1 NUMBER

BBC ACTIVE

We all use numbers as part of our everyday life. Some of the objects that you might use every day such as mobile phones and cash cards need to be protected by powerful codes. Most of the codes used involve very large prime numbers.

Objectives

In this chapter you will:

- find the lowest common multiple and highest common factor of two numbers
- understand the meaning of square root and cube root
- know the correct order to carry out the different arithmetic operations
- use a calculator
- apply the laws of indices.

Before you start

You need to:

- understand and use positive numbers and negative integers
- know how to use a number line
- know your multiplication tables
- know how to find factors and multiples of whole numbers
- be able to identify prime numbers
- understand index notation.

1.1 Understanding prime factors, LCM and HCF

Objectives

- You can express any whole number as the product of its prime factors.
- You can find the HCF of two or three numbers.
- You can find the LCM of two or three numbers.

Why do this?

If burgers come in packs of 4 and buns come in packs of 6, being able to find out the LCM of 4 and 6 is useful to make sure that there are equal numbers of burgers and buns at a barbeque.

Get Ready

1. State whether the following numbers are factors of 18, multiples of 18 or neither.

a 4 **b** 6 **c** 9 **d** 36 **e** 12 **f** 3

2. State whether the following numbers are prime numbers or not.

a 35 **b** 31 **c** 39 **d** 43 **e** 57

Key Points

- Any factor of a number that is a prime number is a **prime factor**. For example, 2 and 3 are the prime factors of 6.
- You can write any number as the product of its prime factors.
- The **Highest Common Factor (HCF)** of two whole numbers is the highest factor that is common to them both. For example, 3, 5 and 15 are all **common factors** of 30 and 45 but 15 is their highest common factor.
- The **Lowest Common Multiple (LCM)** of two whole numbers is the lowest number that is a multiple of both of them. For example, the **common multiples** of 10 and 15 are 30, 60, 90, 120, but 30 is their lowest common multiple.

Example 1

Write 120 as the product of its prime factors.

Method 1

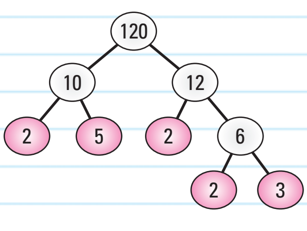
$$\begin{aligned}120 &= 2 \times 60\\ &= 2 \times 2 \times 30\\ &= 2 \times 2 \times 2 \times 15\\ &= 2 \times 2 \times 2 \times 3 \times 5\end{aligned}$$

Divide 120 into factors in stages.
Each time, divide the last number into its lowest prime factor and one other factor.

$$120 = 2^3 \times 3 \times 5$$

$2 \times 2 \times 2$ can be written as 2^3.
The prime factors of 120 are 2, 3 and 5.

Method 2



A factor tree can be used to find prime factors.
Divide 120 up into two numbers that multiply to give 120.
10×12 is not the only way to divide 120.
You could use 2 and 60 or 3 and 40, for example.
Continue dividing up numbers until prime numbers are obtained.

The shaded numbers at the ends of the branches are the prime factors.

$$\begin{aligned}\text{So}\quad 120 &= 2 \times 2 \times 2 \times 3 \times 5\\ &= 2^3 \times 3 \times 5\end{aligned}$$

Example 2 Find **a** the HCF and **b** the LCM of 6 and 10.

a The factors of 6 are 1, **2**, 3, 6.
List all the factors of 6.

The factors of 10 are 1, **2**, 5, 10.
List all the factors of 10.

The HCF of 6 and 10 is 2.
2 is the highest number that appears in both lists.

b Multiples of 6 are 6, 12, 18, 24, **30**, 36 …
List the first few multiples of 6.

Multiples of 10 are 10, 20, **30**, 40, 50, 60 …
List the first few multiples of 10. You will need to continue listing the multiples until there is a number that appears in both lists.

