

Place Value

Place-value chart:

Billions period			Millions period			Thousands period			Ones period		
hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
		6 ,	3	9	2 ,	5	8	0 ,	1	0	1

Expanded form: $6,000,000,000 + 300,000,000 + 90,000,000 + 2,000,000 + 500,000 + 80,000 + 100 + 1$

Standard form: 6,392,580,101

Word form: six billion, three hundred ninety-two million, five hundred eighty thousand, one hundred one

Write the word name for each number and tell the value of the underlined digit.

1. 3,552,308,725

Three billion, five hundred fifty-two million, three hundred eighty thousand, seven hundred twenty-five; fifty million

2. 843,208,732,833

Eight hundred forty-three billion, two hundred eight million, seven hundred thirty-two thousand, eight hundred thirty-three; 800 billion

3. Write $2,000,000,000 + 70,000,000 + 100,000 + 70,000 + 3,000 + 800 + 10$ in standard form.

2,070,173,810

4. **Number Sense** What number is 100,000,000 more than 5,438,724,022?

5,538,724,022

Comparing and Ordering Whole Numbers

Order these numbers from least to greatest:
4,752,213 5,829,302 4,234,295 4,333,209.

Step 1: Write the numbers, lining up places. Begin at the left to find the greatest or least number.

4,752,213
5,829,302
4,234,295
4,333,209

5,829,302 is the greatest.

Step 2: Write the remaining numbers, lining up places. Find the greatest and least of these.

4,752,213 ← greatest
4,234,295 ← least
4,333,209

4,752,213 is the greatest of these.
4,234,295 is the least.

Step 3: Write the numbers from least to greatest.

4,234,295
4,333,209
4,752,213
5,829,302

Complete. Write $>$, $<$, or $=$ in each \bigcirc .

1. 7,642 \bigcirc 7,843

2. 2,858,534 \bigcirc 2,882,201

Order these numbers from least to greatest.

3. 768,265 769,205 739,802

739,802; 768,265; 769,205

4. Write the areas of each country in order from greatest to least.

28,748; 28,450;

27,830; 27,750

Country	Area in Square Kilometers
Albania	28,748
Burundi	27,830
Solomon Islands	28,450
Haiti	27,750

Name _____

Reteaching

1-3

Reteaching 1-3

Decimal Place Value

Here are different ways to represent 2.753149.

Place-value chart:

Ones	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
2	7	5	3	1	4	9

Expanded Form:

$$2 + 0.7 + 0.05 + 0.003 + 0.0001 + 0.00004 + 0.000009$$

Standard form: 2.753149**Word Form:** Two and seven hundred fifty-three thousand one hundred forty-nine millionths

Complete the place-value chart for the following number. Write its word form and tell the value of the underlined digit.

1. 6.324657

Ones	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
6	3	2	4	6	5	7

Six and three hundred twenty-four thousand six hundred fifty-seven millionths;
Five hundred-thousandths

Write each number in standard form.

2. $5 + 0.1 + 0.03 + 0.006 + 0.0007 + 0.00002 + 0.000004$

5.136724

3. Two and seven hundred twenty-four thousand three hundred sixty-five millionths

2.724365

Comparing and Ordering Decimals

List the numbers in order from least to greatest:

6.943, 5.229, 6.825, 6.852, 6.779

Step 1: Write the numbers, lining up places. Begin at the left to find the greatest or least number.

6.943
5.229
6.825
6.852
6.779

5.229 is the least.

Step 2: Write the remaining numbers, lining up places. Find the greatest and least. Order the other numbers.

6.943 ← greatest
6.825] → 6.825
6.852] → 6.852
6.779 ← least

6.779 is the least.
6.943 is the greatest.
6.852 is greater than 6.825.

Step 3: Write the numbers from least to greatest.

5.229
6.779
6.825
6.852
6.943

Complete. Write $>$, $<$, or $=$ for each \bigcirc .

1. $7.539 > 7.344$

2. $9.202 < 9.209$

3. $0.75 = 0.750$

Order these numbers from least to greatest.

4. 3.898 3.827 3.779

3.779, 3.827, 3.898

5. 5.234 5.199 5.002 5.243

5.002, 5.199, 5.234, 5.243

Which had the faster speed?

