Analysis of analytic hierarchy process of some algebra factors affecting the field concept over a commutative ring with identity element definition

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Abstract: In this study first, the theoretical frameworks and application of Analytic Hierarchy Process (AHP) were investigated. Then, the application and the evaluation of Some Algebraic Concepts Affecting The Field Concept Over A Commutative Ring with Identity Element Definition were studied by using AHP, one of the methods of multi criteria decision making. For this purpose Some Algebra Factors Affecting The Field Concept Over A Commutative Ring with Identity Element Definition (FCASF) were assessed and four fundamental factors were found out. These four factors are basic concepts of algebraic structures, transformations of algebraic structures, group and ring . Then each of main and sub factors were ordered in order of priorities by AHP. In addition results and interpretations related to decision making problem were reviewed. Order of priorities of four main factors according to values obtained from AHP was transformations of algebraic structures (0.5669), , ring (0.2732), group (0.1228) and basic concepts of algebraic structures (0.0370). [Sebahat Y. **Analysis of analytic hierarchy process of some algebra factors affecting the field concept over a commutative ring with identity element definition**. *Life Sci J* 2013;10(5s):328-339] (ISSN:1097-8135). http://www.lifesciencesite.com. 60

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1. Introduction

The decision making process played a vital role in human history. These decisions were very important most of the times and they affect the very existence, sufficiency and the success of the people and the institutions. This is defined as the decision making problem. People do not readily accept the fact that the others can help them to take more rational decisions. The brain capacities of the people may not be sufficient to make the complicated decisions. There was a striking increase in the use of human judgment in decision making models [1-3]. One of the methods which use the human judgment is the Analytic Hierarchy Process (AHP) developed and used by Thomas L. Saaty in 1977 [4]. AHP is a mathematical method which takes the priorities of the group or the people into the account and evaluates the qualitative and quantitative variables together. It enables the evaluations of the experts as well as the individual people to be taken into the account. It effectively uses the occupational expertise and the experience of the experts at the decision making stage.

AHP method is intended to make the individuals to give more effective decision. The method has become very popular and used in multi criteria decision making processes [1].

There have been many studies on AHP method [5-22]. In Turkish literature AHP method was first used by Ulengin in 1994 [23]. Here the author graded the benefit / cost ratios of the possible alternatives. Later, AHP method was used in multi criteria decision making problem [24-30]. The AHP method is employed in a very wide range of fields such as economic, administrative, political, social problems and technological problems.

1. 1. Definition

If there is an inverse of every element in a commutative ring with identity elements according to multiplication process it is called a field [31].

2. The general information on analytic hierarchy process (AHP)

2. 1. Decision making

The people have to make decisions in order to continue their lives and become successful in future. If they take scientific criteria into account they can take much more rational decisions [32, 33]. Decision making problem is defined as choosing the most suitable one among the set of choices according to purposes and the criteria imposed.

2. 2. Analytic hierarchy process (AHP)

One of the methods where human judgments are used in decision making process is Analytic Hierarchy Process (AHP) developed by Thomas L. Saaty in 1977. This method was important for the elimination of the confusion in the problems for the people who are supposed to use a complete brain process to make the decision. This process enables the people to give much more efficient and rational decisions. This method has become very popular and used multi criteria decision making problems [34]. In AHP method the decision makers are supposed to decide on the analytical approach to be used.

After deciding the hierarchy of the problem the next step in AHP is the measurement relative superiority of the each element in the hierarchy. The relative superiorities are

converted into number expressed in matrices. After this stage there are valid reasons for the use of mathematics. The methods which try to define the decisions like that are called analytical methods [34-36].

Analytical decision making is based upon the idea to dissociate the problems into smaller sub units in a hierarchical manner for the more effective solution [25]. Apart from its literary meaning hierarchy is the model which shows how the human brain analyses the complicated situations. The incomplete hierarchical models are the ones where the element of certain levels effect only the few of the high level elements [37].

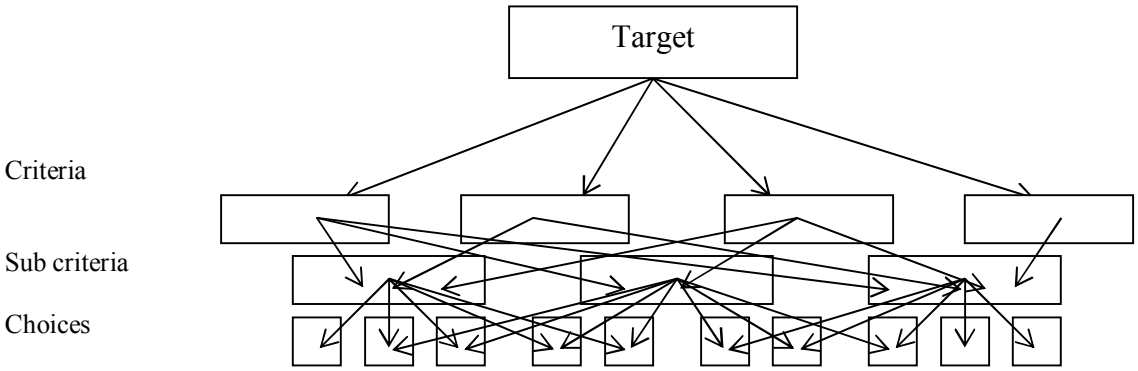

Fig. 1. Incomplete hierarchy model.

According to the empty hierarchy model let the first level is the "complete welfare of the country "Therefore the second level can be "future scenarios. of the country "and the third and fourth levels are "the cities" and the "incomplete transportation plans of the cities." As seen from Fig. 1. neither the scenario nor each transportation plan affect the every city. The purpose is the determination of the priorities of the plans necessary for the complete welfare of the country.

