ATMOPAV NEWSLETTER

PDF edition

ATMOPAV
An affiliate of NCTM & PCTM
Association of Teachers of Mathematics Of Philadelphia And Vicinity

KEEP THESE DATES:

ATMOPAV Fall Conference ~ Hatboro-Horsham High School ~ October 26, 2013
NCTM Regional Conference ~ Baltimore, MD ~ October 16 – 18, 2013
PCTM Conference ~ Seven Springs ~ November 6 - 8, 2013

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ATMOPAV
NEWSLETTER

The Association of Teachers of Mathematics of Philadelphia and Vicinity
An Affiliate of the National Council of Teachers of Mathematics and the Pennsylvania Council of Teachers of Mathematics

President's Message

Susan Negro
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What an exciting evening! On April 4th, ATMOPAV hosted the annual Spring Conference and awards banquet at Germantown Academy. There were over 100 guests, presenters, vendors, and awardees present. We kicked off the conference with ten fantastic, inspiring workshops that included something for everyone. There were preK-12 workshops on technology, pedagogy, and more. Many included hands-on activities to take back to the classroom. The evening concluded with a fabulous meal and awards ceremony. Special guests included Dr. Mike Long, president of PCTM, who opened the banquet with words of encouragement to rally the troops and the award winners.

Our first award of the evening was presented to Herb Greene, by Kathy Herbert, a long time ATMOPAV executive board member. Kathy had this to say about our winner of the Outstanding Contributions to Mathematics Education winner: “With all his teaching and mathematical background and knowledge, I believe Herb’s greatest strength is working with young people and turning them on to mathematics and teaching.” Many of Herb’s students and colleagues were in attendance to support him. Doug Baird, Co-Director of TUteach, praised Herb: “As our first master teacher, Herb set the tone by establishing a ‘culture of caring’ at TUteach. Herb is always there for his students. Herb has made a major contribution to math and science education in our region which will live on through the success of his students.”

Beth Benzing, ATMOPAV first vice president, introduced the teaching award winners: The 2013 Alan Barson Novice Teacher Award winner Kaitlin Valliere, and the Mabel M. Elliott Outstanding Student Teacher award winner Allison Gantt. Fred Hartman, Great Valley’s math department chair, nominated Kaitlin and shared these kind words about her: “She brings her enthusiasm and creativity to the classroom on a daily basis. Her teaching constantly keeps the needs of her students first. She challenges herself and shares teaching methods with her peers to continuously improve herself and the mathematics skills of the students she teaches. This is demonstrated through the activities and teaching modalities which engage her students, aid in their learning and keep her student’s interest active.” Allison Gantt student taught at Central High School in Philadelphia. Her cooperating teacher, Marina Isakowitz, shared: “From the very beginning, I was struck by Allison’s deep thoughtfulness about how students learn and what they need to feel safe in the classroom. Allison’s biggest strength as an educator is her constant reflection on how her students are learning and growing. She is constantly striving towards improvement in her teaching and is not satistified with anything less than her best.”

Mark Wassmansdorf, ATMOPAV treasurer, honored Lynn Hughes with the Outstanding Contribution to ATMOPAV award. It was especially fitting that we honor Lynn this year, as the ATMOPAV Newsletter was selected as this year's winning affiliate publication by the NCTM board. Lynn will continue to be honored when she attends the NCTM conference in Denver later this month. All you need to do is read any copy of this newsletter, and you will know why NCTM selected it as a winner.

(continued on page 16)

The ATMOPAV Newsletter is published three times a year. Contributions in the form of articles, reviews, teaching ideas, humor, and opinionated essays are welcome. Material for the Fall 2013 issue should be submitted no later than September 30, 2013. Please be sure to include contact information (e-mail and/or telephone) if you are not a regular columnist. Send your text to the editor:

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The Association of Teachers of Mathematics of Philadelphia and Vicinity was founded more than 60 years ago. It is an organization of mathematics educators who are dedicated to improving mathematics instruction at every level from kindergarten to college. Through its professional meetings, website, and this publication, ATMOPAV supports, informs, and facilitates communication among teachers, pre-service education students, supervisors, and school administrators from the five-county Philadelphia area. Our awards program recognizes excellence among student teachers, novice teachers, and secondary school students who participate in our annual mathematics competition.

To join ATMOPAV, please complete the membership application in this issue.

Call for Columnists

If you're an educator, you have something to share, and this is a good place to do it. We are looking for more writers of regular columns and of single articles. About what, you ask? Consider these possibilities:

- Activities, especially for grades K - 5
- Mathematics software reviews
- Adapting lessons for students with special needs
- Cross-curricular activities that link math with other subjects
- Diary of a first-year teacher
- Recollections of a retired teacher
- Mathematics-related history
- Useful resource books for teachers
- Math games and
- Literature connections and book reviews
- Reviews of math-related apps
- Teaching with particular software: Scratch, SketchUp, spreadsheets, Geometer's Sketchpad, etc.
- Working with challenged learners
- Parent Night/ Math Night ideas

. . . *and whatever else you might think of.*

We are eager for material that applies to any level of mathematics education. If you don't want to write but know a colleague who may, please pass the word. Contact the editor for further information.

Math-Related Websites for Teachers and Students

Cut the Knot http://cut-the-knot.org ***The Futures Channel*** http://thefutureschannel.com
Making Mathematics http://www2.edc.org/makingmath/default.asp
Don's Math Website http://www.shout.net/~mathman/ ***CoolMath4Kids*** http://www.coolmath4kids.com
JETS website http://www.jets.org/latestnews/JETS_Challenge.cfm
The Virtual Library of Interactive Mathematics http://www.matti.usu.edu/nlvm/nav/vlibrary.html
The Intermath Dictionary http://jwilson.coe.uga.edu/interMath/MainInterMath/Dictionary/welcome/howto.html
Mathematrix http://www.geocities.com/CapeCanaveral/Hangar/7773/
The MacTutor History of Mathematics http://www-groups.dcs.st-and.ac.uk/~history/
Powers of Ten http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html
Mathworld http://mathworld.wolfram.com/ ***RadicalMath*** http://www.radicalmath.org
S.O.S. Mathematics http://www.sosmath.com/ ***Natural Math*** http://www.naturalmath.com/
The Knot Plot Site http://www.cs.ubc.ca/nest/imager/contributions/scharein/KnotPlot.html
Paper Models of Polyhedra http://www.korthalsaltes.com/index.html
Mrs. Glosser's Math Goodies http://www.mathgoodies.com ***Platonic Realms*** http://www.mathacademy.com
The Math Forum@Drexel http://www.mathforum.org/ ***Explore Learning*** http://www.explorelearning.com
Khan Academy http://www.khanacademy.org/ ***XtraMath*** www.xtramath.org
And, of course . . . ***The National Council of Teachers of Mathematics*** http://www.nctm.org/

Little Folks, Big Ideas

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The Miquon School

Let's Play Some Card Games!

