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Radiosity

1 Form factors

Assume a scene is composed of n patches $\mathbf{P}_i$. A subset of these patches encloses the scene. Each patch receives light from other patches and reflects light back into the scene. All reflections are assumed to be diffuse only. The interaction between patch i and patch j is given by a *form factor* f_{ij}. It determines how much light $\mathbf{P}_i$ receives from patch $\mathbf{P}_j$:

$$f_{ij} = \frac{\text{light from } \mathbf{P}_j \text{ to } \mathbf{P}_i}{\text{all light from } \mathbf{P}_j}.$$

We immediately have:

$$\sum_{i=1}^{n} f_{ij} = 1; \quad j = 1, \ldots, n.$$

The f_{ij} are given by

$$f_{ij} = \frac{1}{\|\mathbf{P}_i\|} \int_{\mathbf{P}_i} \int_{\mathbf{P}_j} \frac{\cos\Phi_i \cos\Phi_j \mathrm{d}\mathbf{P}_j}{\pi r^2} \tag{1}$$

where the meaning of the involved terms is from Figure 1.

In order to motivate (**??**), assume that $\mathbf{P}_i$ has very small area. Then, again using Figure 1, we would have

$$f_{ij} = \frac{\cos\Phi_i \cos\Phi_j \|\mathbf{P}_j\|}{r^2}.$$

From this we arrive at (1) by integrating over the areas of $\mathbf{P}_i$ and $\mathbf{P}_j$. Note that we have

$$f_{ij}\|\mathbf{p}_i\| = f_{ji}\|\mathbf{P}_j\|. \tag{2}$$

An easier way to determine the form factors is by the use of a *hemicube*, a simplified model of a hemisphere. Subdivide each of the five faces of the hemicube into squares (pixels), resulting in a total of M pixels. Center the upper half of a cube at a subpatch

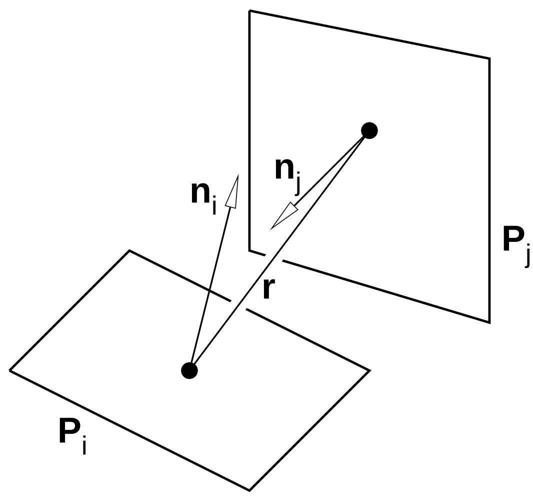

Figure 1: The form factor geometry.

$\mathbf{P}_{i,\rho}$ of $\mathbf{P}_i$ such the top is parallel to $\mathbf{P}_i$. We assume that are R such subpatches.

Now project $\mathbf{P}_j$ onto the hemicube. Count all pixels visible from $\mathbf{P}_{i,\rho}$. Let the number of these pixels be $n_{i,j,\rho}$. Keep in mind that we only count pixels which are not occluded by other objects. We repeat this for all subpatches $\mathbf{P}_{i,\rho}$, which means a recentering of the hemicube. Then

$$f_{ij} = \frac{1}{\|\mathbf{P}_i\|} \sum_{\rho=1}^{R} \frac{\|\mathbf{P}_{i,\rho}\| n_{ij\rho}}{M}$$

2 Setting up the linear system

Let b_i the brightness of patch $\mathbf{P}_i$. Let e_i be the amount of light emitted by $\mathbf{P}_i$, let ρ_i be the reflectivity of $\mathbf{P}_i$, and let a_i be its area. Now the total intensity of light leaving $\mathbf{P}_i$ is given by

$$a_i b_i = a_i e_i + \rho_i \sum_{j=0}^{n} f_{ij} a_j b_j; \quad i = 1, \ldots, n. \tag{3}$$

The first term gives how much light is emitted by $\mathbf{P}_i$. The second term collects all contributions from the other patches; the factor ρ_i shows how much of that is being reflected by $\mathbf{P}_i$.

Invoking (2), we may rewrite (3) as

$$a_i b_i = a_i e_i + \rho_i \sum_{j=0}^{n} f_{ji} a_i b_j; \quad i = 1, \ldots, n$$

and simplify to

$$b_i = e_i + \rho_i \sum_{j=0}^{n} f_{ji} b_j; \quad i = 1, \ldots, n. \tag{4}$$

In matrix form:

$$\mathbf{b} = \mathbf{e} + RF\mathbf{b} \tag{5}$$

or

$$\mathbf{e} = [I - RF]\mathbf{b} \tag{6}$$

where R is a diagonal matrix holding the ρ_i and F is the matrix of form factors. The matrix $I - RF$ has 1's on the diagonal since we can assume $f_{ii} = 0$.

3 Solving the system

This linear system for $\mathbf{b}$ may be huge, but it will always be sparse. For such systems, an iterative solution is most effective. In an iterative scenario, one produces a first guess $\mathbf{b}^0$ for $\mathbf{b}$, e.g., by setting all $b_i = 0.5$. Then a next guess $\mathbf{b}^1$ is found from

$$\mathbf{b}^1 = \mathbf{e} + RF\mathbf{b}^0$$

and this process is continued until convergence happens, which is guaranteed for this scenario. In practice a vector $\mathbf{b}^k$ is called the solution if $\|\mathbf{b}^k - \mathbf{b}^{k-1}\| < \epsilon$ for some tolerance epsilon. This iterative process is known as $Gauss - Jacobi$ iteration.

In that method, all elements of $\mathbf{b}^k$ is updated from $\mathbf{b}^{k-1}$ in one step. A different strategy is usually more effective. Once the first element of $\mathbf{b}^k$ has been computed, the second element of $\mathbf{b}^k$ may be computed by using this new b_1^k instead of the old b_1^{k-1}. Similarly, the b_3^k may be computed using b_1^k and b_2^k. This process of immediate updating is known as *Gauss-Seidel* iteration.

The matrix $I - RF$ is strictly diagonally dominant, with all eigenvalues less than unity in absolute value. For such matrices, a power expansion exists for the inverse:

$$[I - RF]^{-1} = \sum_{i=0}^{\infty} [RF]^i.$$

Thus

$$\mathbf{b} = \sum_{i=0}^{\infty} [RF]^i \mathbf{e} = \mathbf{e} + [RF]\mathbf{e} + [RF]^2\mathbf{e} + \dots$$

Each term corresponds to one level of tracking reflections.