Name: ____________________ *Math 10F&PC H.* Date: ________________

Chapter 2 Trigonometry

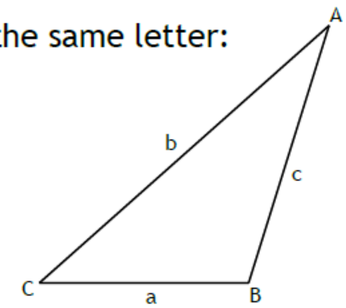
2.8 - The Sine Law

Non-Right Triangles

- Sometimes, you will encounter triangles that are not right triangles.
- Remember, SOH CAH TOA and the Pytheorem **only apply to right triangles**
- For non-right triangles, we need new tools: ***The Sine Law & The Cosine Law***

Labelling Triangles

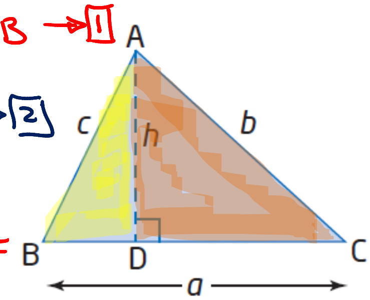
- Remember, an angle and the side opposite have the same letter:



Sine Law

- When finding an angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- When finding a side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Proof: Using the following diagram, prove the Sine Law…



In $\triangle ABD$ $\quad \sin\angle B = \frac{h}{c} \Rightarrow h = c \sin\angle B \rightarrow \boxed{1}$

In $\triangle ACD$ $\quad \sin\angle C = \frac{h}{b} \Rightarrow h = b \sin\angle C \rightarrow \boxed{2}$

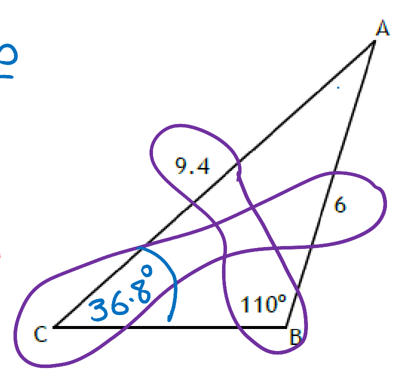
Eq # ① = Eq # ② $\Rightarrow = h$

$\frac{\cancel{c}\sin\angle B}{b\cancel{c}} = \frac{\cancel{b}\sin\angle C}{\cancel{b}c}$ divide by bc

$\frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1– Solve the following triangle.



$\frac{\sin\angle C}{c} = \frac{\sin\angle B}{b}$ $\qquad \sin\angle C = \frac{c \sin\angle b}{b}$

$\angle C = \sin^{-1}\left(\frac{6 \sin 110^\circ}{9.4}\right) = \sin^{-1}(0.5998)$

$\angle C = 36.85 \simeq \boxed{37^\circ}$ $\qquad \angle A = 180 - (110^\circ + 37^\circ)$

$\angle A = 33^\circ$

$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B}$ $\quad \therefore \quad a = \frac{b \sin\angle A}{\sin\angle B}$

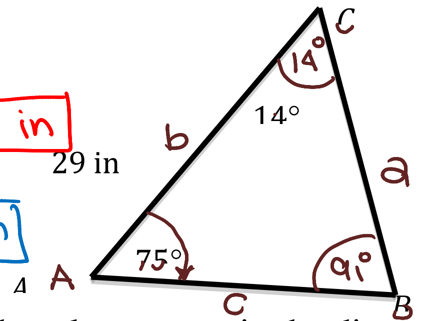
$a = \frac{9.4 \sin 33^\circ}{\sin 110^\circ} = 5.46 \simeq \boxed{5.47}$

Example 2: Identify which side can be found using sine law and calculate its value:

$\angle B = 180^\circ - (14^\circ + 75) = \boxed{91^\circ}$

$\frac{c}{\sin 14^\circ} = \frac{29}{\sin 91}$ $\quad c = \frac{29 \sin 14^\circ}{\sin 91^\circ} = \boxed{7.01 \text{ in}}$

$\frac{a}{\sin 75^\circ} = \frac{29}{\sin 91^\circ}$ $\quad a = \frac{29 \sin 75}{\sin 91} = \boxed{28.01 \text{ in}}$



Example 3: Find the indicated values of the variable and angle measure in the diagram below.

