Name: ____________________ Math 10F&PC H. Date: ________________

Chapter 3 Factors and Products

3.1 - FACTORS AND MULTIPLES OF WHOLE NUMBERS

Focus: Determine prime factors, greatest common factors, and least common multiples of whole numbers.

When a factor of a number has exactly two divisors, 1 and itself, the factor is a *prime factor*.
A ***prime number*** has only two factors: itself and 1.

List the first 10 prime numbers:
Numbers greater than 1 that are not primes are

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 51, 53
59, 61, 67, 71, ------, 97, 101, ..., 199, 201, ... 997

Divisibility by 2   *A whole number is divisible by 2 if the last digit in the number is even (0, 2, 4, 6, 8).*
Divisibility by 3   *A whole number is divisible by 3 if the sum of all its digits is divisible by 3.*
Divisibility by 4   *A whole number is divisible by 4 if the number formed by the last two digits is divisible by 4.*
Divisibility by 5   *A whole number is divisible by 5 if the last digit in the number is 0 or 5.*
Divisibility by 6   *A number is divisible by 6 if it is divisible by 2 and divisible by 3. Thus the last digit in the number must be even and the sum of its digits must be divisible by 3.*
Divisibility by 8   *A whole number is divisible by 8 if the number formed by the last three digits is divisible by 8.*
Divisibility by 9   *A whole number is divisible by 9 if the sum of all its digits is divisible by 9.*
Divisibility by 10   *A whole number is divisible by 10 if the last digit in the number is 0.*

1x12
2x6
3x4

Example, the factors of 12 are 1, 2, 3, 4, 6, and 12.
The prime factors of 12 are __2x3 x2__.
The *prime factorization* of a natural number is the number written as a product of its prime factors.
To determine the prime factorization of 12, write 12 as a product of its prime factors:
__$2 \times 2 \times 3$__ or __$2^2 \times 3$__ . To avoid confusion with the variable *x*, use a dot to represent multiplication instead.

Example 1:
Write the prime factorization of 3300.

Method 1:

1x3300
2x1650
3x1100
4x825
5x660
6x550
10 x330

2 | 3300
2 | 1650
3 | 825
5 | 275
5 | 55
11

Method 2: Factor tree

3300
2, 1650
2, 825
3, 275
5, 55
5, 11

$2 \times 2 \times 3 \times 5 \times 5 \times 11$
$2^2 \times 3 \times 5^2 \times 11$

For 2 or more natural numbers, we can determine their *greatest common factor*, which is the greatest factor the numbers have in common. GCF

Example 2:

Determine the greatest common factor of 138 and 198. ✓

Method 1:

138 ⇒ 1x138 – 2x69 – 3x46 – 6x23

138 = 2x3x23

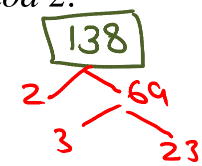
198 ⇒ 1x198 – 2x99 – 3x66 – 6x33

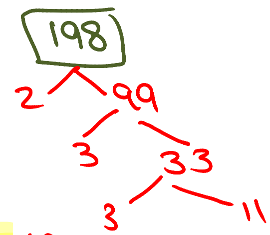
9x22 – 11x18

198 = 2x3x3x11

$\boxed{GCF = 6}$

Method 2:





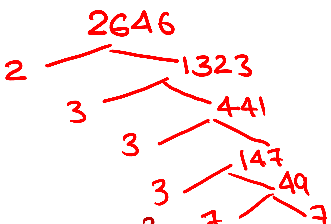
2x3x23

2x3x3x11

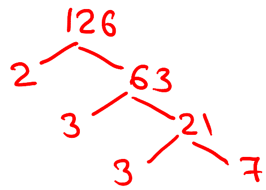
GCF = 2x3 = $\boxed{6}$

Example 3:

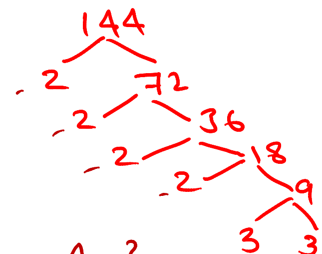
Write the prime factorization of 2646, 126, and 144. Determine the greatest common factor of 126 and 144.



$2646 = 2 \times 3^3 \times 7^2$



$126 = 2 \times 3^2 \times 7$



$144 = 2^4 \times 3^2$

GCF $= 2 \times 3^2 = \boxed{18}$

To generate multiples of a number, multiply the number by the natural numbers; that is, 1, 2, 3, 4, 5, and so on.

For example, some multiples of 26 are:

26·1= 26 26· 2 = 52 26·3 = 78 26·4 =104

For 2 or more natural numbers, we can determine their ***least common multiple*** (LCM), which is the least number that is divisible by each number.

We can determine the least common multiple of 4 and 6 by combining identical copies of each smaller chain to create two chains of equal length.