6. Driver A or Driver D

Driver A

7. Driver C or Driver A

Driver C

Car Racing Winners

Driver	Average Speed (mph)
Driver A	145.155
Driver B	145.827
Driver C	147.956
Driver D	144.809

Problem Solving: Look for a Pattern

Mr. Nagpi works in a machine shop. In the shop, the drill bits are kept in a cabinet with drawers. The drawers are marked with the diameter of the bits as shown on the right. Some of the labels are missing. Help Mr. Nagpi complete the drawer labels.

Drill Bits				
0.10 in.	0.12 in.	0.14 in.	0.16 in.	0.18 in.
0.20 in.	0.22 in.	0.24 in.	0.26 in.	0.28 in.
0.30 in.	0.32 in.	0.34 in.	0.36 in.	0.38 in.

Read and Understand

What do you know?

What are you trying to find?

Some drawers are labeled with decimals.

A way to find the values of the missing labels

Plan and Solve

Find a pattern for the decimals.

- Look for a pattern to the change in the tenth-values across a row or down a column.
- Look for a pattern to the change in the hundredth-values across a row or down a column.
- Use the patterns to complete the table.
- The tenth-values are not increasing across a row. They are increasing by 1 down a column.
- The hundredth-values are increasing by 2 across a row. They are not increasing down a column.
- The missing labels in the third row are 0.36 in. and 0.38 in.

Find the pattern in the table. Then fill in the missing values in the table.

0.20	0.21	0.22	0.23	0.24
0.50	0.51	0.52	0.53	0.54
0.80	0.81	0.82	0.83	0.84

Mental Math

There are several ways that you can add and subtract numbers mentally to solve a problem.

Commutative Property of Addition

You can add two numbers in any order.

$$15 + 27 = 27 + 15$$

Associative Property of Addition

You can change the groupings of addends.

$$17 + (13 + 10) = (17 + 13) + 10$$

Compatible numbers are numbers that are easy to compute mentally.

$$25 + 93 + 75$$

25 and 75 are compatible because they are easy to add.

$$\begin{aligned} 25 + 93 + 75 &= (25 + 75) + 93 \\ &= 100 + 93 = 193 \end{aligned}$$

With **compensation**, you adjust one number to make computations easier and compensate by changing the other number.

$$\begin{array}{r} 320 - 190 \\ + 10 \quad + 10 \\ \hline \downarrow \quad \downarrow \\ 330 - 200 = 130 \end{array}$$

Add or subtract mentally.

1. $265 + 410 + 335 = \underline{1,010}$ 2. $885 - 155 = \underline{730}$
3. $2,500 + 1,730 + 70 = \underline{4,300}$ 4. $1,467 - 397 = \underline{1,070}$

5. **Number Sense** How many more strikeouts did Pitcher A have than Pitcher C?

152 more strikeouts

6. How many strikeouts did Pitcher B and Pitcher E have altogether?

498 strikeouts

7. How many strikeouts were recorded by all five pitchers?

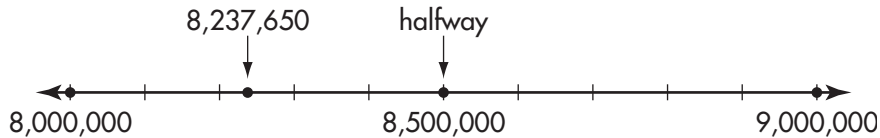
1,265 strikeouts

Strikeout Data

Pitcher	Number of Strikeouts
A	372
B	293
C	220
D	175
E	205

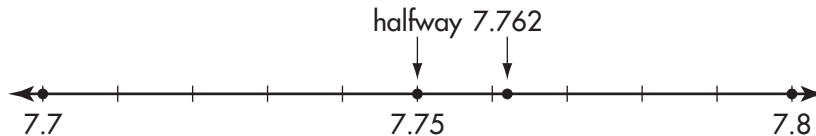
Rounding Whole Numbers and Decimals

You can use the number line below to help you round 8,237,650 to the nearest million. Is 8,237,650 closer to 8,000,000 or 9,000,000?



8,237,650 is less than halfway to 9,000,000. 8,237,650 is closer to 8,000,000.

The number line can also help you round 7.762 to the nearest tenth. Is 7.762 closer to 7.7 or 7.8?