2. 2. 3. Process

The multi criteria problems cover the process which needs the detailed investigation, learning and review the priorities. The evaluation of these processes may take some time. According to Saaty, 2000, AHP may be very useful for shortening this process [34].

2. 3. The measurement and the use of AHP

In AHP applications the opinions of the people directly related to the subject are taken by face to face talks or interviews. For the consistency of the results these people are chosen from the ones with certain degree of expertise about the subject examined, since the results of the AHP depends entirely on the mutual comparison of these people. According to these comparisons one can form superiority, comparative or decision matrix in AHP. This matrix is formed by the transformation of these judgments into numerical values [34, 38, 39].

Saaty et.al. developed and used the 1-9 scale in their studies (Table 1). 1-9 scale gives the best result.

Table 1. Basic measures and their descriptions in AHP

Degree of Importance	Definition	Explanation
1	Equally important	Both options make the equal contribution
3	Moderately important	Experience and judgment makes one criterion a little bit more important than the other
5	Strongly important	Experience and judgment makes one criterion highly important than the other
7	Very strongly important	A criterion is regarded superior to the other and is apparent in application
9	Absolutely important	There is a highly reliable proof showing one criterion is more important than the other
2, 4, 6, 8	Mid values	The values between two consecutive values used when adjudication is necessary
Non zero values	If the comparison of activity i with activity j is determined with a non zero value then j and i have mutual values	A logical estimation

The upper limit is 9 in AHP as seen Table 1. There are many reasons for that [34, 38, 40, 41].

1. This method developed by Saaty gives the best results for 7 criteria out of n<10 criteria. In other words in the solution of multi criteria problems with AHP method there may be big inconsistencies for the case when number of criteria is bigger than 9.

2. If the elements of a matrix are formed by very big numbers then this may cause big inconsistencies.

2. 3. 1. Relative measurement

In this study the pairwise comparisons or comparisons of the two criteria were made by the use of relative scale changing from the people to people. The data of the relative scale were obtained by the use of Table 1.

2. 4. The pairwise comparisons matrix of AHP

AHP is a mathematical method which takes the priorities of the groups of the people into account and evaluates the quantitative and qualitative variables in decision making. Today our globalizing world has so many complicated problems which are closely related to each other. Therefore it is not easy to distinguish the reasons and consequences of the phenomena which take place in the system [34]. That was why the concept or the situation which was of great importance for the one may be of secondary importance for the other. The pairwise comparisons are the most important achievement of AHP. The pairwise comparisons are based upon the measurement technique in psychology. Thurstone, defined the awareness concept in pairwise comparison as follows [34]. "The awareness process is the perception, identifying, distinguishing or reaction of the person."

Pairwise comparisons are the most important stage of the AHP. To obtain pairwise comparisons a relative measure is used. The data obtained from them are transformed into a matrix in AHP. If a_{ij}, is the value of the comparisons of property of i and property of j then the pairwise comparisons matrix is given as follows.

$$A=\begin{vmatrix} a_{11} & a_{12} & . & a_{1n} \\ a_{21} & . & . & . \\ . & . & . & . \\ a_{n1} & . & . & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & . & a_{1n} \\ 1/a_{12} & . & . & . \\ . & . & . & . \\ 1/a_{1n} & . & . & a_{nn} \end{vmatrix}$$

If a_{ji} value is given then this is $a_{ji}= 1 / a_{ij}$. This is known as "being opposite" property [42]. The eigenvalue or priority vector obtained after the solution of pairwise comparisons vector is shown as $W= (w_1, w_2, ..., w_n)$. wj, is defined as priority or eigenvector. These values gives W^* matrix.

$$W^* = \begin{vmatrix} w_1 / w_1 & . & . & w_1 / w_n \\ . & . & . & . \\ . & . & . & . \\ w_n / w_1 & . & . & w_n / w_n \end{vmatrix}$$

If these results are consistent then there should not be very big differences between the element of A and W^* matrices [34, 35, 36].

2. 3. The methods calculating the priority or weighing vectors

After the establishment of pairwise comparison or judgment matrices one has to calculate priority or the weighing vectors. According to AHP methodology the eigenvalues or the eigenvectors of the comparisons matrix help us to define the priority. The eigenvector which gives the highest eigenvalue gives the priority [43].

The elements at each row of the comparisons matrix A is summed up. Then they are normalized by being divided to the general sum. The first element in the vector formed like that shows the priority of the first activity and the second element shows the priority of the second activity and so on.

The eigenvalue approach gives the complete solution of the priorities of the values of matrix A [44]. However the solution of the eigenvalues and the eigenvectors are not easy. Especially the dimension of the n value makes the solution difficult. There are generally no analytical solutions for n≥5 which requires the solution of polynomial equation of 5^{th} degree. According to Saaty (1990) the solution of eigenvectors is the best way to see the most superior one in the Pairwise comparisons [39]. The solution of the eigenvectors is carried by the following steps:

1. Step: The shortest way to obtain the best solution is enlarge the comparison matrix by its powers. That is why its square is taken every time.

2. Step: Then the rows are summed up and normalized. This vector gives the best solution.

3. Step: If the differences between the successive summations are small then the calculation is terminated. Here if the elements of comparison matrix are written in four digits and repeat the calculations, one can see that there was no need for more than one iteration.There were eigenvalues and eigenvalue method used in this study.

2. 4. Consistency index (CI) and randomization index (RI)

The calculated consistencies of the priority vectors can easily be checked. The eigenvalue method helps us quite a lot in the calculation of the consistency of the comparison matrix. The inputs in the a_{ij} values in pairwise comparison matrix cause a great change in the λ_{max} eigenvalue. That was why the difference of λ_{max}-n is a measure of consistency. Saaty defined the normalization of the comparison matrix (n) as the consistency index (CI) as $CI = \lambda_{max} - n / n - 1$ [45].

