

4.2

Fractions, Decimals, and Percents

Focus on...

After this lesson, you will be able to...

- ☐ convert among fractions, decimals, and percents
- ☐ estimate percent values
- ☐ distinguish between terminating and repeating decimals
- ☐ relate fractions to terminating and repeating decimals



Individual statistics from sporting events are often reported as percents or decimal numbers. It may be necessary to convert among fractions, decimals, and percents to better understand these statistics.

Discuss the Math

How can you convert among fractions, decimals, and percents?

1. Look at the statistics in the table. Which hockey goalie do you think is having the best season? Why?

Goalie	Shots on Goal	Saves
A. Auld	673	606
M. Fernandez	586	545
M. Kiprusoff	797	726
D. Hasek	709	658

2. Goalies can be rated on “save percentage.” This statistic is the ratio of saves to shots on goal.

$$\text{Save percentage} = \frac{\text{Number of saves}}{\text{Shots on goal}}$$

- a) Copy the table into your notebook. Extend the table to include two more columns.

Sports Link

Although the statistic is called “save percentage,” the result is a decimal number.

- b) In the first new column, write the save percentage for each goalie as a fraction.
 - c) In the second new column, write the save percentage as a decimal to the nearest thousandth.
 - d) Decide which goalie is having the best season. Explain.
3. a) How does the save percentage help you determine a goalie's performance?
- b) Is it better to have a higher or a lower save percentage? Explain why.
4. The save percentage is usually stated as a decimal.
- a) How are the decimal and fraction forms of the save percentage related?
 - b) Which form is more useful? Why?
 - c) Is either form an actual percent value?

Did You Know?

The first Canadian hockey sticks were modelled on Irish hurley sticks and were made by the Mi'kmaq in Eastern Canada over 100 years ago.

Reflect on Your Findings

5. Summarize methods you can use to
- a) convert a fraction to a decimal
 - b) convert a decimal to a percent

Example 1: Convert From Fractions to Decimals and Percents

The following data were gathered one season for three National Basketball Association (NBA) teams.

Team	Wins	Losses
Miami	59	23
New Jersey	42	40
Los Angeles	34	48

A statistic called “team percentage” is the ratio of team wins to total games.

$$\text{Team percentage} = \frac{\text{Number of wins}}{\text{Total games played}}$$

- a) What is the team percentage for each team? Leave your answer as a fraction.
- b) Change each fraction to a decimal number rounded to the nearest thousandth.
- c) Use your rounded decimal value to show the approximate percent value for each team.

Solution

- a) Total games = wins + losses

Miami:

$$\begin{aligned}\text{Total games} &= 59 + 23 \\ &= 82\end{aligned}$$

$$\text{Team percentage} = \frac{59}{82}$$

New Jersey:

$$\begin{aligned}\text{Total games} &= 42 + 40 \\ &= 82\end{aligned}$$

$$\text{Team percentage} = \frac{42}{82}$$

Los Angeles:

$$\begin{aligned}\text{Total games} &= 34 + 48 \\ &= 82\end{aligned}$$

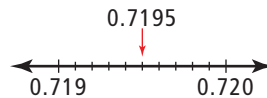
$$\text{Team percentage} = \frac{34}{82}$$

- b) Convert each fraction to a decimal.

Miami:

$$\frac{59}{82} \approx 0.720$$

$$\boxed{C} \ 59 \div 82 = 0.719512191\dots$$

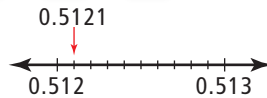


The digit to the right of the thousandths place is 5, so round up.

New Jersey:

$$\frac{42}{82} \approx 0.512$$

$$\boxed{C} \ 42 \div 82 = 0.512195121\dots$$

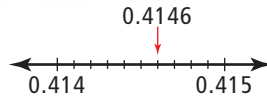


The digit to the right of the thousandths place is 1, so round down.

Los Angeles:

$$\frac{34}{82} \approx 0.415$$

$$\boxed{C} \ 34 \div 82 = 0.414634146\dots$$



The digit to the right of the thousandths place is 6, so round up.

- c) To convert to a percent, multiply the decimal by 100.

$$\begin{aligned}\text{Miami team percentage} &= 0.72 \times 100\% \\ &= 72\%\end{aligned}$$

$$\begin{aligned}\text{New Jersey team percentage} &= 0.512 \times 100\% \\ &= 51.2\%\end{aligned}$$

$$\begin{aligned}\text{Los Angeles team percentage} &= 0.415 \times 100\% \\ &= 41.5\%\end{aligned}$$

Show You Know

Convert each fraction to a decimal number. Round each decimal number to the indicated place value. Then, convert to a percent.

a) $\frac{27}{56}$ (tenths) b) $\frac{125}{396}$ (thousandths) c) $\frac{1496}{2005}$ (hundredths)

Example 2: Change Fractions to Repeating Decimals

Some common fractions may change to **repeating decimal** numbers. These decimal numbers contain one or more digits that repeat over and over without ending.