7.762 is more than halfway to 7.8. 7.762 is closer to 7.8.

Round each number to the place of the underlined digit.

1. 4,725,806

4,700,000

2. 7.049

7

3. 165,023,912

200,000,000

4. 18.692

18.7

5. Round the number of connected computers in Year 2 to the nearest ten million.

40,000,000

**Number of Computers
Connected to the Internet**

Year 1	30,979,376
Year 2	42,199,279
Year 3	63,592,854

6. **Number Sense** Marc earned \$9.37 per hour working at the library. Round his wage to the nearest ten cents.

\$9.40

Estimating Sums and Differences

During one week, Mr. Graham drove a truck to five different towns to make deliveries. Estimate how far he drove in all.

Mr. Graham's Mileage Log

Cities	Mileage
Mansley to Mt. Hazel	243
Mt. Hazel to Perkins	303
Perkins to Alberton	279
Alberton to Fort Maynard	277
Fort Maynard to Mansley	352

To estimate the sum, you can round each number to the nearest hundred miles.

$$\begin{array}{r}
 243 \Rightarrow 200 \\
 303 \Rightarrow 300 \\
 279 \Rightarrow 300 \\
 277 \Rightarrow 300 \\
 +352 \Rightarrow +400 \\
 \hline
 1,500 \text{ mi}
 \end{array}$$

Mr. Graham drove about 1,500 mi.

You can estimate differences in a similar way.

Estimate $7.25 - 4.98$.

You can round each number to the nearest whole number.

$$\begin{array}{r}
 7.25 \Rightarrow 7 \\
 -4.98 \Rightarrow -5 \\
 \hline
 2
 \end{array}$$

The difference is about 2.

Estimate each sum or difference.

1. $19.7 - 6.9$

About 13

2. $59 + 43 + 95$

About 200

3. $582 + 169 + 23$

About 770

4. $87.99 - 52.46$

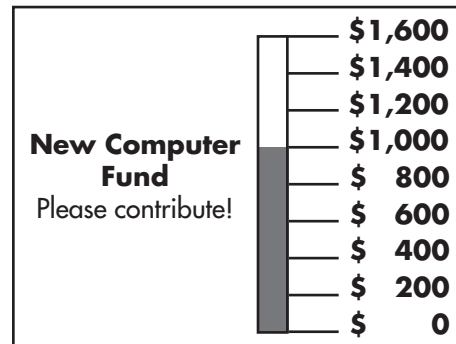
About 36

5. **Estimation** Brigid worked 16.75 h. Kevin worked 12.50 h. About how many more hours did Brigid work than Kevin?

About 4 h

Problem Solving: Draw a Picture and Write an Equation

A community center is raising funds to buy a computer. Here is a picture of the sign they put outside the center. How much more money must the center raise?



How to write an equation number sentence for a problem:

One Way

The goal is \$1,600.

So far, \$1,000 has been raised.

The amount yet to be raised is the unknown.

Think: The amount raised so far and the amount yet to be raised will reach the goal.

Write an equation.

$$1,000 + x = 1,600$$

Think: What number added to 1,000 will result in 1,600?

$$1,000 + \mathbf{600} = 1,600$$

The amount yet to be raised is \$600.

Another Way

The goal is \$1,600.

So far, \$1,000 has been raised.

The amount yet to be raised is the unknown.

Think: The difference between the goal and what has been raised so far is the amount yet to be raised.

Write an equation.

$$1,600 - 1,000 = x$$

Think: What number will result if 1,000 is subtracted from 1,600?

$$1,600 - 1,000 = \mathbf{600}$$

The amount yet to be raised is \$600.

A mason needs 22 bricks to make a stoop. So far he has carried 15 to the site. How many more bricks must he carry?

Draw a picture. Write an equation. Write a number sentence. Solve.



Sample: $22 - 15 = x$;

$22 - 15 = 7$; 7 bricks

Adding and Subtracting

Find $35,996 + 49,801$.