In our blended first/second grade classroom, we supplement our mathematics instruction with games and activities that children can work on during exploration time or when their work is completed. When engaging in these activities and games, besides practicing some of the basic math skills, the children in our class think a lot about the strategy they want to use to win a game, have a chance to practice sportsmanship and fairness, and have a lot of fun in the process. We set the games up on a shelf that allows ready access for the children to find a partner or two and start to play a game. We have regular discussions about appropriate ways to play games with others, and talk about strategies we can use to solve disagreements if they arise during a game. Below are four games that would be fun in the classroom. I included descriptions provided by Gamewright, the manufacturer. The children in our classroom play *Sleeping Queens* and *Frog Juice* often. The games are usually very inexpensive and are readily available. I found these at Amazon.com. Enjoy!

Sleeping Queens - A royalty rousing Card Game. Rise and Shine! The Pancake Queen, the Ladybug Queen, and ten of their closest friends have fallen under a sleeping spell and it's your job to wake them up. Use strategy, quick thinking, and a little luck to wake these napping nobles from their royal slumbers. Play a knight to steal a queen or take a chance on a juggling jester. But watch out for wicked potions and a dastardly dragon! The player who wakes the most queens wins. As you immerse yourself in the Evarts' fantasyland, you will find a game that helps develop memory, strategy, and elementary arithmetic skills. *Game and description by Gamewright.*

Frog Juice - How would you create a magic spell? You might need some unicorn horn or some eye of newt, or maybe some bats or toadstools or star and moon dust. But be forewarned as you're stirring up your potion--you'd better be wary of a black cat or the powerful All-Purpose Witch Wash! One of your opponents might even be brewing up an Uglifying Spell (Results guaranteed to crack mirrors at a single glance!). All of these magical ingredients are part of Frog Juice, an imaginative card game that uses fairy-tale elements from fairy maids to frogs to create a game that kids will enjoy. *Game and description by Gamewright*

There's A Moose In The House - Eeek! There's a moose loose in the kitchen and another in the bathroom! In this silly matching card game, the goal is to keep moose out of your house, while at the same time giving them to your opponents. Use a door to close off empty rooms or if you're lucky, plant a moose trap to keep those loveable but uninvited visitors away! *Game and description by Gamewright*

Too Many Monkees - Primo the Monkey was fast asleep until his friends swung by for a surprise pajama party. Help Primo chase away the primate pranksters by flipping and swapping cards in numerical order. Watch out for elephants and giraffes who want to crash in on the fun. And if you're lucky, draw a wild monkey card to clear out the room in one clean sweep. The first player to stop the monkey mayhem and get Primo back to bed wins. *Game and description by Gamewright.*

MIDPOINT

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Making Sense of Integers

There are lots of things that we should memorize: alphabetical order is one, although our increasing ability to store and search for information in other ways may eventually render that skill obsolete. It's good to know your phone number, your blood type, your social security number, your email address, and (of course) your multiplication facts. But when mathematics students try to memorize things that they can't re-create by reasoning after they forget the rules or algorithms, they're probably headed for trouble.

One of the math textbooks I am using contains a presentation of adding and subtracting positive and negative numbers that includes a lot of use of number lines. This was helpful. Students could see that steps in a positive direction coupled with steps in a negative direction could yield either a positive or negative result, depending on which of the numbers was greater in absolute value. We also used blank plastic tiles with a plus or minus sign written on them with indelible pen. "Negative 3" was represented as 3 negatives, or 3 minus tiles. "Positive 5" was represented by 5 positives, or 5 plus tiles. We referenced the number line experience to agree that a step in one direction followed by a step in the opposite direction left us where we began. That is, positive 1 plus negative 1 has a net value of 0. So we did a lot of simple adding with our tiles, identifying and pulling out what we called *zero pairs* to arrive at a solution for each problem. We also solved problems with pencil and paper using the same symbols and strategy. For example, (-7) + (+5) would look like this:

- - - - - - -
+ + + + +

Students then drew a loop around each zero pair and could see that what was left was the value of the answer:

(-/+) (-/+) (-/+) (-/+) (-/+) - -

All of the students were quickly able to generalize about how integer addition worked and could accurately solve problems with decimal and common fractions as well as whole numbers.

We then used the idea of zero pairs to create symbolic representations of subtraction problems that some students initially found bewildering, including one like this: (+5) – (-3).

Five positives are easy to represent: + + + + +

But there aren't any negatives to subtract. How can we solve it? Several hands went up after some thinking time, all with the same suggestion -- create them by adding zero pairs like this:

+ + + + + (+/-) (+/-) (+/-)

Students agreed that the value of the diagram above was still positive 5, but there now were three negatives that we could remove, ending up with this:

\+ + + + + + + +

So (+5) – (-3) = (+8). The discussion that followed was interesting. Some students were clearly unsure about why this was correct, even though they had understood the use of zero pairs. For most, it made sense when we put it into words: *When we subtract a negative number, the value becomes less negative. If it's less negative, then it's more positive. This connects with the subtraction we have done for years. When we subtract a positive number, the value becomes less positive.* We returned to the number lines in our text. Subtracting negatives moved us upward (or to the right, if the number line was horizontal.) Subtracting positives moved us downward (or to the left). It all fit together. Nothing was done by rote. We could draw it, we could show it with tiles, and we could demonstrate it by moving on a number line. Students could re-create those things for themselves if they felt the language-based rationale fading out in their heads.

So far, so good. But the next section of our text presented multiplication and division. A box at the top of the page said, "You can follow these rules when multiplying or dividing integers:" and then gave eight rules with no additional explanation:

+ × + = +	+ ÷ + = +
+ × – = –	+ ÷ – = –
– × + = –	– ÷ + = –
– × – = +	– ÷ – = +

There was no attempt to connect multiplication to addition or division to subtraction. The eight rules were really only seven rules (since multiplication can be done in any order). Students were just supposed to commit this list to memory and apply it whenever required. If they forgot one of the rules, they had no way to recover it for themselves.

We then divided the class into small discussion groups. Each group was given one of the rules and asked to find a way to justify it -- perhaps by using repeated addition or subtraction in place of multiplication or division, perhaps with logical language, or some other way. After a while, many of the groups arrived at explanations such as these:

Positive three times negative two really means that you have two negatives three times (showing it with tiles), so the answer has to be negative six. A positive number of negatives gives you a whole bunch of negatives. If you turn it around, you have positive three two times, but they're <u>*negative*</u> *times. It's really subtraction. Or you could say, since positive three times positive two is positive six, that putting a negative in there has to make the six negative because it's the opposite.*

Division is kind of confusing, so we decided to do it backwards with multiplication. Three times four is twelve, so twelve divided by three has to be four. That means positive twelve divided by negative three has to be negative four because only negative four times negative three will give you positive twelve. If only one of them is negative, the twelve would come out negative when you multiply them. And all the other combinations work *the same way. You can find it by doing multiplication and see what kind of number you need to make it come out.*

Did this take a while? It sure did. Did every student understand all of the arguments? No, but most found someplace to grab on and see the sense of it; see the connection with earlier concepts. As we worked our way through the exercises in the book and came up with real-life situations in which we might be dealing with integer computation, I could see students jotting simple equations to confirm the signs they were putting on their solutions. Eventually, all of this is likely to become generalized into rules that they can just recite, *e.g.,* a negative divided by a negative gives you a positive quotient. But most will have a strategy for rebuilding the rule if they don't immediately recall it. More important, they will know that there is a clear and understandable mathematical justification for the rules which fits neatly with and rests upon things they have learned in the past. There's no excuse for turning mathematical instruction into a collection of seemingly-arbitrary statements and mysterious algorithms.