Use a calculator to change each fraction to a repeating decimal.

a) $\frac{1}{3}$ b) $\frac{5}{9}$ c) $\frac{5}{6}$

Solution

a) $\frac{1}{3} = 1 \div 3$  $= 0.\overline{3}$

Use a bar over the 3 to show the repeating part.

b) $\frac{5}{9} = 5 \div 9$  $= 0.\overline{5}$

The calculator shows the final digit as 6 because it rounds up. It would show more 5s if it had a larger display.

c) $\frac{5}{6} = 5 \div 6$  $= 0.8\overline{3}$

Place a bar over only the 3 since the 8 does not repeat.

Show You Know

Show the following fractions as repeating decimals.

a) $\frac{2}{3}$ b) $\frac{7}{9}$

repeating decimal

- a decimal number with a digit or group of digits that repeats forever
- repeating digits are shown with a bar, e.g., $0.777 \dots = 0.\overline{7}$

Example 3: Estimate Percents

Paige has answered 94 questions correctly out of 140 questions. Estimate her mark as a percent.



Solution

Think: What is 50% of 140?

Half of 140 is 70.

Think: What is 10% of 140?

$$140 \div 10 = 14$$

Add 50% and 10% parts together to estimate.

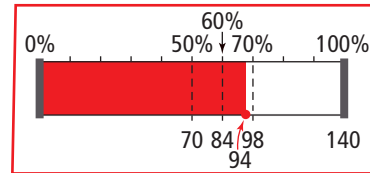
$$50\% + 10\% = 60\% \text{ of } 140$$

$$70 + 14 = 84 \quad \text{Too low.}$$

$$50\% + 10\% + 10\% = 70\% \text{ of } 140$$

$$70 + 14 + 14 = 98 \quad \text{Too high.}$$

The answer is between 60% and 70% but closer to 70%.



Strategies

Guess and Check

Refer to page xvi.

Show You Know

Estimate each of the following as a percent.

- a) 23 out of 80 b) 421 out of 560

Example 4: Change Terminating Decimal Numbers to Fractions

- a) What fraction of a dollar is \$0.75?
b) Change 0.652 to a fraction.

Solution

- a) The decimal number 0.75 is a **terminating decimal**. The last digit is in the hundredths place, so the denominator is 100.

$$0.75 = \frac{75}{100}$$

So, \$0.75 is $\frac{75}{100}$ of a dollar or $\frac{3}{4}$ of a dollar.

- b) The 2 is in the thousandths place, so the denominator is 1000.

$$0.652 = \frac{652}{1000}$$



terminating decimal

- a decimal number in which the digits stop
- examples include 0.4, 0.86, 0.125

Show You Know

Change each terminating decimal number to a fraction.

- a) 0.48 b) 0.078

Key Ideas

- To change a fraction to a decimal number, divide the numerator by the denominator.

$$\frac{3}{8} = 3 \div 8 = 0.375$$
- Repeating decimal numbers can be written using a bar notation.

$$\frac{1}{3} = 0.333... = 0.\overline{3}$$
- To express a terminating decimal number as a fraction, use place value to determine the denominator.

$$0.9 = \frac{9}{10} \quad 0.59 = \frac{59}{100} \quad 1.463 = \frac{1463}{1000}$$
- You can use mental math to estimate percents.

Communicate the Ideas

- Express 0.7 and 0.67 as fractions.
 - How are the number of digits after the decimal point for each number related to the number of zeros in the denominator of each fraction? Explain.
- Vernon was asked to estimate 63 out of 160 as a percent. He said the answer was between 40% and 50%. Do you agree? Justify your answer.
- Eleven out of 15 calves born on a ranch in one week had white faces. What percent of the calves born that week had white faces? Round your answer to the nearest tenth of a percent. Show how you got your answer.
- Kanisha said that $0.6555...$ could be written as $0.\overline{65}$ using bar notation. What mistake has she made? Show how she could write the answer correctly.

Practise

For help with #5 to #7, refer to Example 1 on page 133.

- Use a calculator to change each fraction to a decimal number. Round to the place value indicated.

a) $\frac{13}{25}$ (tenths) b) $\frac{107}{215}$ (hundredths)

c) $\frac{43}{50}$ (tenths) d) $\frac{197}{289}$ (thousandths)

- Use a calculator to change each fraction to a decimal number. Round to the place value indicated.

a) $\frac{11}{25}$ (tenths) b) $\frac{171}{280}$ (thousandths)

c) $\frac{43}{60}$ (hundredths) d) $\frac{19}{313}$ (thousandths)

7. A baseball player's batting average is the ratio of number of hits to times at bat. It is usually expressed as a decimal number.

$$\text{Batting average} = \frac{\text{Number of hits}}{\text{Times at bat}}$$

The following data are for one baseball season.

