cut apart one deck of cards for each pair of students. There are 27 cards to a deck. Three cards (symbolic, pictorial, solution) make a family. There are 9 families in each deck. Key: o = 1 and ø = –1 and  = x

Family Reunion 11 = x	Family Reunion x = (– 2)	Family Reunion x = (– 4)
Family Reunion 0 = x	Family Reunion x = (– 3)	Family Reunion 4 = x
Family Reunion x = 7	Family Reunion x = 2	Family Reunion x = 3

Check It Out KEY

1. Write the equation for the given pictorial model. Indicate if the given steps are the correct process for finding a solution to the equation. If a step is incorrect, correct it. Use symbolic notation to solve the equation. Check the solution.

Pictorial Model



Key: ø = (– 1) and o = 1

Steps to Solve Equation:	Symbolic Model
~~1. Subtract 6 from each side of the equation.~~ Add 6 to both sides of the equation.	$7 + 6 = 3x - 6 + 6$
~~2. Multiply both sides of the equation by 3.~~ Divide both sides of the equation by 3.	$\frac{13}{3} = \frac{3x}{3}$
3. Simplify.	$\frac{13}{3}$ or $4\frac{1}{3} = x$

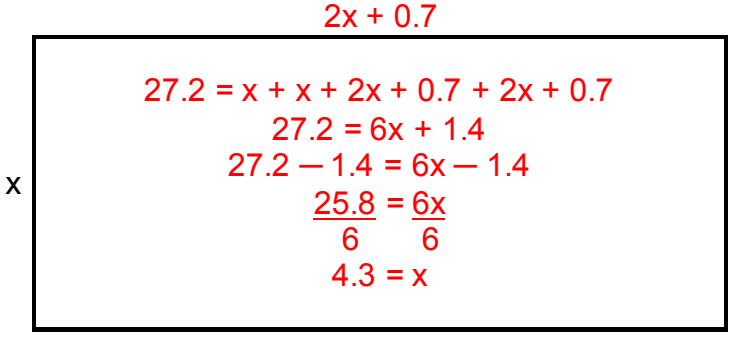
Check:

$7 = 3(\frac{13}{3}) - 6$

$7 = 13 - 6$

$7 = 7$

2. Write an equation that represents the perimeter of the rectangle in the given diagram. The perimeter of the rectangle is 27.2 centimeters. Let x represent the number of centimeters in the width of the rectangle. The length is 0.7 centimeters more than twice the width. Find the value for x that will make the equation true.



$27.2 = x + x + 2x + 0.7 + 2x + 0.7$

$27.2 = 6x + 1.4$

$27.2 - 1.4 = 6x - 1.4$

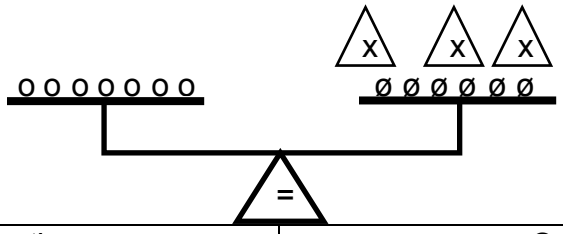
$\frac{25.8}{6} = \frac{6x}{6}$

$4.3 = x$

Check It Out

1. Write the equation for the given pictorial model. Indicate if the given steps are the correct process for finding a solution to the equation. If a step is incorrect, correct it. Use symbolic notation to solve the equation. Check the solution.

Pictorial Model

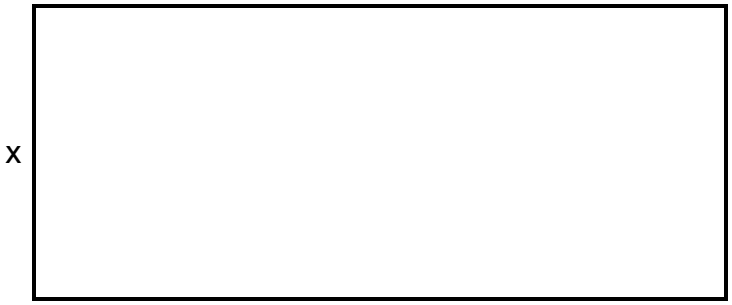


Key: ø = (– 1) and o = 1

Steps to Solve Equation:	Symbolic Model
1. Subtract 6 from each side of the equation.	
2. Multiply both sides of the equation by 3.	
3. Simplify.	