Q: When is a "student" not a student?

A: When it's a teacher joining PCTM at the Student Rate!

Q: Huh?

A: The Pennsylvania Council of Teachers of Mathematics is offering all teachers (including current members who want to renew) a special promotion – two years for $10. You can hardly get two cups of coffee at your local Starbucks for that! The regular rate is $20 a year for teachers, so do the math . . . we'll wait. Yes, it's a bargain. You get a magazine, reduced registration fees for PCTM conferences, professional development opportunities, and more. There is a membership application on page 20. Act now, and pass the word to your colleagues.

Smith Numbers and a good blog to check out . . .

Here's a problem posted on the *Math Mamma Writes* blog (http://mathmamawrites.blogspot.com). It could give your students experience finding prime factorizations and working with digital roots while solving an interesting puzzle.

A Smith number is a composite integer, the sum of whose digits is equal to the sum of the digits in its prime factorization. 58 will help us to get a better grasp on that definition:

The prime factorization of $58 = 2 \times 29$.

$5 + 8 = 13$ and $2 + 2 + 9 = 13$.

So 58 passes the test and is a Smith number.

What is the only Smith number that's less than 10?

There are four more two-digit Smith numbers. Can you find them?

History note: Smith numbers were named by Albert Wilansky of Lehigh University. He noticed that his brother-in-law's 7-digit phone number had this property. Some mathematicians notice stuff like that. His brother-in-law's last name is Smith.

TIMEWASTERS

Don Scheuer
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It occurred to me that as I write the column for this issue that I have now been doing this for a third of a century. That totals up to around 100 columns give or take a few. Based on that statistic, I have decided that I will retire from doing this at the end of the next one-third century or the next 100 columns, give or take a few, whichever comes first.

As I write this on April Fool's Day, I am quietly anticipating the end of the chilly weather and the greening of the world around us. There's nothing like the sight of a freshly mowed fairway (a place on which I spend little time, despite the fact that I am golfing). Maybe this spring will bring more chances for me to explore this part of the course.

One lone problem-solver came through at the last minute with regard to the baubles offered in the last issue. Tom Waltrich of Gwynedd Mercy Academy High School scored a hat trick with the last bunch of puzzlers. Here are the previous problems and a solution for each.

A. **The sequence 13, 20, 32, 47, 70, 104, 157, 240, 372 . . . has terms which are the sum of corresponding terms of two sequences, one of which is an arithmetic progression and the other a Fibonacci-type sequence in which every term after the first two is the sum of the two previous terms. Find the first nine terms of the arithmetic and Fibonacci-type sequences for which this sequence is the sum.**

Solution: If A is the first term of an arithmetic progression and B the common difference, then:

$T_1 = A$, $T_2 = A + B$, $T_3 = A + 2B$, $T_4 = A + 3B$, etc. and

If $U_1 = C$ and $U_2 = D$ are the first two terms for the Fibonacci sequence, then:

$U_3 = C + D$, $U_4 = C + 2D$, $U_5 = 2C + 3D$, etc.

From the given condition regarding the sequences, four equations in four unknowns can be set up.

$A + C = 13$

$A + B + D = 20$

$A + 2B + C + D = 32$

$A + 3B + C + 2D = 47$

The solution set for these equations is $A = 5$, $B = 4$, $C = 8$ and $D = 11$

Then, the first nine terms of the arithmetic progression are:

5, 9, 13, 17, 21, 25, 29, 33, 37 and the first nine terms of the Fibonacci progression are:

8, 11, 19, 30, 49, 79, 128, 207, 335

B. **What is the largest seven-digit number that contains each of the digits 1 through 7 and has the property that the sum of any two consecutive digits is a prime number? What is the smallest such number with the same characteristics?**

Solution: Because the largest such number is required, place the 7 in the first position. Then place the largest number whose sum with 7 is a prime in the next place. That digit is 6. The largest whose sum with 6 is a prime comes next. That is 5. Following this reasoning yields the number 7652341. The smallest such number would start with 1 in the first place and the same reasoning followed for the largest number will result in the smallest, 1234765.

C. **What is the probability that a number chosen at random from the natural numbers from 1 through 1000 inclusive does not have 2, 3, 4, or 5 as a factor?**

Solution: Using the greatest integer function for each of the factors, we can set up the following:

N_2 = number of naturals with 2 as a factor = $\left[\frac{1000}{2}\right] = 500$

N_3 = number of naturals with 3 as a factor = $\left[\frac{1000}{3}\right] = 333$

N_5 = number of naturals with 5 as a factor = $\left[\frac{1000}{5}\right] = 200$

N_6 = number of naturals with 6 as a factor (2 and 3) = $\left[\frac{1000}{6}\right] = 166$

N_{10} = number of naturals with 10 as a factor (2 and 5) = $\left[\frac{1000}{10}\right] = 100$

N_{15} = number of naturals with 15 as a factor (3 and 5) = $\left[\frac{1000}{15}\right] = 66$

N_{30} = number of naturals with 30 as a factor (2,3 and 5) = $\left[\frac{1000}{30}\right] = 33$

(Note: a number that does not have 2 as a factor does not have 4 as a factor)

$N_2 + N_3 + N_5$ would count the numbers with 6, 10 or 15 as factors twice and with 30 as a factor three times.

So, $N_2 + N_3 + N_5 - N_6 - N_{10} - N_{15}$ would take care of those counted twice, but we have to add in those with factors 2, 3 and 5. So the number of naturals from 1 to 1000 inclusive which have 2,3,4 or 5 as factors is:
$N_2 + N_3 + N_5 - N_6 - N_{10} - N_{15} + N_{30} = 734$. The probability of the number NOT having 2,3, 4 or 5 as factors is then 266/1000 = .266.

And now for some fun in the sun.

A. A 5 by 5 square grid contains the numbers 1, 2, 3, 4, . . ., 25 in sequence in the successive rows. Five numbers are picked so that no two of them are in the same row or column. What is their sum? Why?

B. A person has 2 more brothers than sisters, and each of that person's brothers also has 2 more brothers than sisters. How many more brothers than sisters does that person's oldest sister have?

C. If a number is selected at random from the set of all five-digit numbers in which the sum of the digits is 43, what is the probability that this number will be divisible by 11?

