Chapter 4– Trigonometry & the Unit Circle
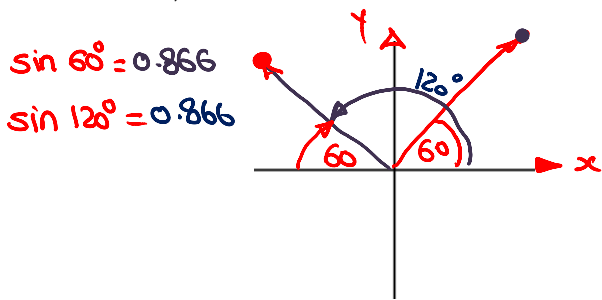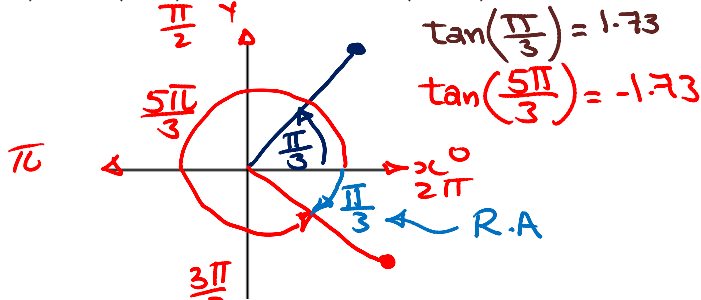### Section 4.3A – Reference Angles

The reference angle of the angle θ, is the acute angle determined by the terminal side of θ and either the positive or negative x-axis.

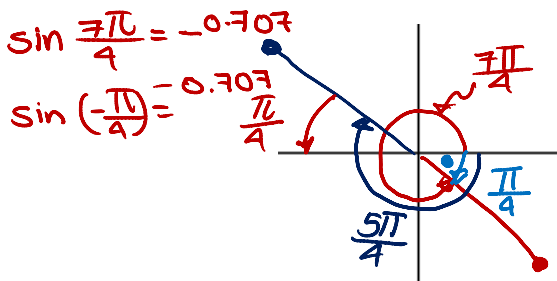**Example 1:** Draw each angle below. Then, evaluate using a calculator.
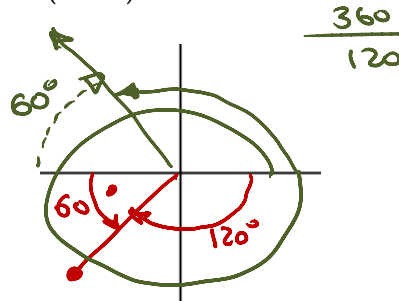
a) $\sin 60°$ & $\sin 120°$

$\sin 60° = 0.866$

$\sin 120° = 0.866$

b) $\tan (\pi/3)$ & $\tan (5\pi/3)$

$\tan\left(\frac{\pi}{3}\right) = 1.73$

$\tan\left(\frac{5\pi}{3}\right) = -1.73$

c) $\sin (7\pi/4)$ & $\sin (-5\pi/4)$

$\sin \frac{7\pi}{4} = -0.707$

$\sin \left(-\frac{\pi}{4}\right) = 0.707$

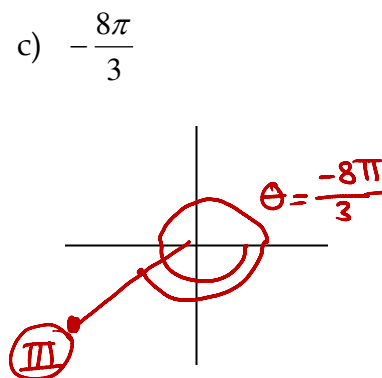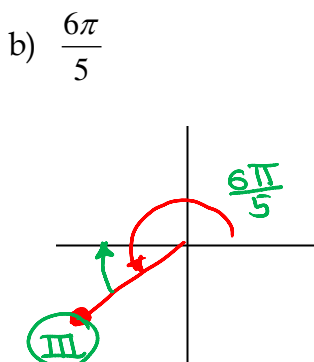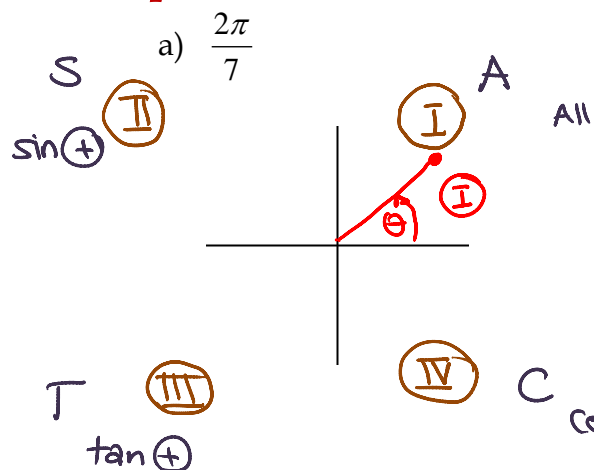d) $\cos (-120°)$ & $\cos 480°$