The LCM of 6 and 10 is 30.
30 is the smallest number that appears in both lists.

ResultsPlus
Examiner's Tip

The method of listing factors and multiples is best used when the given numbers are small.

Example 3 Find **a** the HCF and **b** the LCM of 140 and 210.

$140 = 2 \times 2 \times 5 \times 7$
$210 = 2 \times 3 \times 5 \times 7$

First express both numbers as the product of their prime factors.

Method 1

$140 = \mathbf{2} \times 2 \times \mathbf{5} \times \mathbf{7}$
$210 = \mathbf{2} \times 3 \times \mathbf{5} \times \mathbf{7}$

Identify the common factors; the numbers that appear in both lists.

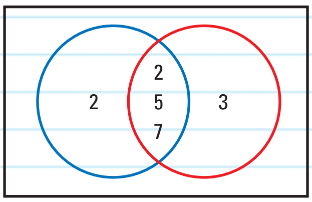
a HCF of 140 and 210 $= 2 \times 5 \times 7$
$= 70$

Multiply the common factors together to get the HCF.

b LCM of 140 and 210 $= 70 \times 2 \times 3$
$= 420$

Multiply the HCF by the numbers in both lists that were not highlighted to get the LCM.

Method 2



Put the prime factors into a Venn diagram.
The prime factors of 140 are in the blue circle.
The prime factors of 210 are in the red circle.
The common factors of 140 and 210 are inside the part of the diagram where the two circles intersect.

a HCF of 140 and 210 $= 2 \times 5 \times 7$
$= 70$

The HCF is the product of the numbers that are inside both circles.

b LCM of 140 and 210 $= 2 \times 2 \times 3 \times 5 \times 7$
$= 420$

The LCM is the product of all the numbers that appear in the Venn diagram.

Exercise 1A

Questions in this chapter are targeted at the grades indicated.

C
A03

*1 Can the sum of two prime numbers be a prime number?
Explain your answer.
[*Hint:* Try adding some pairs of prime numbers.]

2 The number 48 can be written in the form $2^n \times 3$.
Find the value of n.

3 The number 84 can be written in the form $2^n \times m \times p$ where n, m and p are prime numbers.
Find the values of n, m and p.

4 Find the HCF and LCM of the following pairs of numbers.
a 6 and 8 **b** 5 and 10 **c** 4 and 10 **d** 6 and 18

5 **a** Write 24 and 60 as products of their prime factors.
b Find the HCF of 24 and 60. **c** Find the LCM of 24 and 60.

6 **a** Write 72 and 120 as products of their prime factors.
b Find the HCF of 72 and 120. **c** Find the LCM of 72 and 120.

7 Find the HCF and LCM of the following pairs of numbers.
a 36 and 90 **b** 54 and 72 **c** 60 and 96 **d** 144 and 180

8 $x = 2 \times 3^2 \times 5$, $y = 2^3 \times 3 \times 7$
a Find the HCF of x and y. **b** Find the LCM of x and y.

9 $m = 2^4 \times 3^2 \times 5 \times 7$, $n = 2^3 \times 5^3$
a Find the HCF of m and n. **b** Find the LCM of m and n.

B

10 Bertrand's theorem states that 'Between any two numbers n and $2n$, there always lies at least one prime number, providing n is bigger than 1'. Show that Bertrand's theorem is true:
a for $n = 10$ **b** for $n = 20$ **c** for $n = 34$.

A03

11 A ship is at anchor between two lighthouses L and H.
The light from L shines on the ship every 30 seconds.
The light from H shines on the ship every 40 seconds.
Both lights started at the same moment.
How often do both lights shine on the ship at once?

A03

12 Burgers come in boxes of 8.
Buns come in packets of 6.
What is the smallest number of boxes of burgers and packets of buns that Mrs Moore must buy if she wants to ensure that there is a bun for every burger?

A
A03

*13 Sally says that if you multiply two prime numbers then you will always get an odd number.
Is Sally correct? Give a reason for your answer.