4 = 2x2 6 = 2x3

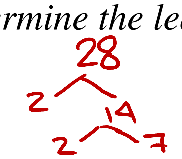
What is the least common multiple of 4 and 6? 2x2x3 = $\boxed{12}$

Example 4:

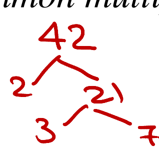
Determine the least common multiple of 18, 20, and 30.

Method 1: *Method 2:*

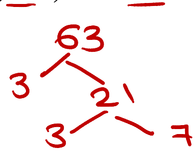
Try this: *Determine the least common multiple of 28, 42, and 63.*



$2^2 \times 7$



$2^1 \times 3^1 \times 7^1$



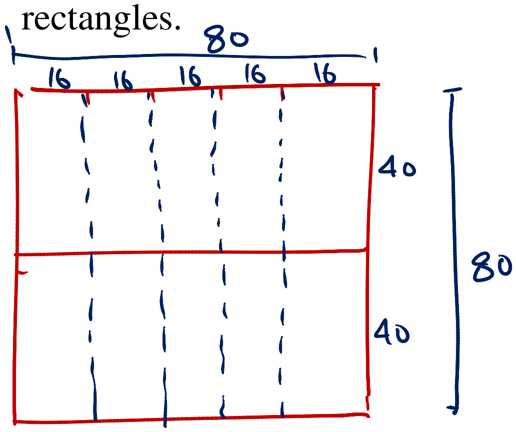
$3^2 \times 7^1$

$LCM = 2^2 \times 3^2 \times 7$

$\boxed{LCM = 252}$

Example 5:

a. What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut. Sketch the square and rectangles.



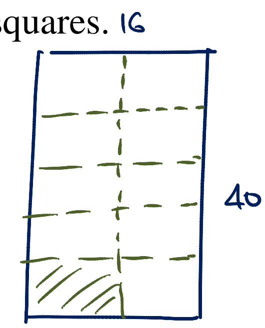
Find the L.C.M between 16 & 40

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

$40 = 2 \times 2 \times 2 \times 5 = 2^3 \cdot 5^1$

$LCM = 2^4 \times 5^1 = 80$ cm

b. What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut. Sketch the rectangle and squares.



Find the GCF between 16 & 40

$16 = 2^4$

$40 = 2^3 \times 5^1$

$GCF = 2^3 = \boxed{8 \text{ cm}}$

Scientific Notation

1. Write the following numbers in scientific notation:

a) 5,500,000,000 5.5×10^9

b) 780 23,010,000 436 7.802301×10^{10}

c) 0.091 9.1×10^{-2}

d) 0.000003004 3.004×10^{-6}

2. Write the following numbers in regular notation:

a) 5.5×10^{-7} 0.00000055

b) 7.1×10^{10} 71,000,000,000

c) 1.0×10^3 1000

3. Compute the following:

a) $10^3 \times 10^5 = 10^8$

b) $4 \times 10^{-3} \times -5 \times 10^{-5} = -2.0 \times 10^{-7}$

$a^x \cdot a^y = a^{(x+y)}$

c) $10^{-3} \times 10^5 = 10^2$

d) $(8.0 \times 10^5)(1.2 \times 10^8) = 9.6 \times 10^{13}$

e) $10^3 \div 10^5 = 10^{-2}$

f) $2.3 \times 10^{-3} \div 1.0 \times 10^{-5} = 2.3 \times 10^2$

g) $10^{-3} \div 10^5 = 10^{-8}$

h) $(8.0 \times 10^5) \div (1.2 \times 10^8) = 6.67 \times 10^{-3}$

i) $(3 \times 10^8)^2 = 9 \times 10^{16}$

j) $\sqrt{4 \times 10^8} = 2 \times 10^4$

4. The mass of an electron is 9.1×10^{-31} kg. What is the mass of 3.2×10^{3} electrons?

$(9.1 \times 10^{-31})(3.2 \times 10^{3}) = 2.912 \times 10^{-27}$

5. The velocity of light is 3.0×10^{8} m/s. How long does it take light to travel to the moon? (The distance from the earth to the moon is 3.84×10^{8} m.) Give your answer in seconds and in minutes.

$t = \frac{d}{v} = \frac{3.84 \times 10^{8}}{3.0 \times 10^{8}} = \boxed{1.28 \text{ sec}}$

Expand.

1. $(x - 2)(x + 4)$

2. $(3x + y)(x - 4y)$ FOIL

3. $(x + 4)^2$

4. $(y - 4)(y + 4)$

5. $(2x - 3y)^2$

6. $(5a + 3b)(5a - 3b)$

7. $(x + 3y)(x + 4y)$

8. $(5m - 2n)(7m - n)$

9. $(4m - 3n)(-n + 6m)$

10. $(2y + 5)(3y^2 - 2y)$

Assignement: p. 140 #4, 6, 8 – 19, 21 (odd letters)