Editor's note: The thump you just heard was not Don's golf ball striking a tree but the sound of the proverbial gauntlet hitting the floor. It's the last problem set of the year. Give those students who always sit in the back of the room something to do. Or tackle it yourself. Send solutions to Don at his email address: mathguy1@verizon.net.

Words from NCTM . . .

What's All This Talk about Rigor?

By NCTM President Linda M. Gojak (Reprinted from NCTM *Summing Up*, February 5, 2013)

Recently, I had a conversation with a group of math coaches who are working with elementary teachers on implementation of the Common Core Standards for Mathematics. The discussion turned to a description of rigor in the classroom. The coaches commented that many of their teachers were confused by exactly what was meant by teaching and learning with rigor. The coaches weren't sure how to respond.

Rigor in the Common Core State Standards

The word "rigor" is widely used in policy discussions, but it's rarely understood or defined, and often it merely passes as code for "better." It is interesting that the term "rigor" does not appear in the Common Core State Standards for Mathematics, although it is certainly implied. "Rigor" appears multiple times in the U.S. Department of Education's paper, "A Blueprint for Reform: The Reauthorization of the Elementary and Secondary Education Act," as well as its recent document, "ESEA Flexibility"—both of which include a call for rigorous academic content standards.

Rigor in Instruction

The coaches and I began our work of exploring the notion of rigor with an online search of the word "rigor." The thesaurus led us to a list of synonyms, including "affliction," "inflexibility," "difficulty," "severity," "rigidity," "suffering," and "traditionalism"— none of which describe characteristics of rigorous mathematics instruction. No wonder the teachers were confused! However, two additional words included in the list —"thoroughness" and "tenacity"— provided avenues for some serious thought about what "rigor" implies. We generated the following chart, which led to an interesting discussion with the classroom teachers. There are certainly other characteristics that can be added to the list.

Learning experiences that involve rigor …	Experiences that do not involve rigor …
challenge students	are more "difficult," with no purpose (for example, adding 7ths and 15ths without a real context)
require effort and tenacity by students	require minimal effort
focus on quality (rich tasks)	focus on quantity (more pages to do)
include entry points and extensions for all students	are offered only to gifted students
are not always tidy, and can have multiple paths to possible solutions	are scripted, with a neat path to a solution
provide connections among mathematical ideas	do not connect to other mathematical ideas

Continued next page

contain rich mathematics that is relevant to students	contain routine procedures with little relevance
develop strategic and flexible thinking	follow a rote procedure
encourage reasoning and sense making	require memorization of rules and procedures without understanding
expect students to be actively involved in their own learning	often involve teachers doing the work while students watch

Rigor Involves Everyone

Rigor involves all partners in teaching and learning. Teachers must consider rigor in planning lessons, tasks, and assignments. Rigorous lessons build on and extend prior knowledge. They encourage productive struggling. Although the objective of a lesson should be clear in the teacher's mind, the lesson should not focus on one correct path to a solution or even one correct answer. A rigorous lesson embraces the messiness of a good mathematics task and the deep learning that it has the potential to achieve.

Students who are successful in a rigorous learning environment take responsibility for their learning. They learn to reflect on their thinking. They persist in solving a problem when the path to solution is not immediately obvious. They recognize when they are not on the correct path and need to switch directions during the solution process. Students must learn to ask productive questions rather than expecting to be shown how to proceed. (And, teachers must answer those questions with just enough information to move students forward while preserving the challenge of the task!

Rigorous teaching and learning require rigorous formative assessment throughout a unit so the teacher knows what the student has learned and can plan additional activities, or adjust them, to address student needs. Students also have a role in formative assessment—they must approach tasks with tenacity and ask clarifying questions when they are unsure how to proceed. All assessments must include opportunities for students to demonstrate the processes and practices in their approach to doing mathematics. Good formative assessment can be incorporated into daily instruction and prepare students for the summative assessments that take place at certain points throughout the unit of study.

Moving toward Rigor

How can we support classroom teachers and pre-service teachers (pre-K–16) in working toward greater rigor in mathematics instruction? Professional development experiences that model rigor through the use of rich tasks, rich discourse, and good questions allow teachers to experience rigorous instruction. When selecting tasks, teachers must be sure that mathematical ideas are explicit and the connections are clear. The days of a few word problems at the end of multiple skill exercises in the textbook are over! Concepts must be introduced and explored in contexts that are interesting and motivating for students. Tasks must provide entry points for all students, offer them well-defined opportunities to make connections to other mathematics, and include both opportunities and expectations for them to develop deeper understanding. The focus and coherence of the Common Core State Standards lead the way to rigorous instruction. It is time for us to begin the journey.

Technology Corner

Apps and More for the Math Classroom

The Commonsense Media organization's website contains reviews of educational apps, games, and websites that they believe are worthwhile. It contains information about the target age group, topic, rating, and platform. Some of their age group suggestions may seem unnecessarily restrictive to you. For example, *Dragon Box* (reviewed in the Winter 2013 issue of this newsletter) is listed for students age 7 to 8 and is not included in their recommendations for anyone older. But it's definitely interesting and suitable for middle school students. For that reason, as you browse their recommendations, you will probably want to look more widely than just for the age range in your classroom. Go to www.commonsensemedia.org for a very helpful list of digital resources, many of which are free. -- LH

The Cottage Problem

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The following is a problem presented to my students which focuses on Binomial Probability. I typically use simulations to get them ***into the problem***.

The Problem: Maria is planning to rent a cottage at the beach the week after graduation. She has invited seven friends to go along (and share the cost) with her. **The cottage will be affordable only if at least five friends agree to go with her.** Maria thinks there is a 75% chance that each of the friends will join her. She also thinks that each person's decision is independent of everyone else's decision.

Simulation: Use simulations to estimate the probability that ***at least five*** friends will say "yes".
The key strokes here apply to the TI-84. randInt(1,100,7)→L_1 (Affordable if 2 or fewer are more than 75.)

Typical individual student groups will report:

NOT affordable: 30 outcomes
that is, more than 2 of the seven numbers generated in L_1 are more than 75)
Affordable: 80 outcomes
(that is, 2 or fewer of the seven numbers generated in L_1 are more than 75)

Students are asked: Based on your simulations, do you think Maria should go ahead with her plans to rent a cottage at the beach for the week after graduation?

The exact theoretical probability that ***at least five*** friends will say "yes" is as follows:

$$P(x \geq 5) = C(7,5)(.75)^5(.25)^2 + C(7,6)(.75)^6(.25)^1 + C(7,7)(.75)^7(.25)^0 = 1 - binomcdf(7,.75,4) = .756$$

An extension of the problem: Find the probability that ***at least five*** friends will say "yes" if she invites:

8 friends
10 friends
11 friends
12 friends

A "LEAST SQUARES" EXERCISE FOR TI CALCULATORS

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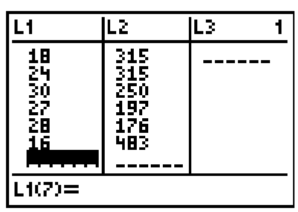
Roman Catholic High School

This is an idea for a little worksheet that might help students get a grip on the least squares concept, the definition of which seems rather vague, and also get some exercise working with lists. Given several pairs of numbers, four possible y-hat equations are given, looking much alike, and the students work out the sum of the squares of the residuals for each to see which sum is the least. Using the calculator to process the steps should give the class an appreciation of the efficiency with which it can work!