1.2 Understanding squares and cubes

Objectives

- You know how to find squares and cubes of whole numbers.
- You understand the meaning of square root.
- You understand the meaning of cube root.

Why do this?

If things are packed in squares you can quickly work out how many you have using square numbers, for example crates of strawberries or eggs.

Get Ready

1. Work out **a** 6×6 **b** $2 \times 2 \times 2$ **c** -3×-3

Key Points

- A **square number** is the result of multiplying a whole number by itself.
  The square numbers can be shown as a pattern of squares.

$1^2 = 1 \times 1 = 1$
1st square number

$2^2 = 2 \times 2 = 4$
2nd square number

$3^2 = 3 \times 3 = 9$
3rd square number

$4^2 = 4 \times 4 = 16$
4th square number

- A **cube number** is the result of multiplying a whole number by itself then multiplying by that number again.
  The cube numbers can be shown as a pattern of cubes.

$1^3 = 1 \times 1 \times 1 = 1$
1st cube number

$2^3 = 2 \times 2 \times 2 = 8$
2nd cube number

$3^3 = 3 \times 3 \times 3 = 27$
3rd cube number

- To find the **square** of any number, multiply the number by itself.
  The square of $-4 = (-4)^2 = -4 \times -4 = 16$.
- $5 \times 5 = 25$, so we say that 5 is the **square root** of 25. It is a number that when multiplied by itself gives 25.
  You can write the square root of 25 as $\sqrt{25}$. The square root of 25 can also be -5 because $-5 \times -5 = 25$.
- To find the **cube** of any number, multiply the number by itself then multiply by the number again.
  The cube of $-2 = (-2)^3 = -2 \times -2 \times -2 = -8$.
- $-2 \times -2 \times -2 = -8$, so we say that -2 is the **cube root** of -8. It is a number that when multiplied by itself, then multiplied by itself again, gives -8. You can write the cube root of -8 as $\sqrt[3]{-8}$.

Example 4 Find **a** the 6th square number
b the 10th cube number.

a The 6th square number is $6^2 = 6 \times 6$
$= 36$

b The 10th cube number is $10^3 = 10 \times 10 \times 10$
$= 1000$

ResultsPlus
Examiner's Tip

You need to know the integer squares and corresponding square roots up to 15×15, and the cubes of 2, 3, 4, 5 and 10.

Exercise 1B

D

1 Write down:

a the first 15 square numbers

b the first 5 cube numbers.

2 From each list write down all the numbers which are:
i square numbers ii cube numbers.

a 50, 20, 64, 30, 1, 80, 8, 49, 9

b 10, 21, 57, 4, 60, 125, 7, 27, 48, 16, 90, 35

c 137, 150, 75, 110, 50, 125, 64, 81, 144

d 90, 180, 125, 100, 81, 75, 140, 169, 64

ResultsPlus
Examiner's Tip

You need to be able to recall
- integer squares from 2×2 up to 15×15 and the corresponding square roots
- the cubes of 2, 3, 4, 5 and 10.

Example 5

Find a $(-3)^2$ b $\sqrt{100}$ c $(-4)^3 + \sqrt[3]{125}$

a $(-3)^2 = -3 \times -3 = 9$

Two signs the same so answer is positive.

b $\sqrt{100} = 10$

c $(-4)^3 = -4 \times -4 \times -4 = -64$

$\sqrt[3]{125} = 5$

$(-4)^3 + \sqrt[3]{125} = -64 + 5$

$= -59$

$5^3 = 125$ so the cube root of 125 is 5.

