Name: ______________________ ***Date:*** ___________________

Math 10F&PC Chapter 3 Factors and Products

3.1 - FACTORS AND MULTIPLES OF WHOLE NUMBERS

Focus: Determine prime factors, greatest common factors, and least common multiples of whole numbers.

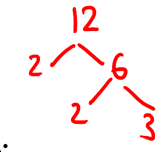
When a factor of a number has exactly two divisors, 1 and itself, the factor is a *prime factor*.
A ***prime number*** has only two factors: itself and 1.

PRIME NUMBERS TO 1000

2	3	5	7	11	13	17	19	23	29	31	37
41	43	47	53	59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131	137	139	149	151
157	163	167	173	179	181	191	193	197	199	211	223
227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409	419	421	431	433
439	443	449	457	461	463	467	479	487	491	499	503
509	521	523	541	547	557	563	569	571	577	587	593
599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743
751	757	761	769	773	787	797	809	811	821	823	827
829	839	853	857	859	863	877	881	883	887	907	911
919	929	937	941	947	953	967	971	977	983	991	997

Divisibility by 2 *A whole number is divisible by 2 if the last digit in the number is even (0, 2, 4, 6, 8).*
Divisibility by 3 *A whole number is divisible by 3 if the sum of all its digits is divisible by 3.*
Divisibility by 4 *A whole number is divisible by 4 if the number formed by the last two digits is divisible by 4.*
Divisibility by 5 *A whole number is divisible by 5 if the last digit in the number is 0 or 5.*
Divisibility by 6 *A number is divisible by 6 if it is divisible by 2 and divisible by 3. Thus the last digit in the number must be even and the sum of its digits must be divisible by 3.*
Divisibility by 8 *A whole number is divisible by 8 if the number formed by the last three digits is divisible by 8.*
Divisibility by 9 *A whole number is divisible by 9 if the sum of all its digits is divisible by 9.*
Divisibility by 10 *A whole number is divisible by 10 if the last digit in the number is 0.*

1x12
2x6
3x4



Example, the factors of 12 are 1, 2, 3, 4, 6, and 12.

The prime factors of 12 are 2x2x3.

The *prime factorization* of a natural number is the number written as a product of its prime factors.

To determine the prime factorization of 12, write 12 as a product of its prime factors: $2.2.3$ or $2^2.3$. To avoid confusion with the variable *x*, use a dot to represent multiplication instead.

2x2x3 2xx3x
2.2.3 2x.3x

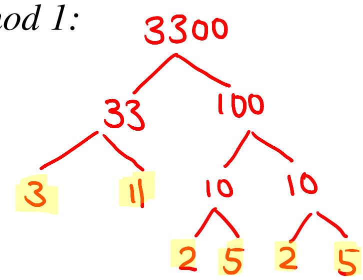
List the first 10 prime numbers: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23

Numbers greater than 1 that are not primes are composites.

Example 1:

Write the prime factorization of 3300. $= 2.2.3.5.5.11 = 2^2.3.5^2.11$

Method 1:



Method 2:

2 | 3300
2 | 1650
3 | 825
5 | 275
5 | 55
11

For 2 or more natural numbers, we can determine their *greatest common factor*, GCF which is the greatest factor the numbers have in common.

Example 2:

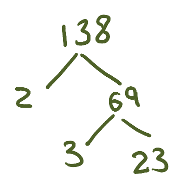
Determine the greatest common factor of 138 and 198.

Method 1:

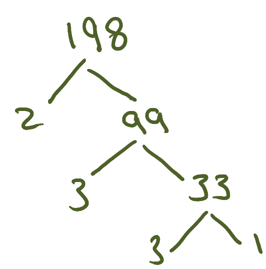
138: 1x138, 2x69, 3x46, 6x23
1, 2, 3, 6, 23, 46, 69, 138

198: 1x198, 2x99, 3x66, 9x22, 11x18
1, 2, 3, 6, 11, 18, 22, 33, 66, 99, 198

Method 2:



2.3.23

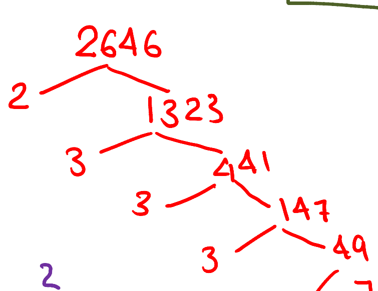


2.3.3.11

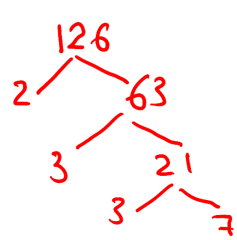
G.C.F = 2x3 = $\boxed{6}$

Example 3:

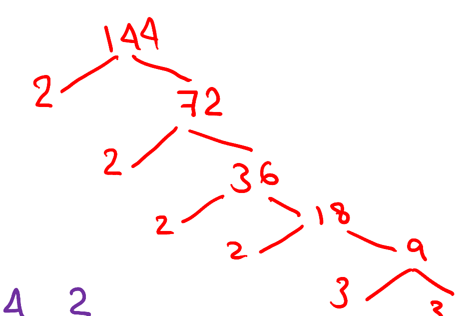
Write the prime factorization of 2646, 126, and 144. Determine the greatest common factor of 126 and 144.