Player	Times at Bat	Hits
A	605	194
B	624	197



- What is the batting average for each player? Round each answer to the nearest thousandth.
- Which player had a better batting average for the season? Justify your response.

For help with #8 to #11, refer to Example 2 on page 135.

- Write each repeating decimal number using bar notation.
 - 0.555555...
 - 0.090909...
 - 0.1877777...
 - 2.0151515...
- Write each repeating decimal number using bar notation.
 - 0.4444444...
 - 0.2666666...
 - 0.1851851...
 - 1.0626262...
- Change each fraction to a repeating decimal. Then use bar notation to show the repeating part.
 - $\frac{5}{6}$
 - $\frac{2}{3}$
 - $\frac{45}{99}$
 - $\frac{7}{11}$
- Change each fraction to a repeating decimal. Then use bar notation to show the repeating part.
 - $\frac{1}{6}$
 - $\frac{3}{7}$
 - $\frac{25}{99}$
 - $\frac{4}{11}$

For help with #12 and #13, refer to Example 3 on page 136.

- Estimate each of the following as a percent.
 - 36 out of 70
 - 125 out of 300
- Estimate each of the following as a percent.
 - 275 out of 500
 - 46 out of 90



For help with #14 and #15, refer to Example 4 on page 136.

- Change each decimal number to a fraction.
 - 0.95
 - 0.3
 - 0.243
 - 0.08
- Change each decimal numbers to a fraction.
 - 0.80
 - 0.2
 - 0.18
 - 0.455

Apply

16.



- Express the value of each coin as a fraction of a dollar. Write the fractions in ascending order.
 - Express the value of each coin as a decimal number. Write the decimal numbers in ascending order.
 - Write a statement using percent that compares the three coins.
- For each of the following statements, rewrite each percent or decimal as a fraction.
 - 0.71 of Earth's surface is covered with water or ice.
 - Matthew's batting average is .421.
 - Nitrogen makes up approximately 78% of the air we breathe.

18. There are 81 girls and 59 boys in River City Day Care.

- Estimate the percent of the children in the day care that are girls.
- Show how you made your estimate.

19. A small town in southern British Columbia has a population of 270. Of these people, 85 are over the age of 60.

- Estimate the percent of the town's population that is older than 60.
- Show the number of people over 60 as a fraction of the town's population. Express your fraction as a decimal number correct to 3 decimal places.
- What is your decimal value from b) expressed as a percent? How does this value compare with your estimate in a)?

20. Rachel and Tim notice a pattern when changing ninths into decimal numbers. Here is what they see on their calculator.

$$\frac{1}{9} = 0.11111111 \quad \frac{2}{9} = 0.22222222$$

$$\frac{3}{9} = 0.33333333$$

- Without dividing, what do you think $\frac{4}{9}$ is as a decimal number? Write your answer using bar notation.
- Predict the decimal equivalent for $\frac{8}{9}$.
Check your answer using a calculator.

Extend

21. Look at each repeating decimal and its fraction equivalent.

$$0.363636... = \frac{36}{99} = \frac{4}{11}$$

$$0.545454... = \frac{54}{99} = \frac{6}{11}$$

$$0.636363... = \frac{63}{99} = \frac{7}{11}$$

- What would the fraction equivalent be for 0.272727...? 0.909090...? 0.818181...?
 - Write a rule to convert each repeating decimal to a fraction. Hint: Write the repeating decimals using bar notation.
 - Write four other repeating decimals that would follow the same pattern.
- 22. a)** Use a calculator to find repeating decimal numbers for $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.
- Add the numbers in the first half of each repeating part to the numbers in the last half. For example, $\frac{1}{7} = 0.\overline{142857}$, so add 142 to 857.
 - What pattern did you find?
 - What do you think will happen with $\frac{7}{13}$? $\frac{4}{11}$? Do you get the same results?

MATH LINK

Each of the headlines shown contains a fraction, a decimal, or a percent.

Which headlines would be better written using a different representation? Rewrite them with a more appropriate representation. Explain your reasoning.

Headlines

- Baseball Ticket Sales Up 0.06
- Profits Fall 31%
- $\frac{22}{100}$ of Young Teens See Parents as Role Models
- Bicycle Helmets Can Prevent $\frac{4}{5}$ Serious Head Injuries
- Prices Increase by 0.25
- Almost 0.5 Canadians Read a Daily Newspaper