Saaty et.al. formed a randomization index (RI) series in order to calculate consistency ratio. This randomization index of the matrices with 1-15 dimension were randomly filled by 100 values for each elements in each dimensions as 1/9, 1/8, ..., 1, ..., 8 and 9 to calculate the CI values. Then the average values of the consistency indices for each dimension are computed to form the randomization indexes.

The consistency ration (CR) is the ratio of the consistency index to the randomization index at the same dimension. This ratio is CR=CI/RI. Saaty proposes that this value should be less than 10 %. That was why 10% is taken as the upper limit [46]. If the CR value is greater than 10 % then the decision makers are to revise their judgments in order to decrease the consistency ratio to the desired value. The table below lists the random consistency indices according to n.

Table 2. Random index numbers

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.52	0. 89	1.11	1.25	1.35	1.40	1.45	1.49	1.51	1.54	1.56	1.57	1.58

3. AHP application

3. 1. Definition of the problem

In this study some factors affecting the field concept over a commutative ring with identity element definition (FCASF) such as the basic concepts of algebraic structures, transformations of the algebraic structures, group and ring concepts are investigated. Each of these main factors may have an equal or different effect on the field. Any one of the sub criteria of these main factors completely affects the field. Therefore there was a need of establishing the link between these factors and AHP. The goal of the use of this method is to determine the superiorities of the main and sub factors towards each other by taking target into account. That is also called factor prioritization.

3. 2. Accepted variables for hierarchical of some factors affecting the field concept

(FCASF) and presentation the problem in hierarchical order

Hierarchical Structure of FCASF was presented as the three stages in Fig. 2. The first stage shows target. The second stage shows the main criteria and third stage shows sub criteria.

In application the prioritization of FCASF were evaluated on five qualitative criteria suggested by the experts. The main criteria and sub criteria are given in Fig. 2.

3. 3. Calculation

This section deals the AHP solution of the problem which hierarchical structure is given in Fig. 2.

3. 3. 1. The determination of the priority of the main criteria according to the target

Here first the priority vector for the main criteria is calculated. After discussing with the experts there were pairwise comparison matrix or judgement matrix was formed for four main criteria (Table 3). The comparisons are made by considering the upper level or the target. When the judgement was made the following question is asked. “When we consider the general target, how much the each criterion is important than the other?” The judgments were made by the experts are the main measurement of the AHP (Table 1).

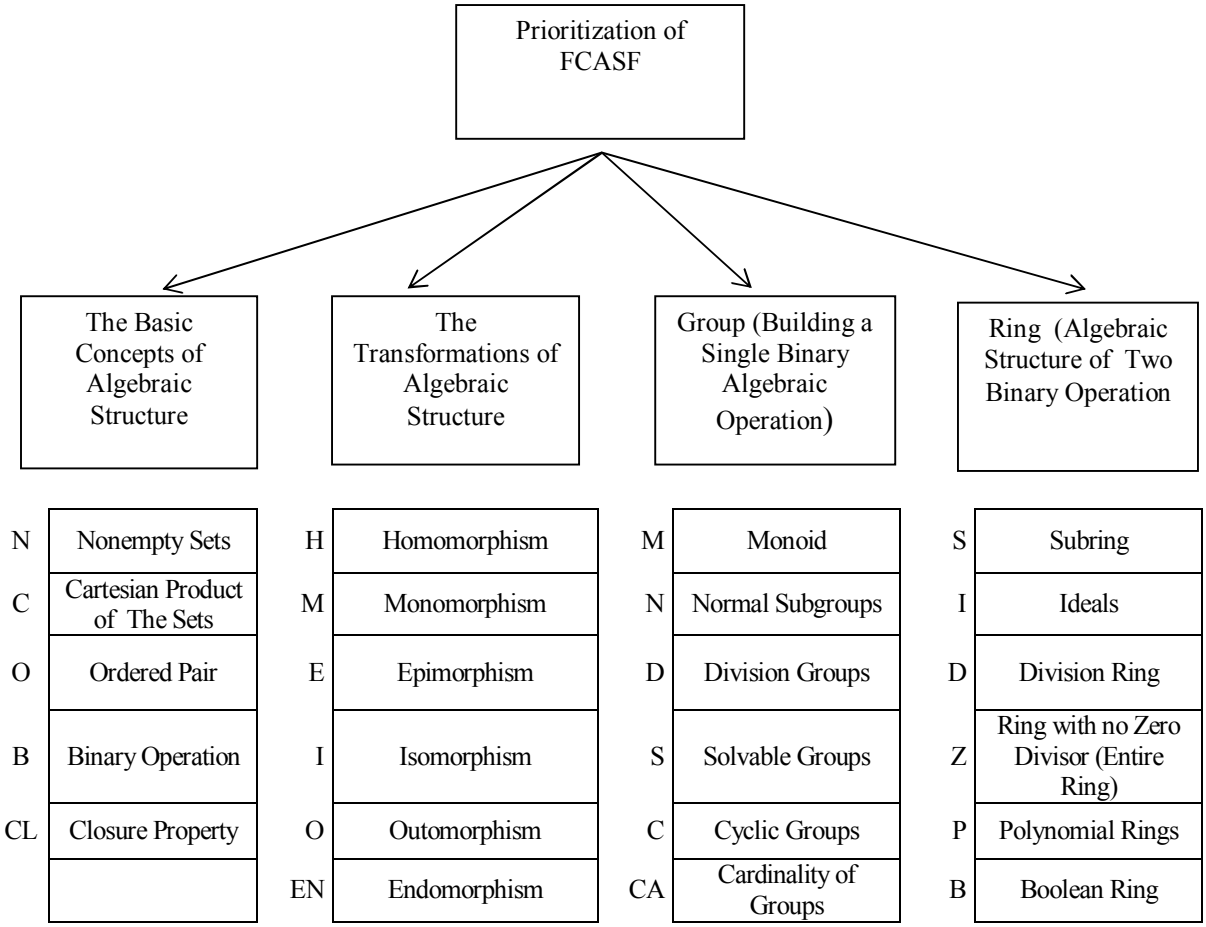

Fig. 2. Non complete hierarchical structure of some factors affecting the field concept.