ResultsPlus
Examiner's Tip

Remember, when multiplying or dividing:
two signs the same give a +
two different signs give a −

Exercise 1C

1 Work out

a 3^2 b 7^2 c 4^3 d 10^3 e 11^2

2 Write down

a $\sqrt{36}$ b $\sqrt{16}$ c $\sqrt{81}$ d $\sqrt{1}$ e $\sqrt{64}$

D

3 Work out

a $(-6)^2$ b $(-2)^3$ c $(-9)^2$ d $(-1)^3$ e $(-12)^2$

C

4 Write down

a $\sqrt[3]{8}$ b $\sqrt[3]{-27}$ c $\sqrt[3]{-1}$ d $\sqrt[3]{64}$ e $\sqrt[3]{1000}$

B

5 Work out

a $3^2 + 2^3$ b $\sqrt{4} \times 5^2$ c $5^2 \times \sqrt{100}$ d $\sqrt[3]{-8} + 4^2$

e $\sqrt[3]{1000} \div \sqrt{100}$ f $4^3 \div 2^3$ g $(-1)^3 + 2^3 - (-3)^3$ h $4^2 + (-3)^3$

i $\dfrac{6^2}{2^2}$ j $5^2 \times \dfrac{\sqrt{16}}{\sqrt[3]{8}}$ k $2^3 \times \dfrac{\sqrt{100}}{\sqrt{64}}$ l $\dfrac{4^2 - \sqrt[3]{-8}}{\sqrt{9}}$

1.3 Understanding the order of operations

Objective

- You know and can apply the order of operations.

Why do this?

When following a recipe, you need to add the ingredients in the right order. The same is true of calculations such as $3 \times 4 + 2 \times 5$. The operations must be carried out in the correct order or the answer will be wrong.

Get Ready

1. Work out **a** 6×3 **b** 4^2 **c** $70 \div 7$

Key Points

- **BIDMAS** gives the order in which each **operation** should be carried out.
- Remember that B I D M A S stands for:

Brackets	If there are brackets, work out the **value** of the expression inside the brackets first.
Indices	Indices include square roots, cube roots and **powers**.
Divide **M**ultiply	If there are no brackets, do dividing and multiplying before adding and subtracting, no matter where they come in the expression.
Add **S**ubtract	If an expression has only adding and subtracting then work it out from left to right.

Example 6

$$\begin{aligned}10 \times 2^2 - 5 \times 3 &= 10 \times 4 - 5 \times 3\\ &= 40 - 15\\ &= 25\end{aligned}$$

Work out 2^2 first, then do all the multiplying before the subtraction.

Example 7

$$\begin{aligned}(12 - 2 \times 5)^3 &= (12 - 10)^3\\ &= 2^3\\ &= 8\end{aligned}$$

The sum in the bracket is worked out first. Work out 2×5 and then do the subtraction.

Exercise 1D

1 Work out

a $5 \times (2 + 3)$ **b** $5 \times 2 + 3$ **c** $20 \div 4 + 1$ **d** $20 \div (4 + 1)$
e $(6 + 4) \div -2$ **f** $6 + 4 \div 2$ **g** $24 \div (6 - 2)$ **h** $24 \div 6 - 2$
i $7 - (4 + 2)$ **j** $7 - 4 + 2$ **k** $5 \times 4 - 2 \times 3$ **l** $28 - 4 \times -6$
m $14 + 3 \times 6$ **n** $6 + 3 \times 5 - 12 \div 2$ **o** $25 - 5 \times 4 + 3$ **p** $(15 - 5) \times (4 + 3)$

2 Work out

a $(3 + 4)^2$ **b** $3^2 + 4^2$ **c** $3 \times (4 + 5)^2$ **d** $3 \times 4^2 + 3 \times 5^2$
e $2 \times (4 + 2)^2$ **f** $3 \times \sqrt{25} + 2 \times 3^3$ **g** $\dfrac{(2 + 5)^2}{3^2 - 2}$ **h** $\dfrac{5^2 - 2^2}{-3}$

D

B

3 Work out

a $(2 + 3)^3 \div \sqrt{25}$

b $((15 - 5) \times 4) \div ((2 + 3) \times 2)$

c $2^3 + 6^2 \div \sqrt{9} - 4 \times 3$

d $(\sqrt[3]{-27} - 2)^2 + \sqrt{3^2 \times 2^2}$

1.4 Using a calculator

Objectives

- You can use a calculator.
- You can find and use reciprocals.