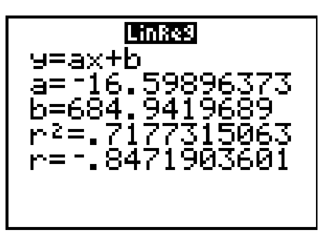
Give the students maybe six pairs of numbers and four "equation-candidates" for best fit. Set up a table for each equation (like the table three or four paragraphs below). Let the class take it from there.

What follows immediately is a description of the process on the TI grapher.

Here are my six pairs of data, shown entered into L1 and L2. To enter them, choose 1:Edit in the first STAT menu. (To delete any data already in the lists, select the heading for each column, press CLEAR, hit ENTER, and you're set.)



We, the teachers, of course, will know that the "real" equation for y-hat is, in this example, $y = -16.599x + 684.942$, rounded to three decimal places. To verify this, press STAT, CALC, 4:LinReg(ax + b), ENTER, then L1 (2ND 1) for the Xlist, a comma, and L2 (2ND 2) for the Ylist. Press ENTER until you get to this screen:



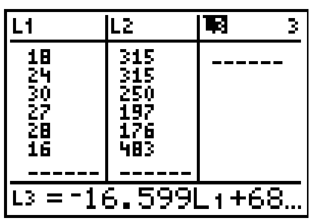
Compare the values for a and b with the "real" equation above. Of course, the students can do this, too, but it didn't register with my crew until I pointed it out.

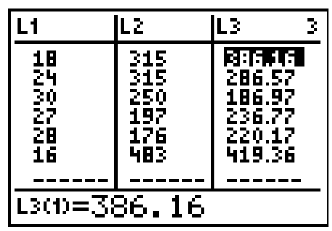
What we want to do is give four possibilities for y-hat, including the "real" one, with the values for a and b in the other three equations close to the correct numbers. Here is the process. (This table is more detailed than the ones to be printed on the worksheet.)

L1 *x*	L2 *y*	L3 (predicted value) *y*	L4 (residual) *y* – *y*	L5 (residual squared) $(y-y)^2$
		-16.599 L1+684.942	L2 – L3	$(L4)^2$
18	315	386.16	-71.16	5063.74
24	315	286.57	28.434	808.49
…	…	…	…	…
			sum of residuals >	17427.63

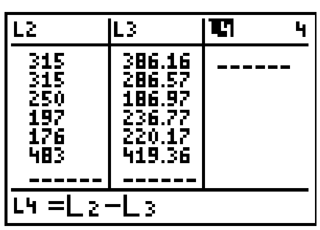
For the third through fifth lists, select the heading of the column, as shown below, then fill in the needed expression (which will be seen below the lists) and press ENTER.

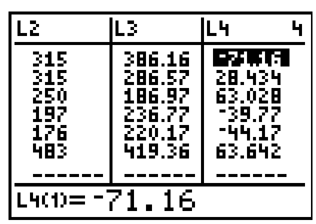
L3 yields the predicted values for the equation we're testing, corresponding to the x-values in the first list. Be sure to use L1, not X. Select the heading for L3. This enables you to type in the equation at the bottom of the screen. Press ENTER and the column is filled up.



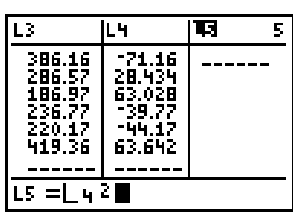


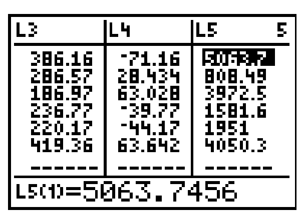
L4 calculates the residual for each *x*, *y* – *y* .



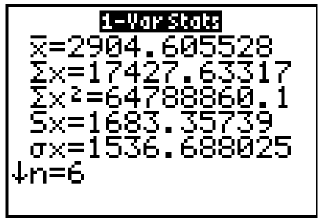


L5 squares the residuals.





To get **the sum of the squares of the residuals**, that is, the sum of the entries in list 5, select STAT, CALC, 1:1-Var Stats, ENTER, L5, ENTER. The number in the second row, $\Sigma x = 17427.63317$, is our quarry. See this in the next figure. Or you could just add them up!



Keeping L1 and L2 and processing $y = -17.010x + 684.940$ yields a sum of 18030.8501; $y = -16.571x + 684.941$, sum 17430.41627 (very close, but no cigar!); and $y = -16.672x + 684.941$, sum 17446.69151.

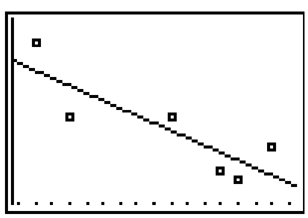
The tables the students will fill in on the worksheet don't have to include L1 and L2, since these are the same for all. Just print something like my first figure with instructions at the top of the page, and then construct tables for just L3, L4, and L5:

L3	L4	L5
	Sum of squares of residuals:	

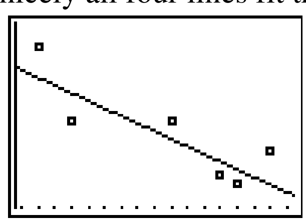
The four tables will fit on one side of a sheet of paper!

The *last* step is to ask the students to identify the line with the least sum, which would have the best fit of the four equations given.

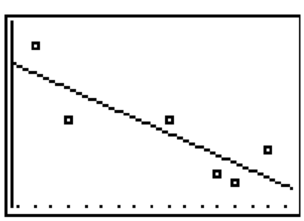
Have the class plot the points with each line and see how nicely all four lines fit the six points:



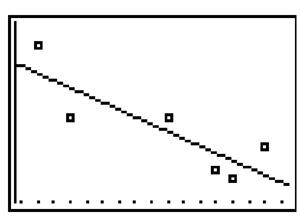
$y = -16.599x + 684.942$



$y = -17.01x + 684.94$



$y = -16.571x + 684.941$



$y = -16.672x + 684.941$

My class *assured* me that they found this exercise helpful in understanding the concept.
Some notes:

When I started this article, I used $y = -16.60x + 384.92$, only rounding to two decimal places. I was startled when my next equation had a smaller sum! That slope rounded more closely to the actual slope than my estimate had.

Sometimes the entries in the lists will be given in scientific notation, say, $9.1\text{E}4 = 9.1 \times 10^4$.

If you select an entry in a list for some reason, it will also appear at the bottom of the screen, but with many more decimal places.