$\frac{360}{120}$

To reduce the complexity of angles, <u>reference</u> <u>angles</u> are used for trig calculation.

For angle (θ) in standard form position, It's reference angle is the angle between the terminal arm & the nearest x-axis.

Before we spend more time on the different trigonometric ratios (e.g. sine, cosine, tangent, etc…), it is important to know (and be familiar with) which of the **four quadrants** a given standard angle is located – especially when the angle is in radian.

Example 2: Draw each angle. Then, state the quadrant in which it is located.
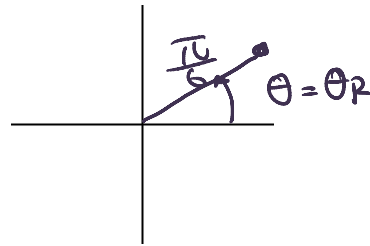
a) $\dfrac{2\pi}{7}$

S $\sin \oplus$ II   I

T   III   IV   C $\cos \oplus$

$\tan \oplus$ All   A

b) $\dfrac{6\pi}{5}$

$\frac{6\pi}{5}$ III

c) $-\dfrac{8\pi}{3}$

$\theta = \dfrac{-8\pi}{3}$ III

Example 3: Draw each angle, then <u>state</u> its **reference angle**.

a) 120°

RA = 180 − 120°
= 60°

$\theta_R = 60°$

b) $\dfrac{\pi}{6}$

$\theta = \theta_R$
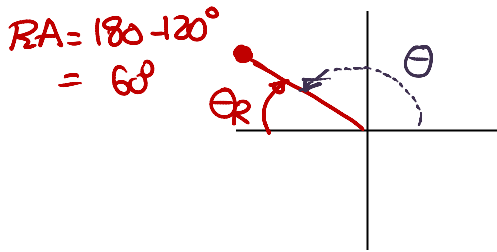
$\theta_R = \dfrac{\pi}{6}$

c) 3.5
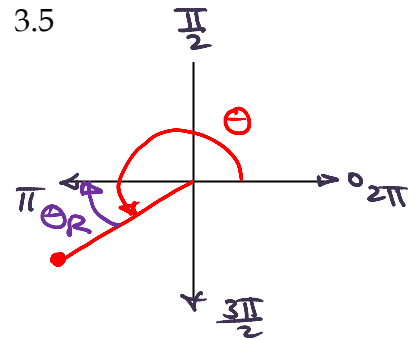
$\theta_R = 3.5 - \pi$
$= 0.36$ (rad.)

d) 310°

$\theta_R = 360° - 310$
$= 50°$

e) $-\dfrac{5\pi}{4}$
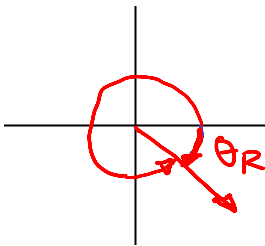
$-\dfrac{5\pi}{4}$
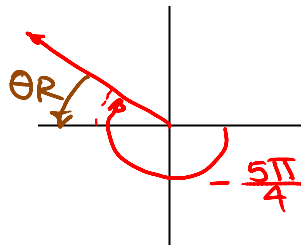
$\theta_R = \dfrac{\pi}{4}$

f) 472°
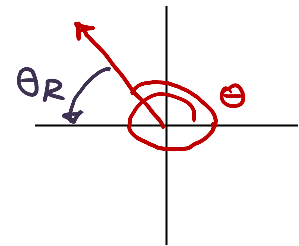
$\theta_R = 540° - 472°$
$= 68°$