Check:

2. Write an equation that represents the perimeter of the rectangle in the given diagram. The perimeter of the rectangle is 27.2 centimeters. Let x represent the number of centimeters in the width of the rectangle. The length is 0.7 centimeters more than twice the width. Find the value for x that will make the equation true.



Balancing Practice KEY

For problems 1 – 2 use the following symbols to draw a pictorial model for each equation:
o = 1 ø = (– 1) ____ = *x*

1. (– 8) = 2*x* + 4

ø ø ø ø ø ø ø ø = ____ ____ o o o o

2. 3 = 2*x* — 3

o o o = ____ ____ ø ø ø

For problems 3 – 4, draw a pictorial model for the equation and indicate what first step would be done to solve each equation. o = 1 ø = (– 1) ____ = *x*

3. 3*x* — 8 = (– 9) ____ ____ ____ ø ø ø ø ø ø ø ø = ø ø ø ø ø ø ø ø ø

Add 8 positives to both sides of the equal sign or remove 8 negatives from both sides of the = sign

4. (– 8) = 6*x* + 4 øøøøøøøø = ____ ____ ____ ____ ____ ____ oooo

Add 4 negatives to both sides of the = sign

For problems 5 – 6, use the given pictorial model and equation to find the value of *x*.
o = 1 ø = (– 1) ____ = *x*