I hope you find this useful!

And another note:

A follow-up on the last issue's article concerning how different TI-84s could be:

A reader, Stephen Demos, e-mailed me to share this with me: "Any TI-84 can be updated to the newest operating system with simple link to a calculator that has it. (Just Send OS.)" He goes on to say you can tell which version you have by choosing memory, and about. I thank him, his students, and colleagues for sharing this. I humbly apologize for losing the name of your high school, Steve!

A continuing dialogue on . . .

Pedagogy in the Mathematics Classroom

Editor's note: One of the suggestions to improve our newsletter that was made by the NCTM Affliliate Services Committee was to publish more articles on *pedagogy*, or "the art and science of teaching; education; instructional methods" (source: dictionary.com). Although some would argue that this term applies only to teacher-directed, didactic instruction, we choose to interpret it as a label for what we should be doing to make our classrooms joyous, student-focused, centers of durable and independent learning – however that may be achieved. So we invite you to share your methodology, perhaps talk a bit about the population that you serve, and give us some insight into why it works for them. Thanks very much to Sherry Paris for offering her ideas below. We hope other teachers will send us anything from a few sentences to a full article to be published in future issues of this newsletter.

Pedagogical Ideas, Chapter 2

Sherry Paris

Strath Haven High School

Act it out: Whenever possible, have students come up to the front of the classroom to dramatically model a real-life situation. This helps them understand the scenario, helps them see a relevance to the content, and helps solidify the concept. For example, in a lesson on graphing absolute value, I have one student stand at the front of the room as a "tower building" and another walk ("drive at a constant speed") past the tower, starting at some distance to the right and ending at some distance to the left of the tower. Students then plot the distance versus time. Another great idea is to play "math pictionary" or "Geometry Draw-it" where one student acts out or draws the geometry topic and another student guesses.

More "getting physical": One of my favorite activities is to have students model functions (and their end behavior) with their arms and hands. We all stand and face the front of the room. With arms held out at shoulder level, the right arm is indicated as the positive "x" direction. The left arm is indicated as the negative "x" direction. From there, we hold up arms to represent linear, quadratic, cubic, radical, absolute value, and even rational functions. As the teacher, I look over my shoulder to assess student progress and call out statements like "no chicken wings!" if students barely hold their arms up for quadratics.

Get musical: On a rare instance when I'm inspired, I make up a song about a math topic (such as standard deviation or graphing rational functions). I call them "math raps" and my students love them! I usually ask for one or more students to help keep the beat by clapping or drumming or doing "beatbox" while I sing (recite). Then, I have students repeat the song with me to help them remember the mathematical ideas. Challenge yourself to try it!

Cooperation Counts: I structure my classroom around cooperative groups of 4. Students help each other with problem-solving and process. Sometimes, I will work with one student on an item and then ask them to teach it to their group-mates. A great activity that I learned from a colleague is to give students a set of review problems where they each work individually for 15 minutes then then collaborate with each other to make sure all answers are correct since only one paper is checked. This promotes great mathematical communication!

Students activate selves, activate peers: There are some awesome Formative Assessment techniques. Here are a few of my favorites: calling on randomly selected volunteers by using craft sticks with student names on them; student self-assessment on topics using traffic light colors (green = good to go; yellow= slow down and practice; red=stop and learn); asking students a question and having them respond with thumbs up, thumbs down or thumbs sideways (partial understanding); using mini whiteboards to have students solve a few quick problems as a group; having students write practice problems for a neighbor and then swap (and check each other's work which increases communication and ownership); using entrance tickets or exit tickets to ask a quick question.

Collaborate with Colleagues: What is your favorite classroom activity? Do your colleagues know that? Can we make time to communicate and collaborate with each other regularly to share best practices? I have learned about some great activities from colleagues which help my students learn.

President's Message (continued)

The final award of the evening was presented to my friend and mentor Ruth Carver, winner of her second Past President's Award. The most impressive aspect of Ruth's leadership is that she has taken the words of Ralph Nader very seriously, "Start with the premise that the function of leadership is to produce more leaders, not more followers." Tom Waltrich (our newest board member), Bob Lochel (our second vice president), and Beth Benzing (our first vice president) were each recruited and groomed by Ruth to insure the stability of our organization. She maintains high expectations for all of those who work with her. Whether you are on a board with Ruth or you teach with her, you will rise to the occasion. She makes each of us better.

In closing, I would like to encourage you to embrace April as *Mathematics Awareness Month*. Let the ATMOPAV award winners and presenters *be* our inspiration to set professional development goals this month. Get out there and do something for yourself and your students. Visit the National Museum of Mathematics in Manhattan, make plans now to attend the PCTM fall conference (or better yet, submit a workshop proposal), apply for an ATMOPAV mini-grant for an innovative classroom activity, or visit the Math Counts website.

See you all on October 26th at Hatboro-Horsham High School for the Fall Conference!

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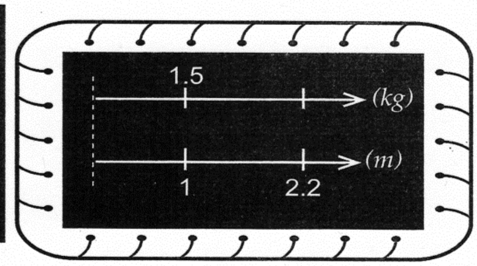
Double Number Line Diagrams: A Visual Reasoning Tool

by Tad Watanabe, Kennesaw State University
twatanab@kennesaw.edu



In the December 2012 issue of the *ComMuniCator,* I discussed what tape diagrams are and how they can be used to solve problems. In this article I discuss double number line diagrams and how they can also be used to solve problems.

In the Common Core State Standards (CCSS) for Grade 6, you will find the following standard:

> *6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.*

Many teachers are familiar with ratio tables and equations, but may not be familiar with double number line diagrams and how these representations can support students' ratio and rate reasoning.

Since there is no entry in the CCSS Glossary on double number line diagrams, why are they included explicitly in this standard? Although these diagrams may not be familiar to teachers in North America, they are often standard features of some Asian mathematics curriculum materials; see, for example, Watanabe, et al. (2010) for how Japanese elementary school materials develop these models. Furthermore, these models are gaining more attention in North America (Beckmann and Fuson 2008). The purpose of this article is to provide a brief explanation of these diagrams and to illustrate how they may be used and for what types of problem situations. These diagrams may become powerful reasoning tools for students that go beyond ratio and rate reasoning in the 6th grade. While several examples from Japanese textbooks will be included, similar problems are found in textbooks in North America.

Double Number Line Diagrams

As the name suggests, a double number line diagram involves a pair of number lines. If the rectangular coordinate system is made up of two intersecting number lines with 0 as the point of intersection, a double number line diagram is a pair of parallel number lines that are "hinged" at 0 as shown in ***Figure 1***.