Example 4: Draw each angle, then <u>state</u> its **reference angle** $\theta = \dfrac{79\pi}{42}$

$\theta_R = \dfrac{79\pi}{42} - \dfrac{84\pi}{42}$

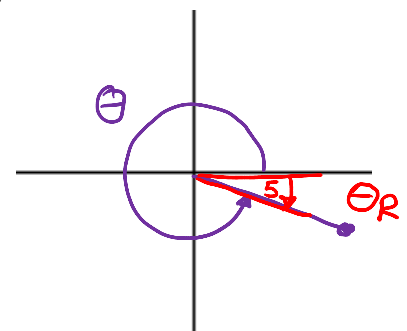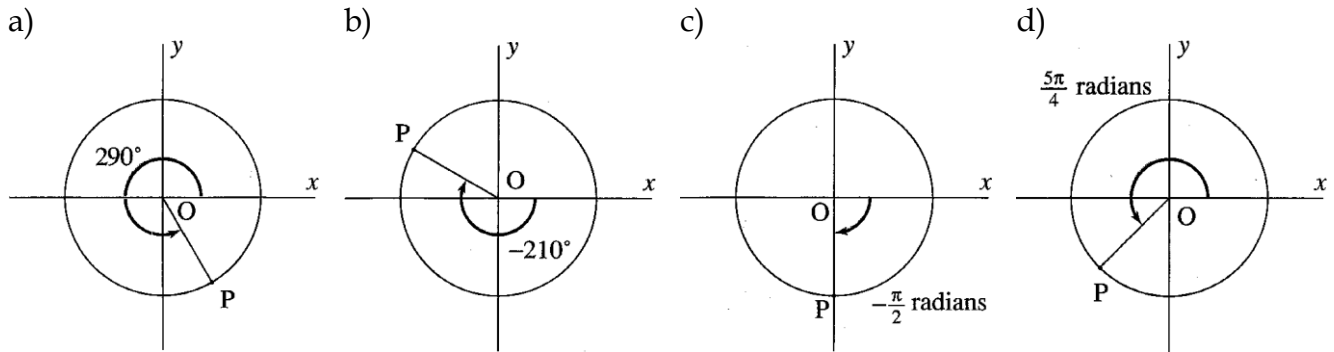$\theta_R = \dfrac{-5\pi}{42}$

Section 4.3A – Reference Angles WS

1. **State the location (Quadrant I, II, III, or IV) for each angle below. Draw the angle.**

    a) 30°   b) 135°   c) 60°   d) −120°   e) 450°   f) 270°

    g) $\dfrac{3\pi}{4}$ h) $\dfrac{4\pi}{3}$ i) $-\dfrac{\pi}{6}$ j) $\dfrac{-2\pi}{3}$ k) $\dfrac{16\pi}{3}$ l) $\dfrac{21\pi}{-4}$

2. **For each angle in standard position, determine the reference angle.**

    a)   b)   c)   d)

3. **Find the reference angle. Express your answer in terms of π.**

    a) 120°   b) 240°   c) 330°   d) 30°   e) −60°   f) −30°

    g) 180°   h) 270°   i) −135°   j) −225°   k) −90°   l) −180°

4. **For each of the angles below,**
    i) find the reference angle; and
    ii) find one positive and one negative coterminal angle.

    a) 155°   b) 270°   c) 40°   d) −200°   e) −60°   f) 312°

    g) $\dfrac{3\pi}{4}$ h) $\dfrac{11\pi}{6}$ i) $\dfrac{2\pi}{3}$ j) $-\dfrac{5\pi}{8}$ k) $-\dfrac{8\pi}{5}$ l) $-\dfrac{\pi}{2}$

Answers:

1. a) I   b) II   c) I   d) III   e) y-axis   f) y-axis   g) II   h) III   i) IV   j) III   k) III   l) II
2. a) 70°   b) 30°   c) π/2   d) π/4
3. a) π/3   b) π/3   c) π/6   d) π/6   e) π/3   f) π/6
    g) 0π   h) π/2   i) π/4   j) π/4   k) π/2   l) 0π
4i). a) 25°   b) 90°   c) 40°   d) 20°   e) 60°   f) 48°
    g) π/4   h) π/6   i) π/3   j) 3π/8   k) 2π/5   l) π/2
4ii) a) to l)   Answers may vary.