Table 3 was rewritten as a four digit matrix. A comparison matrix was as follows:

$$A = \begin{array}{c|cccc} & C & T & G & R \\ C & 1.0000 & 0.1111 & 0.2000 & 0.1428 \\ T & 9.0000 & 1.0000 & 5.0000 & 3.0000 \\ G & 5.0000 & 0.2000 & 1.0000 & 0.3333 \\ R & 7.0000 & 0.3333 & 3.0000 & 1.0000 \end{array}$$

For n= 4 the randomization index = 0.89

λ_{max} = 4.0990 (The calculation of this value will be shown latter)

CI = $(\lambda_{max} - n) / n-1 = (4.0990-4) / (4-1) = 0.0990 / 3 = 0.0330$

CR = CI / RI = 0.0330 / 0.89 = 0.0371 since this value is less than 10% it can conveniently be claimed that the judgement matrix is self consistent. Since the judgment matrix A shows self consistentcy the final result is obtained by the of this matrix which gives the priority of the criteria. This was done by the use of eigenvalue and eigenvector method developed by Saaty 1991 [46]. This method has three steps as follows:

First step: Taking the square of matrix A.

Second step: summing up and the normalization of the rows of A^2 matrix.

Third step; If necessary repeating these steps.

Table 3. Pairwise comparison matrix of the main criteria according to the target

Main Criteria	The Basic Concepts of Algebraic Structure (C)	The Transformations of Algebraic Structure (T)	Group (Building a Single Binary Algebraic Operation) (G)	Ring (Algebraic Structure of Two Binary Operation (R)
The Basic Concepts of Algebraic Structure (C)	1	1/9	1/5	1/7
The Transformations of Algebraic Structure (T)	9	1	5	3
Group (Building a Single Binary Algebraic Operation) (G)	5	1/5	1	1/3
Ring (Algebraic Structure of Two Binary Operation (R)	7	1/3	3	1

However if these A matrix is taken in digitalized four digit manner there are no differences between the first and second iterations as regards to priorities . That was why only first and second steps were used in this study. 1st step: Taking the square of matrix A.

Normalization process; is obtained by the division of summations of each row with the general sum. For instance the normalization of the basic concepts of the algebraic structures is 6.3788 / 172.4204 = 0.0370. The normalized values give order of the importance or the priorities. The eigenvector corresponding to the highest eigenvalue (λ_{max}) W= (6.3788; 97.7515; 21.1802; 47.1099). Normalize eigenvector W= (0.0370; 0.5669; 0.1228; 0.2733) gives the priority according to the target criteria. According to it the criterion with the highest value of 0.5669 is obtained for the transformation of the algebraic structures. This was followed by ring (0.2733), group (0.1228) and the basic concepts of algebraic structure (0.0370).

One can look at the difference between the A judgment matrix and W* matrix which can be obtained from the priority vector W= (W_1, W_2, W_3, W_4). This difference can reach big dimensions in the case of inconsistency. The consistency of the judgment matrix has a positive effect on the differences.

$$A^2 = \begin{array}{c|cccc} & C & T & G & R \\ C & 3.9995 & 0.3098 & 1.3839 & 0.6856 \\ T & 64.0000 & 3.9998 & 20.8000 & 8.9517 \\ G & 14.1331 & 1.0666 & 3.9999 & 1.9806 \\ R & 31.9997 & 2.0443 & 9.0665 & 3.9994 \end{array}$$

2. Step: Taken the summations of the rows and normalizing A^2 matrix;

Row Sums	Normalized Row Sums (Priorities)	Criteria
6.3788	0.0370	The Basic Concepts of Algebraic Structure
97.7515	0.5669	The Transformations of Algebraic Structure*
21.1802	0.1228	Group
47.1099	0.2733	Ring
Total: 172.4204	1.0000	

There was no need for the establishment of the W* matrices corresponding for the each judgment matrix. Since according AHP method the judgement matrices are self consistent the differences between W* matrices are not very big.

$$W^* = \begin{vmatrix} w_1/w_1 & \dots & w_1/w_4 \\ w_2/w_1 & \dots & w_2/w_4 \\ w_3/w_1 & \dots & w_3/w_4 \\ w_4/w_1 & \dots & w_4/w_4 \end{vmatrix} = \begin{vmatrix} 1.0000 & \dots & 0.1354 \\ 15.3244 & \dots & 2.0750 \\ 3.3204 & \dots & 0.4496 \\ 7.3854 & \dots & 1.0000 \end{vmatrix}$$

Priority option*

λ_{max}, values are obtained as follows from summation of the column values resulted after the matrix products .This gives

A, matrix				W, column vector		Product matrix	
1	1/9	1/5	1/7	0.0370		0.1636	
9	1	5	3	0.5669		2.3335	
5	1/5	1	1/3	0.1228	=	0.5123	
7	1/3	3	1	0.2732		1.0896	
					λ_{max} =	4.0990	

as the column matrix. When we sum the column terms we obtain λ_{max} = 4.0990. The number of criteria n = 4 and λ_{max} value are very close to each other .We can say that the judgments are self consistent. However the definite evaluations should be done by looking at the consistency ratios.