Why do this?

Many jobs require the accurate use of calculators, such as working in a bank or as an accountant.

Get Ready

1. Work out an estimate for: **a** 234×89 **b** $318.2 \div 2.98$ **c** $(7.2)^2$

Key Points

- A scientific calculator can be used to work out arithmetic calculations or to find the value of arithmetic expressions.
- Scientific calculators have special keys to work out squares and square roots. Some have special keys for cubes and cube roots.
- To work out other powers, your calculator will have a $\boxed{y^x}$ or $\boxed{x^y}$ or $\boxed{\wedge}$ key.
- The inverse of x^2 is $\sqrt{x}$ or $x^{\frac{1}{2}}$, and the inverse of x^3 is $\sqrt[3]{x}$ or $x^{\frac{1}{3}}$.
- The **inverse operation** of x^y is $x^{\frac{1}{y}}$.
- You can use the calculator's memory to help with more complicated numbers.

Example 8

Work out **a** $4.6^2 + \sqrt{37}$ **b** $\dfrac{1.2^3 + 12.5}{(3.7 - 2.1)^2}$

Give your answers correct to 3 significant figures.

a $4.6^2 + \sqrt{37}$ — Key in 4 . 6 x^2 + √ 3 7 =

$4.6^2 + \sqrt{37} = 27.242\,762\ldots$

$= 27.2$ — Round your answer to 3 significant figures.

b $\dfrac{1.2^3 + 12.5}{(3.7 - 2.1)^2}$ — Work out the sum on the top of the fraction.

$1.2^3 + 12.5 = 14.228$ — Key in 1 . 2 x^3 + 1 2 . 5 =

$(3.7 - 2.1)^2 = 2.56$ — Work out the sum on the bottom of the fraction. Key in (3 . 7 − 2 . 1) x^2

$14.228 \div 2.56 = 5.557\,8125$ — Divide your answers.

$= 5.56$ — Round the final answer to 3 significant figures.

ResultsPlus Watch Out!

Do not round your numbers part way through a calculation; use all the figures shown on your calculator. Only round the final answer.

Exercise 1E

D

1 Work out:

a $\sqrt{961}$ b $\sqrt{40.96}$ c $\sqrt[3]{4913}$ d $\sqrt[3]{3.375}$ e $\sqrt{1024}$

2 Work out:

a $(3.7 + 5.9) \times 4.1$ b $3.1^2 + 4.8^2$ c $(-8.7 + 6.3)^2$ d $4.5^3 + 8^2$

3 Work out, giving your answers correct to one decimal place.

a $3.2^3 \times 6.7$ b $\sqrt{24} + 6.7^3$ c $9.2^2 \div \sqrt{14}$ d $7.5^3 - \sqrt{120}$

4 Work out, giving your answers correct to three significant figures.

a $\dfrac{5.63}{2.8 - 1.71}$ b $\dfrac{9.84 \times 2.6}{2.8 \times 1.71}$ c $\dfrac{6.78 + 9.2}{7.8 - 2.75}$ d $\dfrac{6.7^2}{5.6^2 - 2.1^2}$

C

5 Work out, giving your answers correct to three significant figures.

a $\sqrt{11.62} - \dfrac{6.3}{9.8}$ b $\dfrac{5.63}{2.8} + \dfrac{1.7}{0.3}$ c $\dfrac{\sqrt{342}}{1.8 - 1.71}$ d $\left(\sqrt{\dfrac{56}{0.18}} + 657\right)^2$

6 Work out, giving your answers correct to three significant figures.

a $\dfrac{\sqrt{45} + 6.3^2}{79.1 - 28.5}$ b $\sqrt{\dfrac{8.9 \times 2.3}{9.6 + 7.8}}$ c $\dfrac{4.2^3}{\sqrt{7.8^2 + 3.5^2}}$ d $\dfrac{(23.5 + 8.7)^2}{\sqrt{65^2 + 82}}$

Reciprocals

Key Points

- The **reciprocal** of the number n is $\frac{1}{n}$. It can also be written as n^{-1}.
- When a number is multiplied by its reciprocal the answer is always 1.
- All numbers, except 0, have a reciprocal.
- The reciprocal button on a calculator is usually $\boxed{1/x}$ or $\boxed{x^{-1}}$.