$\angle A = 50^\circ$
$\angle B = ?$
$b = 4.0$
$a = 4.5$
$c = ?$
$\angle C = ?$

[1] Find $\angle B = \theta$

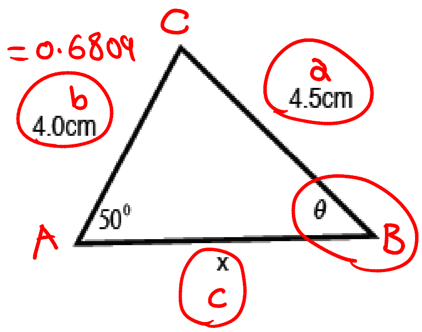
$\frac{\sin B}{b} = \frac{\sin A}{a}$ $\quad \sin B = \frac{4.0 \sin 50^\circ}{4.5} = 0.6809$

$\angle B = \sin^{-1} 0.6809 = 42.9 \simeq \boxed{43^\circ}$

[3] Find $\angle C = 180 - (50^\circ + 43) = \boxed{87^\circ}$

[3] Find side $c = x$ $\quad \frac{x}{\sin 87} = \frac{4.5}{\sin 50}$

$x = \frac{4.5 \sin 87}{\sin 50} = \boxed{5.86}$



Example 4

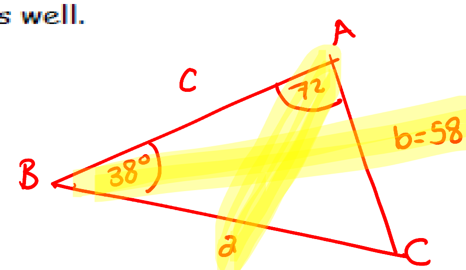
$\triangle ABC$ has the following angle measures and edge lengths. b=58cm, $\angle B = 38^\circ$, $\angle A = 72^\circ$. Draw and label a diagram then determine the length of a and the measure of $\angle C$. Find the Area of the triangle as well.

$b = 58$ cm
$\angle B = 38^\circ$
$a = ?$
$\angle A = 72^\circ$
$\angle C = ?$

[1] Find $\angle C = 180 - (38 + 58) = \boxed{70^\circ}$

[2] Find side a $\quad \frac{a}{\sin a} = \frac{b}{\sin B}$

$a = \frac{58 \sin 72^\circ}{\sin 38} = 90.9 \simeq \boxed{91}$



Example 5– A surveyor measures a base line PQ 440 m long. He takes measurements of a landmark R from P and Q and finds that $\angle$QPR=46^{O} and $\angle$PQR = 75^{O}

a) Calculate the perimeter of ΔPQR to the nearest metre. → Add All sides

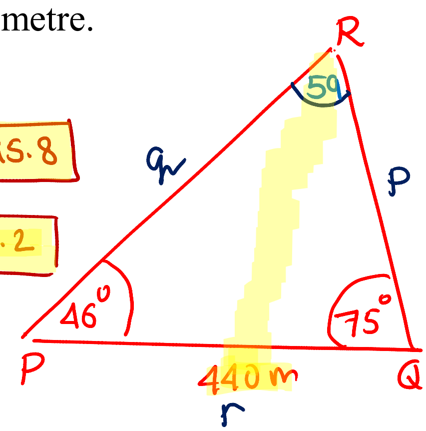
b) Calculate the area of ΔPQR to the nearest square metre.