3. 3. 2. Determination of the order priority of the sub criteria

The calculations in this part are related to the determination of the priority of the third level or the sub criteria according to the upper criteria. The third level will be compared with the second level elements one by one .According to the AHP methodology the lower level is compared with the upper level [34]. In the comparison of the options of the basic concepts of algebraic structures according to their factor the first thing was the establishment of the pairwise comparison matrix. Since the basic concepts in algebraic structures and the related sub criteria are qualitative variables. The comparison matrix is formed by the experts according to 1-9 measure. The resulting priorities determine the level of importance of the sub factor as regards basic concepts of algebraic structure factors which play an effective role in FCASF.

In other 4 criteria the same procedure defined by Saaty will be repeated in order prioritize sub factors effecting to the factors which effect the concept of vector space. Firstly in Table 4 was given the pairwise comparisons of the basic concepts in algebraic structures according to the target as the hierarchical sub factor .

Table 4. The pairwise comparison matrix of the factors of the basic concepts in algebraic structures according to the target

Sub Criteria	Nonempty Sets (N)	Cartesian Product of The Sets (C)	Ordered Pair (O)	Binary Operation (B)	Closure Property (CL)
Nonempty Sets (N)	1	1/5	1/6	1/7	1/9
Cartesian Product of The Sets (C)	5	1	1/2	1/4	1/5
Ordered Pair (O)	6	2	1	1/3	1/5
Binary Operation (B)	7	4	3	1	1/3
Closure Property (CL)	9	5	5	3	1

Table 4 the matrix was re written in four digits. The A_1 matrix;

A_1 =

	N	C	O	B	CL
N	1.0000	0.2000	0.1667	0.1429	0.1111
C	5.0000	1.0000	0.5000	0.2500	0.2000
O	6.0000	2.0000	1.0000	0.3333	0.2000
B	7.0000	4.0000	3.0000	1.0000	0.3333
CL	9.0000	5.0000	5.0000	3.0000	1.0000

For n= 5 the Random index = 1.11; λ_{max} = 5.2065 (The calculation of this value will be shown latter); CI = (λ_{max} - n) / n-1 = (5.2065-5) / (5-1) = 0.2065 / 4 = 0.0516; CR = C.I / R.I = 0.0516 / 1.11 = 0.0465, since this value is less than 10% we can conveniently claim that A_1 judgement matrix is self consistent. Although it is highly difficult that achieve the full consistency for the comparison matrices obtained from the judgment, the matrix formed by the experts were found to be fully consistent.

The matrix terms were found as follows: For instance the term a_{11} is obtained by the comparison of nonempty sets of nonempty sets as regards to the basic concepts of algebraic structures. The value of

this parameter is naturally 1. The 1/5 value of the a_{12} is obtained by the comparison of the Cartesian products of the sets of nonempty sets as regards to the basic concepts in algebraic structures .The other elements of the matrix are obtained in the similar manner. All the comparison matrices of the other 5 criteria used in this study are obtained by the use of the same method. Since the square and row summation of the matrix are necessary and sufficient for the determination of the priorities of the criteria and sub criteria the processes were carried out in successive manner to determine the importance of the sub criteria. Square matrix:

$A^2_1 =$

	N	C	O	B	CL
N	5.0004	1.8605	1.4176	0.7247	0.3432
C	16.5500	5.0000	3.5835	1.9812	1.1388
O	26.1331	7.5332	5.0001	2.6240	1.5777
B	54.9997	17.0665	10.8334	5.0001	2.8443
CL	94.0000	33.8000	23.0003	10.2026	4.9998

Row Sums	Normalized Row Sums (Priorities)	Sub Criteria
9.3464	0.0277	N
28.2535	0.0838	C
42.8681	0.1271	O
90.7440	0.2691	B
166.0027	0.4923	CL*
Total: 337.2147	1.0000	

These matrix procedures enable us to prioritize the sub factors of the basic concepts in algebraic structures. According to it when as regard to the basic concepts in algebraic structures are compared with the others the closure property is the most important factor affecting to FCASF. When we come to the estimation of λ_{max} value:

A_1, matrix					W_1, column vector		Product matrix
1	1/5	1/6	1/7	1/9	0.0277		0.1588
5	1	1/2	1/4	1/5	0.0838		0.4516
6	2	1	1/3	1/5	0.1271	=	0.6491
7	4	3	1	1/3	0.2691		1.3436
9	5	5	3	1	0.4923		2.6034
						λ_{max} =	5.2065

According to it the number of criteria n= 5 and λ_{max} = 5.2065 values are quite close to each other. We can conveniently conclude that the judgments are self-consistent.

Secondly the options were compared according to the transformations in the algebraic structure factors. That enables us to obtain the prioritization of the sub factors as regards to transformation of the algebraic structure. Since the variables based on the transformations of the algebraic structure are qualitative variables the A_2 comparison matrix is formed by the experts according to 1-9 scale and if A_2 given in four digits then the matrix is written:

For n = 6 the Randomization index= 1.25

λ_{max} = 6.2681 and number of criteria n=6 are quite close to each other . Therefore we can conclude that the judgments are self consistent

CI= (λ_{max}-n) / (n-1) = (6.2681 - 6) / (6-1) = 0.0536 and the consistency ratio:

CR= CI / RI= 0.0536 / 1.25= 0.0429. Since this value is less than 10% we can say that A_2 judgment matrix is self-consistent.