Step 1: Write the numbers, lining up places. Add the ones and then the tens.

$$\begin{array}{r}
 \text{ten thousands} \\
 \text{thousands} \\
 \text{hundreds} \\
 \text{tens} \\
 \text{ones} \\
 35,996 \\
 + 49,801 \\
 \hline
 97
 \end{array}$$

Step 2: Continue adding hundreds, thousands and ten thousands. Regroup as needed.

$$\begin{array}{r}
 11 \\
 35,996 \\
 + 49,801 \\
 \hline
 85,797
 \end{array}$$

Reminder: When the sum of a column is greater than 10, write the ones of the sum and regroup the tens onto the next column.

So $35,996 + 49,801 = 85,797$.

Find $35,996 - 17,902$.

Step 1: Write the numbers, lining up places. Subtract the ones, tens, and hundreds.

$$\begin{array}{r}
 \text{ten thousands} \\
 \text{thousands} \\
 \text{hundreds} \\
 \text{tens} \\
 \text{ones} \\
 35,996 \\
 - 17,902 \\
 \hline
 094
 \end{array}$$

Step 2: Continue by subtracting thousands. Regroup as needed.

$$\begin{array}{r}
 2 \ 15 \\
 \cancel{3}5,996 \\
 - 17,902 \\
 \hline
 18,094
 \end{array}$$

Reminder: The 3 in the ten thousands place is regrouped to 2 ten thousands and 10 thousands.

So $35,996 - 17,902 = 18,094$.

Add or subtract

1.
$$\begin{array}{r}
 7,502 \\
 + 9,909 \\
 \hline
 \end{array}$$

17,411

2.
$$\begin{array}{r}
 64,782 \\
 - 33,925 \\
 \hline
 \end{array}$$

30,857

3.
$$\begin{array}{r}
 835,029 \\
 - 26,332 \\
 \hline
 \end{array}$$

808,697

Myronville School District has 23,081 students, and Saddleton School District has 45,035 students.

4. **Number Sense** How many more students are there in Saddleton than in Myronville?

21,954 more students

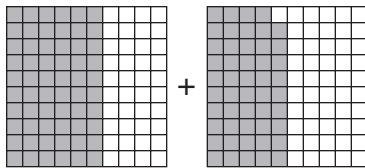
Adding Decimals

In February, Chantell ran a 5K race in 0.6 hour. She ran another 5K race in May in 0.49 hour. What was her combined time for the two races?

Step 1: Write the numbers, lining up the decimal points. Include the zeros to show place value.

$$\begin{array}{r} 0.60 \\ + 0.49 \\ \hline \end{array}$$

You can use decimal squares to represent this addition problem.



Step 2: Add the hundredths.

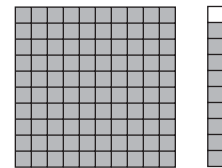
$$\begin{array}{r} 0.60 \\ + 0.49 \\ \hline 9 \end{array}$$



Step 3: Add the tenths.

Remember to write the decimal point in your answer.

$$\begin{array}{r} 1 \\ 0.60 \\ + 0.49 \\ \hline 1.09 \end{array}$$



Chantell's combined time for the two races was 1.09 hours.

Add.

1. $2.97 + 0.35 = \underline{3.32}$ 2. $13.88 + 7.694 = \underline{21.574}$

3. $39.488 + 26.7 = \underline{66.188}$ 4. $88.8 + 4.277 + 78.95 = \underline{172.027}$

5. **Number Sense** Is 16.7 a reasonable sum for $7.5 + 9.2$? Explain.

Yes; 7.5 rounds to 8.0; 9.2 rounds to 9.0;

$8.0 + 9.0 = 17.0.$

6. How much combined snowfall was there in Milwaukee and Oklahoma City?

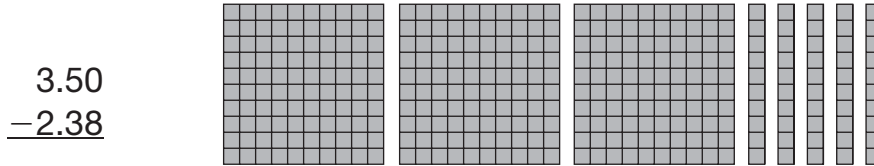
$\underline{105.1 \text{ in.}}$

City	Snowfall (inches) in 2000
Milwaukee, WI	87.8
Baltimore, MD	27.2
Oklahoma City, OK	17.3