[1] $\angle r = 180^\circ - (46^\circ + 75^\circ) = \boxed{59^\circ}$

$r = 440$ m
$\angle P = 46^\circ$
$\angle Q = 75^\circ$
Find q
Find p

- $\frac{q}{\sin Q} = \frac{r}{\sin R}$ $\quad q = \frac{440 \sin 75^\circ}{\sin 59} = \boxed{495.8}$
- $\frac{p}{\sin P} = \frac{r}{\sin R}$ $\quad p = \frac{440 \sin 46^\circ}{\sin 59^\circ} = \boxed{369.2}$

(a) Perimeter $= p + q + r = 369.2 + 495.8 + 440$
$= \boxed{1305 \text{ m}}$



Name: ____________________ *Math 10F&PC H.* **Date:** ________________

Chapter 2 Trigonometry

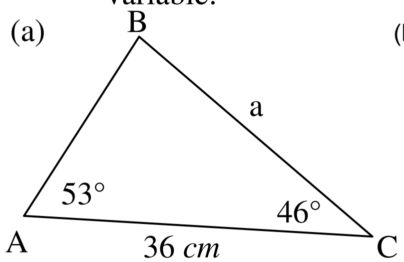
Sine Law Worksheet Assignment

1. Solve for the given variable (correct to 1 decimal place) in each of the following:

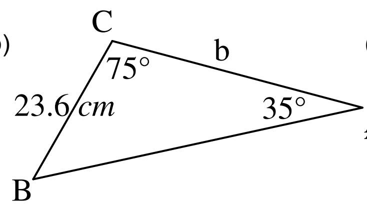
(a) $\dfrac{a}{\sin 35^\circ} = \dfrac{10}{\sin 40^\circ}$ (b) $\dfrac{65}{\sin 75^\circ} = \dfrac{b}{\sin 48^\circ}$ (c) $\dfrac{75}{\sin 55^\circ} = \dfrac{c}{\sin 80^\circ}$

2. For each of the following diagrams write the equation you would use to solve for the indicated variable:

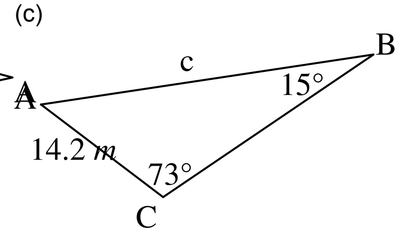
(a)



(b)



(c)



3. Solve for each of the required variables from Question #2.

4. For each of the following triangle descriptions you should make a sketch and then find the indicated side rounded correctly to one decimal place.

(a) In ΔABC, given that ∠A = 57°, ∠B = 73°, and AB = 24 cm. Find the length of AC

(b) In ΔABC, given that ∠B = 38°, ∠C = 56°, and BC = 63 cm. Find the length of AB

(c) In ΔABC, given that ∠A = 50°, ∠B = 50°, and AC = 27 m. Find the length of AB

(d) In ΔABC, given that ∠A = 23°, ∠C = 78°, and AB = 15 cm. Find the length of BC

(e) In ΔABC, given that ∠A = 55°, ∠B = 32°, and BC = 77 cm. Find the length of AC

(f) In ΔABC, given that ∠B = 14°, ∠C = 78°, and AC = 36 m. Find the length of BC

Solutions:

1. (a) 8.9 *units* (b) 50.0 *units* (c) 90.2 *units*
2. (a) $\dfrac{a}{\sin 53^\circ} = \dfrac{36}{\sin 81^\circ}$ (b) $\dfrac{23.6}{\sin 35^\circ} = \dfrac{b}{\sin 70^\circ}$ (c) $\dfrac{14.2}{\sin 15^\circ} = \dfrac{c}{\sin 73^\circ}$
3. (a) 29.1 *cm* (b) 38.7 *cm* (c) 52.5 *m*
4. (a) 30.0 *cm* (b) 52.4 *cm* (c) 34.7 *m* (d) 6.0 *cm* (e) 49.8 *cm* (f) 148.7 *m*