Example 9 Work out the reciprocal of a 8 b 0.25 c $\frac{1}{4^3}$.

a $\frac{1}{8} = 0.125$

b $1 \div 0.25 = 4$

c 4^3

Exercise 1F

D

1 Find the reciprocal of each of the following numbers.

a 4 b 0.625 c 6.4 d $\dfrac{2}{2^4}$

1.5 Understanding the index laws

Objectives

- You can use index notation.
- You can use index laws.

Why do this?

Using the index laws you can work out that you have $2^5 = 32$ great great great grandparents.

Get Ready

1. Work out 2^5 **2.** Work out 5^3 **3.** Work out $27^4 \div 27^2$

Key Points

- A number written in the form a^n is an **index number**.
- The **laws of indices** are:

$a^m \times a^n = a^{m+n}$ To multiply two powers of the same number add the indices.

$a^m \div a^n = a^{m-n}$ To divide two powers of the same number subtract the indices.

$(a^m)^n = a^{m \times n}$ To raise a power to a further power multiply the indices together.

You will encounter negative and fractional indices in Chapter 25.

Example 10 Work out **a** 3^4 **b** 2^6

a $3^4 = 3 \times 3 \times 3 \times 3$
$= 81$

b $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 64$

ResultsPlus
Watch Out!

Remember that a^3 means that you multiply three as together.
It does not mean $a \times 3$.

Example 11 Write each expression as a power of 5. **a** $5^6 \times 5^4$ **b** $5^{12} \div 5^4$ **c** $(5^3)^2$

a $5^6 \times 5^4 = 5^{4+6}$
$= 5^{10}$

Use the index law $a^m \times a^n = a^{m+n}$

b $5^{12} \div 5^4 = 5^{12-4}$
$= 5^8$

Use the index law $a^m \div a^n = a^{m-n}$

c $(5^3)^2 = 5^{3 \times 2}$
$= 5^6$

Use the index law $(a^m)^n = a^{m \times n}$

Example 12 Work out $\frac{4^7 \times 4}{4^5}$

$\frac{4^7 \times 4}{4^5} = \frac{4^7 \times 4^1}{4^5}$

$= \frac{4^8}{4^5}$

Simplify the top of the fraction, add 7 and 1.

$= 4^3$

$= 4 \times 4 \times 4 = 64$

As the question asks you to 'Work out', the final answer must be a number.

ResultsPlus
Examiner's Tip

'Work out' means 'evaluate' the expression, rather than leaving the answer as a power.

ResultsPlus
Watch Out!

Remember that a is the same as a^1.

Exercise 1G

1 Write as a power of a single number

a $6^5 \times 6^7$ b $4^7 \div 4^2$ c $(7^2)^3$ d $5^9 \div 5^3$ e $3^8 \times 3^2$

2 Work out

a $10^2 \times 10^3$ b $5^7 \div 5^4$ c $(2^3)^2$ d $3^4 \div 3^2$ e 4×4^2

3 Find the value of n

a $3^n \div 3^2 = 3^3$ b $8^5 \div 8^n = 8^2$ c $2^5 \times 2^n = 2^{10}$ d $3^n \times 3^5 = 3^9$ e $2^6 \times 2^3 = 2^n$