Subtracting Decimals

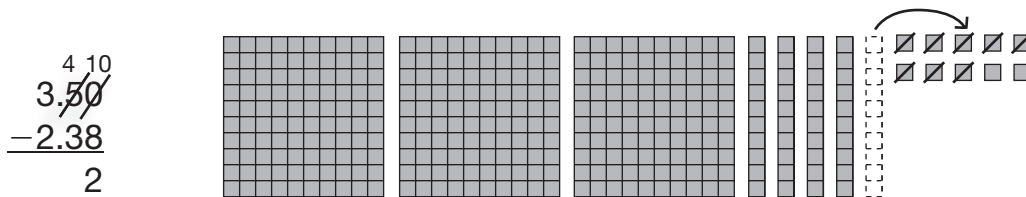
Mr. Montoya bought 3.5 lb of ground beef. He used 2.38 lb to make hamburgers. How much ground beef does he have left?

Step 1: Write the numbers, lining up the decimal points. Include the zeros to show place value.

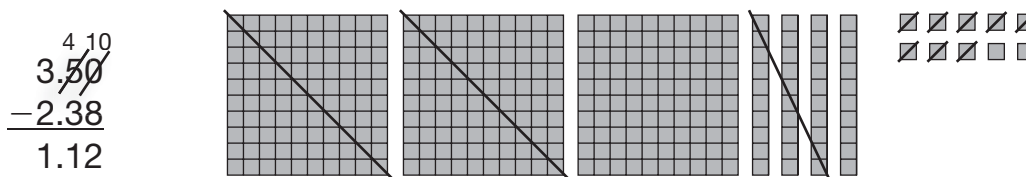


You can use decimal squares to represent this subtraction problem.

Step 2: Subtract the hundredths. Regroup if you need to.



Step 3: Subtract the tenths and the ones. Remember to write the decimal point in your answer.



Mr. Montoya has 1.12 lb of ground beef left over.

Subtract.

1.
$$\begin{array}{r} 82.7 \\ -5.59 \\ \hline \end{array}$$

77.11

2.
$$\begin{array}{r} 43.3 \\ -12.82 \\ \hline \end{array}$$

30.48

3.
$$\begin{array}{r} 7.28 \\ -4.928 \\ \hline \end{array}$$

2.352

Problem Solving: Multiple-Step Problems

Kim has a \$10 bill, a \$20 bill, and 2 \$5 gift certificates. She uses the gift certificates toward the purchase of a CD for \$14.00. How much money does Kim have left after buying the CD?

Read and Understand

What do you know?

Kim has a ten-dollar bill, a twenty-dollar bill, and two five-dollar gift certificates.

She uses the 2 certificates toward the purchase of a CD that costs \$14.00.

What are you trying to find?

How much money does Kim have left after she buys the CD?

Plan and Solve

Answer these hidden questions.

How much money does Kim have?

$$\$20.00 + \$10.00 = \$30.00$$

How much are the two certificates worth?

$$\$5.00 + \$5.00 = \$10.00$$

How much cash will Kim need to buy the CD?

$$\$14.00 - \$10.00 = \$4.00$$

Solve the problem.

$$\begin{aligned} \text{Money} - \text{cash paid for CD} &= \text{Money left} \\ \$30.00 - \$4.00 &= \$26.00 \end{aligned}$$

Write the answer in a complete sentence.

Kim has \$26 left after buying the CD.

Look Back and Check

Is your answer correct?

$$\text{Yes, } \$4.00 + \$26.00 = \$30.00$$

1. You can also find how much money Kim has left by completing the following expression.

$$\$10.00 + \$20.00 + \$5.00 + \$5.00 - \underline{\$14.00}$$

Multiplication Properties

You can use multiplication properties to help you multiply more easily.

Associative Property of Multiplication

You can change the grouping of the factors. The product stays the same.

$$\begin{array}{ccc}
 (3 \times 4) \times 4 = 48 & & 3 \times (4 \times 4) = 48 \\
 \downarrow \downarrow \downarrow & & \downarrow \downarrow \downarrow \\
 \text{Factors} & & \text{Factors} \\
 \uparrow \uparrow & & \uparrow \uparrow \\
 12 \times 4 = 48 & & 3 \times 16 = 48
 \end{array}$$

Commutative Property of Multiplication

You can change the order of the factors. The product stays the same.