5. This model represents the equation: 4*x* + 3 = (– 1). What is the value of *x*?

____ ____ ____ ____ o o o = ø

____ = ø x = (– 1)

6. This model represents the equation: 6 = 3*x* — 6. What is the value of *x*?

o o o o o o = ____ ____ ____ ø ø ø ø ø ø

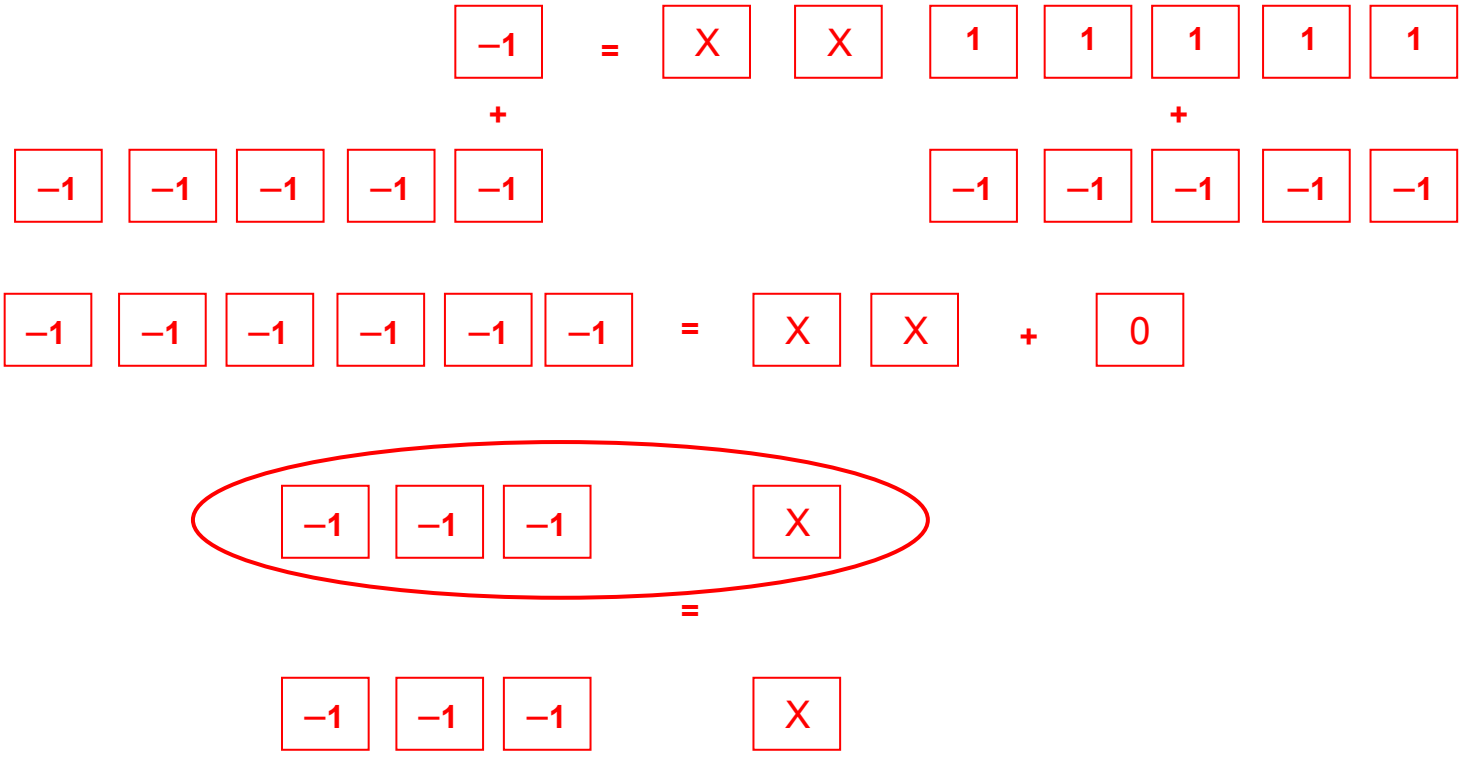
o o o o = ____ 4 = *x*

Balancing Practice Key

For problems 7 – 8, use the given equation to draw a pictorial model of the equation, estimate a reasonable solution and find the value of x. Write a statement to justify the reasonableness of the solution.