Priority option*

A_2 =

	H	M	E	İ	O	EN
H	1.0000	0.3333	0.3333	0.1111	0.1429	2.0000
M	3.0000	1.0000	0.5000	0.1429	0.2500	3.0000
E	3.0000	2.0000	1.0000	0.1429	0.2500	3.0000
İ	9.0000	7.0000	7.0000	1.0000	2.0000	5.0000
O	7.0000	4.0000	4.0000	0.5000	1.0000	4.0000
EN	0.5000	0.3333	0.3333	0.2000	0.2500	1.0000

The isomorphism factor of 0.4611 obtained after taking the square of A_2 matrix and the rows of A_2^2 are summed up is the most important factor resulting FCASF as regard to transformations of algebraic structures. This parts deals with the prioritization of the sub criteria among themselves. Here if the A_3 matrix is written in four digits;

A_3 =

	M	N	D	S	C	CA
M	1.0000	0.2500	0.2000	0.1429	0.2000	0.5000
N	4.0000	1.0000	0.3333	0.2000	0.5000	2.0000
D	5.0000	3.0000	1.0000	0.3333	0.5000	2.0000
S	7.0000	5.0000	3.0000	1.0000	4.0000	4.0000
C	5.0000	2.0000	2.0000	0.2500	1.0000	0.5000
CA	2.0000	0.5000	0.5000	0.2500	2.0000	1.0000

For n = 6 the randomization index = 1.25

λ_{max} = 6.5776 (Since the value of number of criteria n=6 and λ_{max}= 6.5776 are very close one can easily concede that these judgement are self-consistent)

CI = (λ_{max}-n) / (n-1) = (6.5776-6) / (6-1) = 0.5776 / 5 = 0.1155 and the consistency ratio:

CR = CI / RI = 0.1155 / 1.25 = 0.0924. Since this value is less than 10% one can conclude that A_3 judgment matrix is self consistent. Since there are qualitative evaluations done at this level all the matrices are subjected to the consistency analysis.

After taking the square of A_3 and computing the row sums of A_3^2 and then normalized and the solvable groups priority factor of 0.4218 are the most important factor effecting to FCASF as regards to group factor.

If we want to compare the options according to ring factor and for A_4 comparison matrix in four digits.

A_4 =

	S	I	D	Z	P	BO
S	1.0000	0.3333	0.2500	0.2000	0.1429	0.1111
I	3.0000	1.0000	0.5000	0.3333	0.1667	0.1429
D	4.0000	2.0000	1.0000	0.5000	0.2500	0.2000
Z	5.0000	3.0000	2.0000	1.0000	0.3333	0.2000
P	7.0000	6.0000	4.0000	3.0000	1.0000	0.3333
BO	9.0000	7.0000	5.0000	5.0000	3.0000	1.0000

For n = 6 the randomization index = 1.25

λ_{max} = 6.1979 value shows that the judgement are self consistent.

CI = (λ_{max}-n) / (n-1) = (6.1979-6) / (6-1) = 0.0396 and consistency ratio: CR = CI / RI = 0.0396 / 1.25 = 0.0317. Since this value is less than 10 % one can claim that the judgement matrix of A_4 is self-consistent. Since the whole of the matrices give qualitative analysis, there was no consistency test made at this part of the study.

Square of A_4 matrix is taken, the row sums of A_4^2 matrix are computed then the normalization process is carried out. The value of 0.4554 for the Boolean ring factor shows that Boolean ring has the highest effect on FCASF when the other sub factors of the ring are compared.

In conclusion the most prioritized factors are given Table 5. These values used in the calculation of the estimated affects are given in Fig. 3.

Table 5. The Factors with the Highest Weight

Risk factors	The factors with the highest weight	Priority value
General Factors	The Transformation of Algebraic Structures	0. 5669
The Basic Concepts of Algebraic Structure	Closeness Feature	0.4923
The Transformations of Algebraic Structures	Isomorphism	0.4611
Group (Building a Single Binary Algebraic Operation)	Solvable Groups	0.4218
Ring (Algebraic Structure of Two Binary Operation	Boolean Ring	0.4554

4. Results

In this study the AHP technique developed by Thomas L. Saaty was investigated and used in multi criteria decision making processes. It was found to be very effective in multi criteria decision making problems. Especially the inclusion of the personal judgement and experiences increased its sensitivity a lot. This is the biggest difference between AHP and other decision making processes.