4 Write as a power of a single number

a $\frac{3^3 \times 3^5}{3^4}$ b $\frac{5^6 \times 5^7}{5^4}$ c $\frac{2^8 \times 2^5}{2^7}$ d $\frac{6^{15}}{6 \times 6^9}$ e $\frac{4^2 \times 4^7}{4^3 \times 4^4}$

5 Work out

a $\frac{3^3 \times 3^5}{3^6}$ b $\frac{2^6 \times 2^2}{2^4}$ c $\frac{4^7}{4 \times 4^4}$ d $\frac{10^5 \times 10^6}{10^7}$ e $\frac{7^8 \times 7}{7^3 \times 7^4}$

6 Work out the value of n in the following

a $40 = 5 \times 2^n$ b $32 = 2^n$ c $20 = 2^n \times 5$ d $48 = 3 \times 2^n$ e $54 = 2 \times 3^n$

C

B

Chapter review

Key Points

- Any factor of a number that is a prime number is a **prime factor**.
  You can write any number as the product of its prime factors.
- The **Highest Common Factor (HCF)** of two whole numbers is the highest factor that is common to them both.
- The **Lowest Common Multiple (LCM)** of two whole numbers is the lowest number that is a multiple of both of them.
- A **square number** is the result of multiplying a whole number by itself.
- A **cube number** is the result of multiplying a whole number by itself then multiplying by that number again.
- To find the **square** of any number, multiply the number by itself.
- The **square root** of 25 is a number that when multiplied by itself gives 25.
  You can write the square root of 25 as $\sqrt{25}$.
  The square root of 25 can also be -5 because $-5 \times -5 = 25$.
- To find the **cube** of any number, multiply the number by itself then multiply by the number again.
- The **cube root** of -8 is a number that when multiplied by itself, then multiplied by itself again, gives -8.
  You can write the cube root of -8 as $\sqrt[3]{-8}$.
- **BIDMAS** gives the order in which **operations** should be carried out.

- Remember that **BIDMAS** stands for:

Brackets	If there are brackets, work out the **value** of the expression inside the brackets first.
Indices	Indices include square roots, cube roots and **powers**.
Divide	If there are no brackets, do dividing and multiplying before adding and subtracting, no
Multiply	matter where they come in the expression.
Add	If an expression has only adding and subtracting then work it out from left to right.
Subtract	

- A scientific calculator can be used to work out arithmetic calculations or to find the value of arithmetic expressions.
- Scientific calculators have special keys to work out squares and square roots. Some have a special key for cubes and cube roots.
- To work out other powers, your calculator will have a $\boxed{y^x}$ or $\boxed{x^y}$ or $\boxed{\wedge}$ key.
- The inverse of x^2 is $\sqrt{x}$ or $x^{\frac{1}{2}}$, and the inverse of x^3 is $\sqrt[3]{x}$ or $x^{\frac{1}{3}}$.
- The **inverse operation** of x^y is $x^{\frac{1}{y}}$.
- You can use the calculator's memory to help with more complicated numbers.
- The **reciprocal** of the number n is $\frac{1}{n}$. It can also be written as n^{-1}.
- When a number is multiplied by its reciprocal the answer is always 1.
- All numbers, except 0, have a reciprocal.
- The reciprocal button on a calculator is usually $\boxed{1/x}$ or $\boxed{x^{-1}}$.
- A number written in the form a^n is an **index number**.
- The **laws of indices** are:

$a^m \times a^n = a^{m+n}$	To multiply two powers of the same number add the indices.
$a^m \div a^n = a^{m-n}$	To divide two powers of the same number subtract the indices.
$(a^m)^n = a^{m \times n}$	To raise a power to a further power multiply the indices together.

Review exercise

Except where indicated.

A03

1 Jim writes down the numbers from 1 to 100. Ben puts a red spot on all the even numbers and Helen puts a blue spot on all the multiples of 3.

a What is the largest number that has both a red and a blue spot?

b How many numbers have neither a blue nor a red spot?