1	= 1	–1	= –1	X	= x

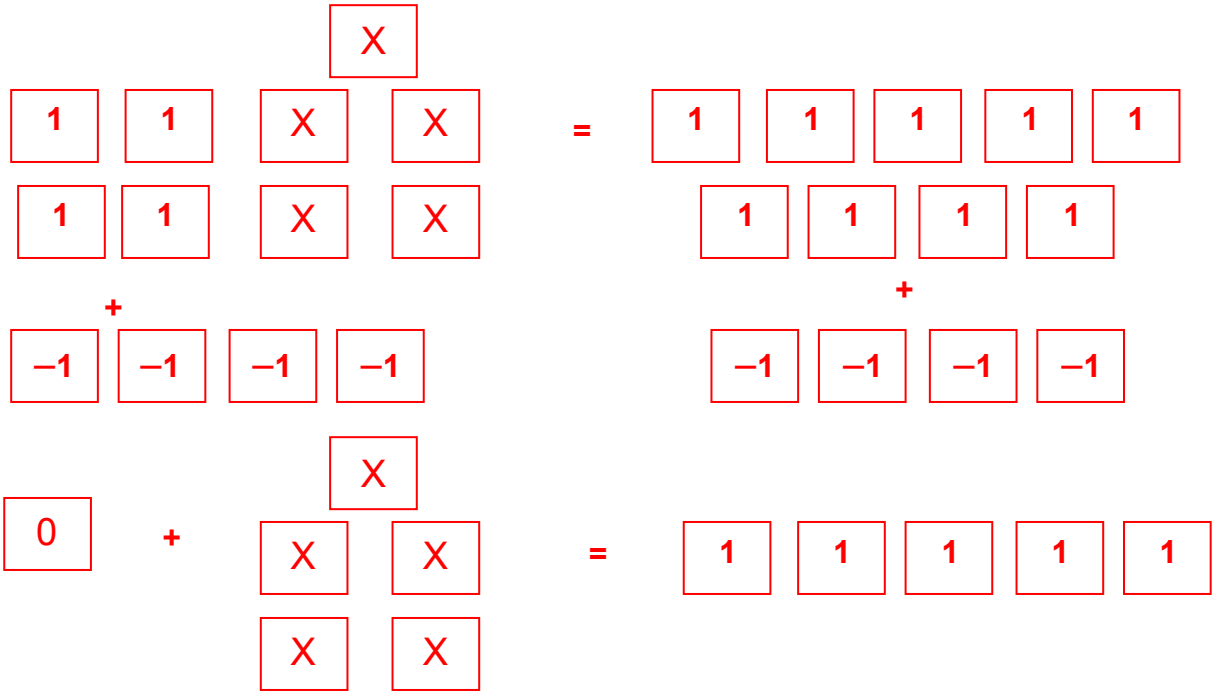
7. (– 1) = 2*x* + 5

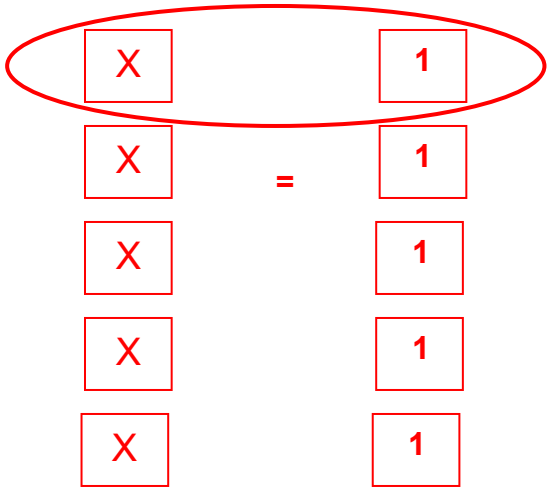


Estimate: (– 6) + 5 = (– 1) so *x* = (– 3) because 2(– 3) = (– 6)
Solution: (– 1) + (– 5) = 2*x* + 5 + (– 5)
(– 6) ÷ 2 = 2*x* ÷ 2
(– 3) = *x*
Statement: The estimated solution was *x* = (– 3) because 2*x* had to = (– 6) and after solving the equation, *x* = (– 6).

Balancing Practice KEY

8. $4 + 5x = 9$





Estimate: $4 + 5 = 9$ so $x = 1$ because $5(1) = 5$
Solution: $4 + 5x + (-4) = 9 + (-4)$
$5x \div 5 = 5 \div 5$
$x = 1$
Statement: The estimated solution was $x = 1$ because $5x$ had to $= 5$ and after solving the equation, $x = 1$

Balancing Practice

For problems 1 - 2 use the following symbols to draw a pictorial model for each equation:
o = 1 ø = (– 1) ____ = *x*

1. (– 8) = 2*x* + 4

2. 3 = 2*x* — 3

For problems 3 – 4, draw a pictorial model for the equation and indicate what first step would be done to solve each equation. o = 1 ø = (– 1) ____ = *x*

3. 3*x* — 8 = (– 9)

4. (– 8) = 6*x* + 4

For problems 5 – 6, use the given pictorial model and equation to solve each equation.
o = 1 ø = (– 1) ____ = *x*

5. This model represents the equation: 4*x* + 3 = (– 1). What is the value of *x*?

____ ____ ____ ____ o o o = ø

6. This model represents the equation: 6 = 3*x* — 6. What is the value of *x*?

o o o o o o = ____ ____ ____ ø ø ø ø ø ø

For problems 7 – 8, use the given equation to draw a pictorial model of the equation, estimate a reasonable solution and find the value of x. Write a statement to justify the reasonableness of the solution.

1	**= 1**		**–1**	**= –1**		*X*	**=** *x*

7. (– 1) = 2*x* + 5

8. 4 + 5*x* = 9