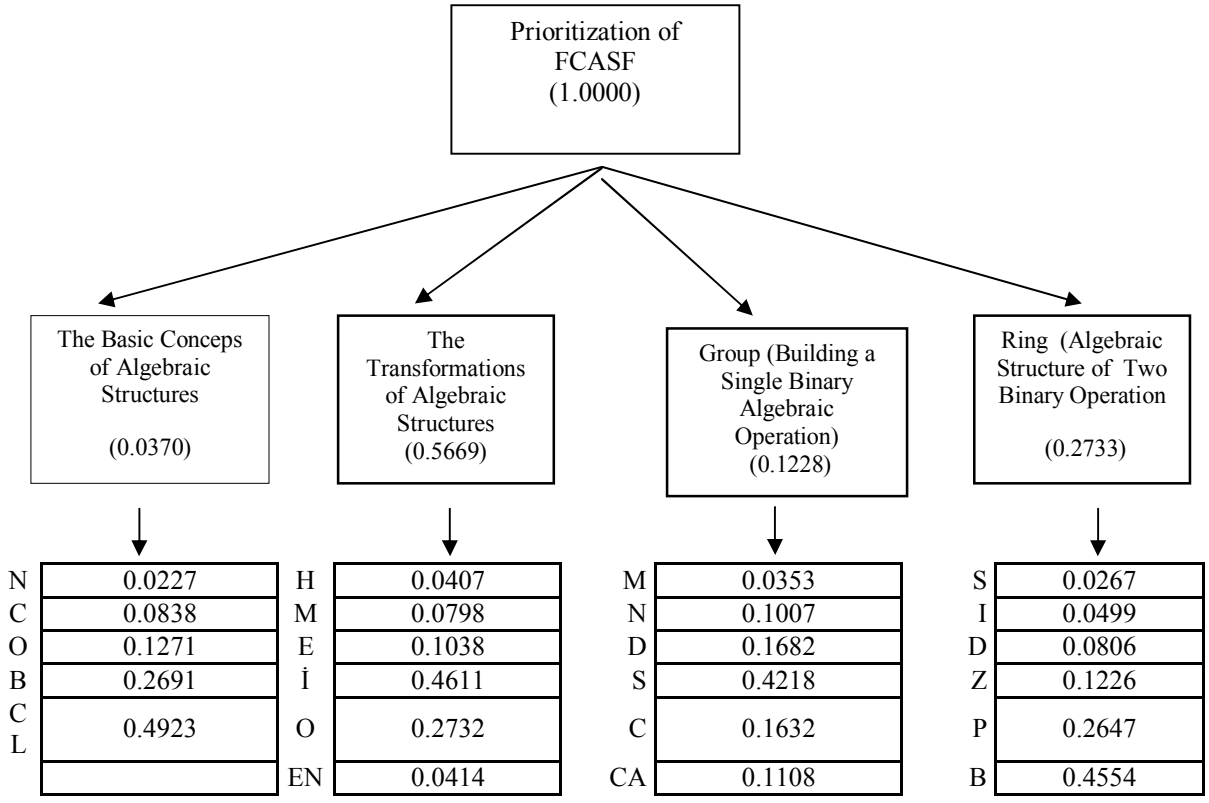

Fig. 3. The priorities or the importance of the main and sub-criteria in hierarchical model.

The first process in the application is finding an expert in data engineering. Then after the discussion with the experts it was decided how the system should be constructed. The next stage was finding the some factors affecting the field defined on the commutative ring with identity element and determines their superiorities against each other. For this purpose the main and sub criteria effecting to the field were determined. Then the decision problem was shown with a three stage hierarchical model. The first stage represents the target and second and third levels represent the main and sub criteria. Before the final solution the consistency ratios of the judgement matrices were checked and found satisfactory. This is very important because the results of the AHP methodology is depend upon these judgement matrices. If there was any inconsistency in the judgment made this should be discussed with the

experts and necessary modification should be made to decreases it down to the acceptable level of less than 10 %.

In this study the prioritization of FCASF problem was solved by the use of AHP method and the reliability of the results were tested by the use of the same method. It was found that very result obtained was consistent. The priority values obtained were tabulated in the table showing the values with the highest load.

The correct evaluation of the data in this study will make great contribution to the science of mathematics and its teaching and learning processes. This determines which criteria should be given to the student on which prioritization should be carried out. In other words the
teaching of field factors by the use of correct priority hierarchy will make the teaching process quite easy and decrease the negative attitude of the students towards it. This will pave the way for healthy and permanent learning of mathematics.

5. Suggestions

According to the data obtained in this study we can easily propose the use of AHP for the people who are facing multi criteria decision problems. It became apparent that the efforts of the theoreticians are not enough for development and go to result, the establishment and application of such methods badly needed in practice. It was found that the interest, field knowledge and the assistance of the applicants were also very important.

In the solution of the multi criteria decision making problems with the use of AHP the use of more than one expert in both the establishment of the hierarchical model and making the judgement will increase the validity of the research. Carrying out similar research in the other fields of mathematics will be very beneficial for the mathematicians and the students.

It is also important that the as well as the expert the opinions of the student should also be taken. Also the data of this study should be carefully evaluated by the experts and the theoreticians of AHP. The problem hierarchic structure of which was given in Fig. 2 is converted from the prioritization problem to the decision problem if the options are added as the IV level. This may be a subject of another paper.

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References

[1] Saaty T. Fundamentals of Decision Making and Priority Theory, RWS Publications, Vol: 6, Pittsburgh, U.S.A, 2000.
[2] Crawford G B. The geometric mean procedure for estimating the scale of a judgment matrix. Math Model 1987;9:327-334.
[3] Weiss EN. Using the analytic hierarchy process in a dynamic environment, Math Model 1987;9:285-292.
[4] Dyers JS. Remarks on the analytic hierarchy process, Manage Sci 1990;36:249-259.
[5] De Gram JG. Technical Report M.F.A. 80-3, National Institute for Water Supply Leidshcendamthe Netherlands, Presented at EURO IV Cambridge, July 22-25, 1980.
[6] Aczel J, Saaty T. Procedure for synthesizing ratio judgments. J Math Psych 1983;27:93-102.
[7] Belton V, Gear T. On a shortcomings of Saaty's method of analytic hierarchies. Omega 1983;11:228-230.
[8] Saaty T, Vargas L. The Allocation of Intangible Resources: The Analytic Hierarchy Process and Linear Programming, University of Pittsburgh, Pittsburgh, U.S.A, 1984.
[9] Dyer JF, Wendell RE. A Critique of the AHP, Working Paper, Department of Management, The University of Texas Austin TX, 1985 84:85:4-24.
[10] Lunda JR, Palmer RN. Subject Evaluation: Linguistics Scales in Pairwise Comparison Methods, Reprints of the ISAHP, Tianjin University, China, Sept. 6-9, 1986;561-568.
[11] Belton V. A comparison of the AHP and a simple multi-attribute value function. EJOR 1986;26:17-21.
[12] Madu CN. An economic decision model for technology transfer. EMI 1988;5:153-62.
[13] Golden BL, Wang Q. An Alternate Measure of Consistency, College of Business and Management University of Maryland College Park, 1989.
[14] Badiru AB, Friends. A multiattribute spreadsheet model for manufacturing technology justification. Comput Ind Eng 1991;21:29-33.
[15] Suresh NC, Kaparthi S. Flexible automation investment: a synthesis of two multi-objection modeling approaches. Comput Ind Eng 1992;22:257-272.
[16] Datta V, Friends. Multi-attribute decision model using the AHP for justification of manufacturing systems. IJPE 1992 28:227-234.
[17] Madu CN. On the total productivity management of a maintenance float system through AHP applications. IJPE 1994;34:2201-207.