$2.3^3.7^2$



$2.3^2.7$



$2^4.3^2$

GCF = 2x3x3 = $\boxed{18}$

To generate multiples of a number, multiply the number by the natural numbers; that is, 1, 2, 3, 4, 5, and so on.

For example, some multiples of 26 are:

$26 \cdot 1 = 26$ $26 \cdot 2 = 52$ $26 \cdot 3 = 78$ $26 \cdot 4 = 104$

For 2 or more natural numbers, we can determine their *least common multiple*, which is the least number that is divisible by each number. LCM

We can determine the least common multiple of 4 and 6 by combining identical copies of each smaller chain to create two chains of equal length.

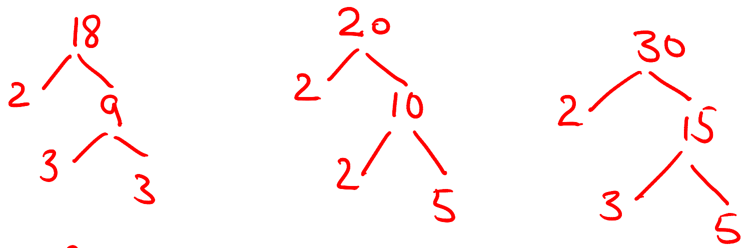
2×2 2×3

What is the least common multiple of 4 and 6? $2 \times 2 \times 3 = \boxed{12}$

Example 4:

Determine the least common multiple of 18, 20, and 30.

Method 1: *Method 2:*



$2 \cdot 3 \cdot 3$ $2 \cdot 2 \cdot 5$ $2 \cdot 3 \cdot 5$

$2 \cdot 3^2$ $2^2 \cdot 5$

$L.C.M = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = \boxed{180}$

Try this: *Determine the least common multiple of 28, 42, and 63.*

$28 = 2^2 \cdot 7$

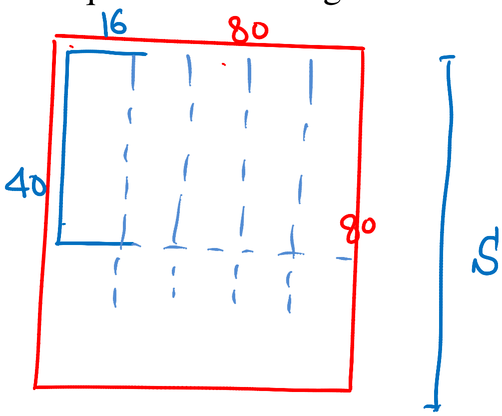
$42 = 2 \cdot 3 \cdot 7$

$63 = 3^2 \cdot 7$

$L.C.M = 2^2 \cdot 3^2 \cdot 7 = \boxed{252}$

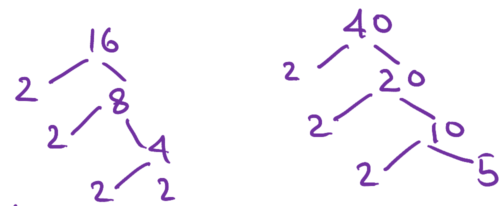
Example 5:

a. What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut. Sketch the square and rectangles.



* lets Find the side of the square

we need to Find LCm between 16, 40

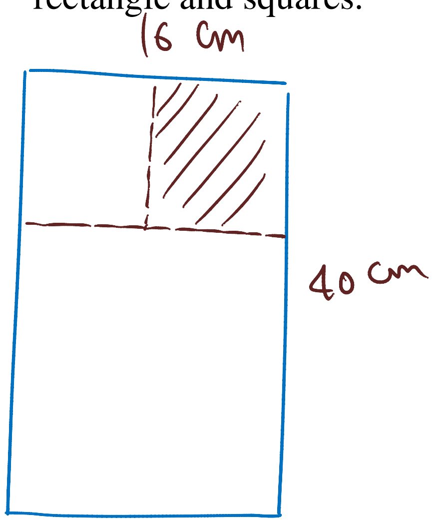


$16 = 2^4$

$40 = 2^3 \cdot 5$

$LCm = 2^4 \cdot 5^1 = 80$ cm

b. What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut. Sketch the rectangle and squares.



Find the G.C.F between 16, 40

$16 = 2^4 = 2 \times 2 \times 2 \times 2$

$40 = 2^3 \cdot 5 = 2 \times 2 \times 2 \times 5$

$g.C.F = 2 \cdot 2 \cdot 2 = \boxed{8 \text{ cm}}$

Assignment: p. 140 #4, 6, 8 – 19, 21 (odd letters)