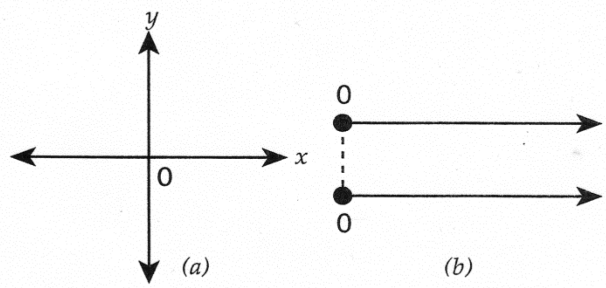


Figure 1. *A rectangular coordinate (a) and a double number line diagram (b).*

How is a double number line diagram used? Consider the following word problem from a Japanese elementary school textbook (Hironaka and Sugiyama 2006, 5A, p. 29).

> *There is a 1 m pipe that weighs 2.17 kg. How much would 2.8 m of the pipe weigh?*

Since this problem involves quantities with two different units (or referent), a single number line is not sufficient. By using a double number line, with one number line for weight and the other for length, the relationship among the quantities in this problem situation may be represented as shown in ***Figure 2***.

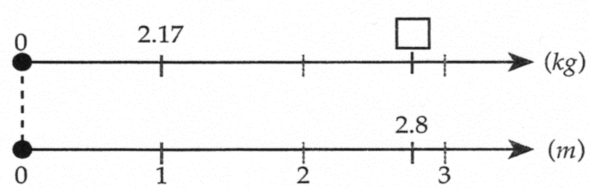


Figure 2. *A double number line diagram to represent a multiplication of a decimal number problem.*

Although this double number line diagram includes tick marks for 2, 3, and their corresponding points on the other number line, they are not necessary. What this diagram shows is that, since the length has become 2.8 times as long, the weight should also be 2.8 times as heavy.

Various researchers have shown that students often have difficulty in deciding the appropriate operation when a problem involves multiplying or dividing by a number less than 1, but greater than 0. One way that double number line diagrams may support students' problem solving is by providing a visual cue for the selection of the appropriate operation. One of the well-known misconceptions middle grade students often have is "multiplication makes bigger and division makes smaller." In other words, students often have difficulty when a problem involves multiplication or division by a positive number less than 1. A double number line diagram can visually illustrate the relationship between the product and the multiplier or the quotient and the divisor. The problems in ***Figure 3*** are both division problems (240 ÷ 1.2 and 240 ÷ 0.8) since they are determining the multiplicand, that is, cost per 1 meter. The diagrams visually represent why division by a positive number less than 1 will result in a quotient (the cost corresponding to 1 meter) that is greater than the dividend.

The white ribbon costs 240 yen for 1.2 m and the blue ribbon costs 240 yen for 0.8 m. Which ribbon costs more for 1 m?

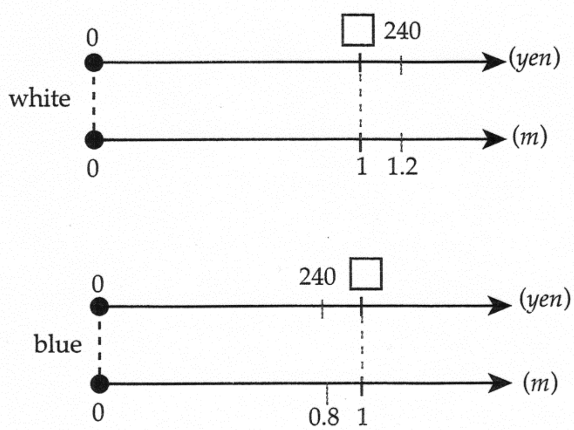


Figure 3 *These double number line diagrams visually illustrate why the quotient (the value corresponding to 1) will be greater than the dividend (240) when the divisor is less than 1 (Hironaka and Sugiyama 2006, 5A, p. 43).*

Although the use of double number line diagrams in Japanese curriculum materials begins with multiplication and division in middle grades, they can be used to represent situations in which two quantities are proportionally related. In fact, Japanese mathematics educators will argue that a major reason for teaching multiplication and division of rational numbers is to help students understand those operations from a proportion perspective. Some Japanese mathematics educators will refer to double number line diagrams as proportion number line diagrams.

Because double number line diagrams are used to represent proportional relationships, they are hinged at 0. Moreover, when quantities are represented on a double number line, the multiplicative relationships that are true among quantities on one number line will be true among the corresponding points on the other number line. It is also the case that the multiplicative relationship between two corresponding numbers will be constant across the double number line. Those relationships are summarized in ***Figure 4***.

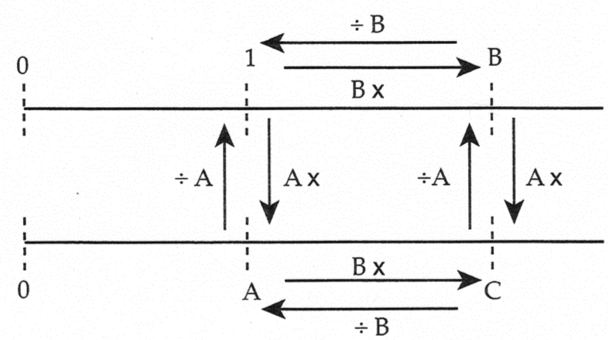


Figure 4 *A multiplicative relationship between two numbers on the top number line also applies to the two corresponding numbers on the bottom number line.*

Although double number line diagrams for a multiplication or a division problem situation will usually include a "1" on one of the number lines, that is not the case in general proportion problems. For example, consider the following word problem.

With 6 gallons of gasoline, Mike's car can travel 250 miles. How far can he travel with 4 gallons of gasoline?

This is a typical missing value proportion problem. This problem situation can be represented on a double number line diagram like the one shown in ***Figure 5***. Note that the two number lines may be reversed, putting miles on top and gallons on the bottom.

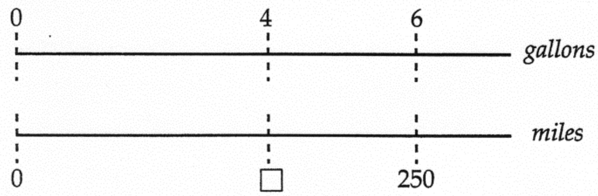


Figure 5. A double number line diagram for a missing value proportion problem.

One superficial way of using a double number line diagram will be to use it as a visual cue to set up a proportion, $\frac{4}{x}=\frac{6}{250}$, or $\frac{4}{6}=\frac{x}{250}$. However, if students can make double number line diagrams as their own thinking tools, some of them might be able to reason as follows. An obvious difference between this diagram and others that involved multiplication or division is that there is no "1" in this diagram. However, there is a point corresponding to 1 on each number line. So, what if we write the 1 on one of them, say on the number line for gallons. Then, if we ignore the correspondence between 4 and the unknown quantity, □, they know that the number corresponding to 1 can be found by dividing 250 by 6 (***Figure 6a***). Once they calculate $250 \div 6 = 41\frac{4}{6}$ they can ignore the correspondence between 6 and 250. Now they can see that the missing quantity can be found by multiplying $41\frac{4}{6}$ by 4 (***Figure 6b***).