Sophie puts a green spot on all the multiples of 5.

c How many numbers have exactly two coloured spots on them?

D

2 Find the missing numbers in each case.

a $? \times 3 = -12$ **b** $(-20) \div (-5) = ?$ **c** $(-6) + ? = (-8)$

d $(-5) \times ? = (-20)$ **e** $6 - ? = 8$

3 Find the missing numbers in each case.

a $2 \times ? + (-3) = (-7)$ **b** $(-4) \times ? + 5 = (-3)$ **c** $? \div 2 + 4 = (-4)$

4 Neal works part time in a local supermarket, stacking shelves.
He has been asked to use the pattern below to advertise a new brand of beans.

This stack is 3 cans high.

a How many cans will he need to build a stack 10 cans high?

b If he has been given 200 cans, how many cans high would his stack be?

Next he is asked to stack cans of tomato soup in a similar shape, but this time it is two cans deep.

Use your answers to parts a and b to answer the following questions.

c How many cans will he need to build a stack 10 cans high?

d If he has been given 400 cans, how many cans high would his stack be?

FS

D AO2 AO3

5 A chocolate company wishes to produce a presentation box of 36 chocolates for Valentine's Day.
It decides that a rectangular shaped box is the most efficient, but needs to decide how to arrange the chocolates.
How many different possible arrangements are there:

a using one layer

b using two layers

c using three layers.

Which one do you think would look best?

FS

AO2 AO3

C AO2

6 The number 1 is a square number and a cube number. Find another number which is a square number and a cube number.

7 $4^2 \times 6^2 = 576$

Work out a $40^2 \times 60^2$ b $400^2 \times 6^2$ c $5760 \div 6^2$ d $4^2 \times 60^2$ e $4^3 \times 6^2$

8 Work out a $2 + 4 \div 4$ b $5^3 \div 5 + 5$ c $(2^2)^3 - (2^3)^2$

9 Simplify a $\frac{3^5 \times 3^3}{3^6}$ b $\frac{4^4 \times 4^7}{4^{10}}$ c $(2^4)^3$ d $\frac{5^{12}}{5^7 \times 5^3}$

10 a Express 252 as a product of its prime factors.

b Express 6×252 as a product of prime factors.

C
A03

11 James thinks of two numbers.
He says 'The highest common factor (HCF) of my two numbers is 3.
The lowest common multiple (LCM) of my two numbers is 45'.
Write down the two numbers James could be thinking of.

ResultsPlus
Exam Question Report

75% of students answered this sort of question well because they chose the right method to answer the question.

June 2008

12 Write 84 as a product of its prime factors.
Hence or otherwise write 168^2 as a product of its prime factors.

A03

13 A car's service book states that the air filter must be replaced every 10 000 miles and the diesel fuel filter every 24 000 miles. FS
After how many miles will both need replacing at the same time?

14 Use your calculator to work out $\dfrac{\sqrt{19.2 + 2.6^2}}{2.7 \times 1.5}$
Write down all the figures on your calculator display.

B
A03

15 $2^{30} \div 8^9 = 2^x$
Work out the value of x. *Nov 2007*

A*
A03

16 Write whether each of the following statements is true or false. If the statement is false give an example to show it.
a The sum of two prime numbers is always a prime number.
b The sum of two square numbers is never a prime number.
c The difference between consecutive prime numbers is never 2.
d The product of two prime numbers is always a prime number.
e No prime number is a square number.

17 **a** Take a piece of scrap A4 paper. FS
If you fold it in half you create two equal pieces. Fold it in half again; you now have four equal pieces.
It is said that no matter how large and how thin you make the paper, it cannot be folded more than seven times. Try it.
If you fold it seven times, how many equal pieces does the paper now have?
b In 2001, there were two rabbits left on an island.
A simple growth model predicts that in 2002 there will be four rabbits and in 2003, eight rabbits.
The population of rabbits continues to double every year.
How long is it before there are 1 million rabbits on the island?