[18] Davies MAP. A multi-criteria decision model application for managing group decision. JORS 1994;45:47-58.
[19] Ramanathan R, Gansh LS. Using AHP for resource allocation problems. EJOR 1995;80:410-417.
[20] Tavana M, Snehamay B. Strategic assessment model (SAM): A multiple criteria decision support system for evaluation of strategic alternativest. Dec Sci 1995;26:119–143.
[21] Jain B. Nag B. A decision support model for investment decisions in new ventures. European J Operat Res 1996;90:473–486.
[22] Rosenbloom ESA. Probabilistic interpretation of the final ranking in AHP. EJOR 1996;371-378.
[23] Ülengin F. Ulaşım Problemlerine Analitik Hiyerarşi Yaklaşımı: İstanbul İçin Bir Uygulama, TMMOB İstanbul 2. Kent İçi Ulaşım Kongresi Bildirileri, 1994; 103-121.
[24] Aladağ Z, Ulusoy S. Lastik Sektöründe Analitik Hiyerarşi Yöntemi ile Ürün Geliştirme Sürecinin İncelenmesi, Kara Harp Okulu 1 nci Sistem Mühendisliği and Savunma Uygulamaları Sempozyumu, Bildiriler - I, 1995; 673-683.
[25] Albayrak C, Albayrak E. Kredi Talepleri'nin Değerlendirilmesinde Analitik Hiyerarşi Yönteminin Kullanılması, Kara Harp Okulu, 1 nci Sistem Mühendisliği and Savunma Uygulamaları Sempozyumu, Bildiriler-1, 1995; 593-599.
[26] Öner A, Ülengin F. Silah Seçiminde AHP Yaklaşımı, Kara Harp Okulu, 1. Sistem Mühendisliği and Savunma Uygulamaları Sempozyumu, Bildiriler–II, 1995; 1109-1121.
[27] Eltez A, Moralı N. Analitik Hiyerarşi Sürecinin Sistem Tasarımında Kullanımı ve Denizli Uygulaması, Araştırma Sempozyumu'97 Bildirileri D.İ.E. – Ankara, 1997.
[28] Emel G G, Emel E. Bir İşletmenin Pazar Gücünün Belirlenmesinde Çok Kriterli Bir Yaklaşım, Verimlilik Dergisi, MPM, 1998; 2:53.
[29] Uyar Y. İş Güvenliği Yatırımlarında Etkin Karar Verme Stratejileri Oluşturmak için Analitik Hiyerarşi Prosesi, Doktora Tezi, G.Ü. Fen Bilimleri Enstitüsü, Ankara, 2001.
[30] Ulusam S. Karpal Tünel Sendromuna Yakalanma Riskinin Geliştirilen Bir Uzman Sistem ile Tahmin Edilmesi, Yüksek Lisans Tezi, G.Ü. Fen Bilimleri Enstitüsü, Ankara, 2001.
[31] Taşçı D. Soyut Cebir, Alp Yayınevi, Ankara, 2007, 310.
[32] Vargas L G. An Overview of the Analytic Hierarchy Process and Its Application, EJOR, 1990; 48: 1, 2-8.
[33] Zahedi F. A Utility Approach to the Analytic Hierarchy Process, Mathematical Modeling, 1987; 9: 3-5, 387-395.
[34] Saaty T. The Analytic Hierarchy and Analytic Network Processes, MCDM XV-th International Conference, Ankara, Turkey, 2000.
[35] Saaty T. The Seven Pillars of the Analytic Hierarchy Process, 322 Mervis Hall, Pittsburgh, U.S.A, 1999.
[36] Barbeau E. Reciprocal Matrices of Order 4, Mathematical Modeling, 1987; 9:3-5, 321-326.
[37] Saaty T. The Analytic Hiyerarchy Process (AHP) For Decision Making, Kobe, Japan. 1980.
[38] Harker P T, Vargas, L G. The Theory of Ratio Scale Estimation: Saaty's Analytic Hierarchy Process, Management Science, 1987; 33: 11, 1383-1403.
[39] Saaty T. Physic as a Decision Theory, EJOR, North – Holland, 1990; 48: 98-104.
[40] Saaty T. Fundamentals of the Analytic Network Process, ISAHP, Kobe, Japan, 1999.
[41] Miller G A. The Magic Number Seven, Plus or Minus Two: Some Limits on our Capacity for Processing Information, Psychological Review, 1956; 63: 81-97.
[42] Vargas L G. Utility Theory and Reciprocal Pairwise Comparisons: The Eigenvector Method, Socio-Economic Planning Sciences, 1986; 20: 6, 387-391.
[43] Saaty T. The Analytic Hiyerarchy Process, McGraw-Hill International Book Company, U.S.A, 1980.
[44] http://www.expertchoice.com
[45] Golden B L, Wasil E A and Harker P T. (Eds) The Analytic Hierarchy Process, Applications and Studies. Berlin: Springer-Verlag, 1989.
[46] Saaty T. Some mathematical concepts of the analytic hierarchy process, Behaviormetrica. 1991;29:1-9.

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