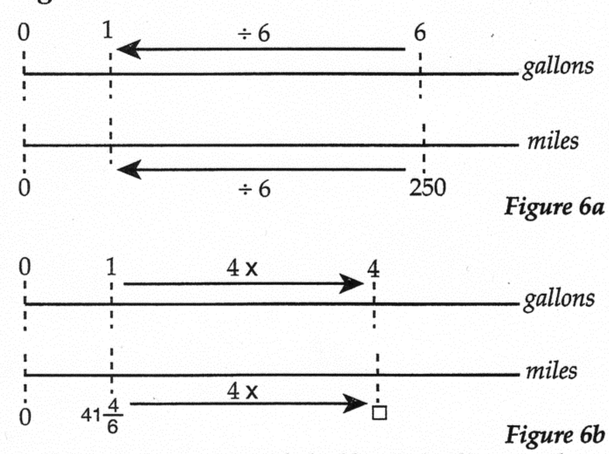


Figure 6. *Reasoning with double number lines to solve the missing value proportion problem.*

In Closing

The examples and explanations in this article are designed to provide you with some basic ideas of double number line diagrams. The Japanese textbook series from which many of the examples came tries to help students make these models as their own reasoning tools, not just teachers' explanation tools (Watanabe, et al. 2010). Such an effort takes time, across grades.

The examples in this article demonstrate the potential power of double number line diagrams. However, if we want students to make the most out of this tool in middle school, they will have to be familiar with these diagrams. Thus, if your students have not seen these diagrams previously, it is important that you provide opportunities for them to become familiar with these diagrams, using mathematics that they have already learned. Moreover, as the examples in the article illustrate, these models can be used in working with problems in earlier grades. Thus it may be useful for teachers across grades to collaborate and think about when and how these diagrams are introduced and how to support students' development of these models as their own reasoning tools.

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MATHCOUNTS 2013

Pennsylvania State MATHCOUNTS Competition – March 23, 2013

Individual Student Results (Top Ten Students)

Place	Student	Chapter/County	School
1*	Yogeshwar Velingker	Lehigh Co.	Orefield Middle School
2*	Michael Wu	Delaware Co.	Garnet Valley Middle School
3*	Vamsi Saladi	Chester Co.	Valley Forge Middle School
4*	Jonathan Xu	Chester Co.	Tredyffrin Easttown M. School
5	Maxwell Aires	Montgomery Co.	Cedarbrook Middle School
6	Joseph Feffer	Central Chapter	Mt. Nittany Middle School
7	Brady Munroe	Chester Co.	Tredyffrin Easttown M. School
8	Eric Wei	Chester Co.	Valley Forge Middle School
9	Thomas Huck	Chester Co.	Tredyffrin Easttown M. School
10	Christopher Yang	Montgomery Co.	Pennfield Middle School

**The top four students will represent Pennsylvania at the National MATHCOUNTS Competition in Washington DC, May 9-12, 2013. (Alternate student is #5.)*

Special Awards (Written Competition)

	Individual	*Chapter/County*	*School*
Highest Scoring Female:	Erica Wang	Harrisburg Ch.	Hershey MS
Highest Scoring Male:	Yogeshwar Velingker	Lehigh Co.	Orefield MS
Highest Scoring in Grade 7:	Joseph Feffer	Central Ch.	Mt. Nittany MS
Highest Scoring in Grade 6:	Asher Joy	Westmoreland Ch.	Greensburg Salem MS

Top Three (School) Pennsylvania Winning Teams

Place	School Team	Chapter/County
1**	Valley Forge MS	Chester County
2	Tredyffrin Easttown MS	Chester County
3	Hershey MS	Harrisburg Chapter

** *Valley Forge MS coach Allison Long will coach PA team in National Competition*.

Winners of County MATHCOUNTS Competitions (5 Counties) – Feb., 2013

Individual Students

Bucks: John Rauen (Pennwood MS); Melissa Lu (Richboro MS); Jimmy Kim (Pennwood MS); Rishi Mago (Sol Feinstone Elem S)

Chester: Victoria Pan (Lionville MS); Thomas Huck (Tredyffrin-Easttown MS); Rohan Jhunjhunwala (Malvern Prep. S); Satyan Alex (Lionville MS)

Delaware: Niles Huang (Radnor MS); Patrick Stoyer (Garnet Valley MS); Michael Wu (Garnet Valley MS); Marina Zhang (Garnet Valley MS); Aaron Zhou (Garnet Valley MS)

Montgomery: Christopher Yang (Pennfield MS); Maxwell Aires (Cedarbrook MS); Joel White (Cedarbrook MS); Steven Qiang (Wissahickon MS)

Philadelphia: Reiyuan Chiu (JR Masterman MS); Daniel Leonard (The Philadelphia S); Sukie Chek (JR Masterman MS); Jenny Chan (JR Masterman MS)

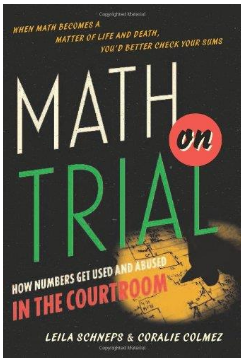
School Winning Teams (5 Counties) – Feb., 2013

Bucks: Pennwood MS **Chester**: Tredyffrin Easttown MS **Delaware**: Garnet Valley MS **Montgomery**.: Cedarbrook MS
Philadelphia: JR Masterman MS

Book Review

Math on Trial: How Numbers Get Used and Abused in the Courtroom

Authors: Leila Schneps and Corlie Colmez



This is a well-written, highly-engaging collection of carefully-analyzed cases in which mathematical arguments were used to influence the outcome of a trial. The authors, who are both mathematicians, have classified the cases by the kind of error or misunderstanding that was made. Sometimes the people presenting the numbers were confused about proper procedure, and sometimes the judges were unable to understand the validity of the argument. In many of the chapters, the story of the case is prefaced by an unrelated anecdote that helps to illustrate the mathematical concept and/or likelihood of making a mistake.

Some of the criminal and civil cases presented would be quite suitable for review and discussion in a high school classroom, although others may be inappropriate because of the details of the events. Students are likely to be more interested in the real-world application of probability to a murder trial than in yet another coin-tossing or dice-rolling experiment, and they may discover that their own confused assumptions could lead them to make a wrong courtroom decision in a matter of life and death.

The cases are a mix of well-known and obscure trials that took place in various parts of the world during the last 200 years or so, and they are given a detailed historical context. Although one need not be a mathematician to understand and enjoy this book, it will certainly give mathematics teachers assurance that their lessons are relevant and useful to all of their students – especially those who may find themselves in a courtroom where mathematical reasoning is offered along with the evidence.

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Scheps, Leila and Colmez, Corlie. ***Math on Trial.*** Basic Books. 2013.
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