$$\begin{array}{ccc}
 7 \times 4 = 28 & & 4 \times 7 = 28 \\
 \downarrow \downarrow & & \downarrow \downarrow \\
 \text{Factors} & & \text{Factors} \\
 & & \downarrow \\
 & & \text{Product}
 \end{array}$$

Zero Property of Multiplication

When one of the factors is 0, the product is always 0.

$$\begin{array}{ccc}
 3 \times 0 = 0 & & 0 \times 3 = 0 \\
 \downarrow \downarrow & & \downarrow \downarrow \\
 \text{Factors} & & \text{Factors} \\
 & & \downarrow \\
 & & \text{Product}
 \end{array}$$

Identity Property of Multiplication

When one of the factors is 1, the product is always the other factor.

Identify the multiplication property or properties used in each equation.

- $100 \times 0 = 0$ zero
- $7 \times 2 = 2 \times 7$ commutative
- $1 \times 55 = 55$ identity
- $(6 \times 7) \times 9 = 6 \times (7 \times 9)$ associative

Reasoning Use the multiplication properties to determine what number must be in the box.

- $5 \times 4 = \boxed{4} \times 5$
- $99 \times \boxed{1} = 99$
- $(3 \times 12) \times \boxed{8} = 3 \times (12 \times 8)$
- $\boxed{0} \times 1 = 0$
- $\boxed{50} \times 2 = 2 \times 50$
- $(16 \times \boxed{33}) \times 25 = 16 \times (33 \times 25)$

Using Mental Math to Multiply

You can also use patterns to multiply mentally.

Fact: $6 \times 8 = 48$

$$60 \times 8 = 480$$

$$6 \times 80 = 480$$

$$600 \times 8 = 4,800$$

$$60 \times 80 = 4,800$$

$$6,000 \times 8 = 48,000$$

$$600 \times 80 = 48,000$$

$$60,000 \times 8 = 480,000$$

$$6,000 \times 80 = 480,000$$

Pattern: Notice that the product is always the digits 48 followed by the total number of zeros that are in the factors.

Find $30 \times 3 \times 50$.

Use the Associative Property of Multiplication to regroup.

$$(30 \times 50) \times 3$$

$$1,500 \times 3 = 4,500$$

Commutative Property of Multiplication	Associative Property of Multiplication
You can multiply factors in any order.	You can change the grouping of factors.
$15 \times 9 = 9 \times 15$	$(8 \times 20) \times 5 = 8 \times (20 \times 5)$

Find each product. Use patterns and properties to compute mentally.

1. $80 \times 90 = \underline{7,200}$ 2. $40 \times 800 = \underline{32,000}$

3. $5 \times 10 \times 20 = \underline{1,000}$ 4. $4 \times 30 \times 25 = \underline{3,000}$

5. **Number Sense** You know that $6 \times 7 = 42$. How can you find 60×700 ?

Sample answer: Multiply 6×7 , then place the total number of zeros in both factors after the product. So, $60 \times 700 = 42,000$.

Estimating Products

A bus service drives passengers between Milwaukee and Chicago every day. They travel from city to city a total of 8 times each day. The distance between the two cities is 89 mi. In the month of February, there are 28 days. The company's budget allows for 28,000 total miles for February. Is 28,000 mi a reasonable budget mileage amount?

One Way to Estimate

Estimate $28 \times 8 \times 89$.

Use rounding.

You can round 89 to 100 and 8 to 10. Then multiply.

$$28 \times 10 \times 100 = 280 \times 100 = 28,000$$

Because this is an overestimate, there are enough miles.

Another Way to Estimate

Estimate $28 \times 8 \times 89$.

Use compatible numbers.

Replace 28 with 30, 89 with 90, and 8 with 10. 30, 90, and 10 are compatible numbers because they are close to the actual numbers in the problem and they are easier to multiply. Now the problem becomes $30 \times 90 \times 10$.

$30 \times 90 = 2,700$ Multiply 3×9 , then place two zeros after the product.

$2,700 \times 10 = 27,000$ Multiply 27×1 using the Identity Property of Multiplication, then place three zeros after the product.

In the estimate, we used numbers greater than the original numbers, so the answer is an overestimate.

28,000 total miles is a reasonable budget amount.

Estimate each product. Use rounding or compatible numbers.

1. $42 \times 5 \times 90 = \underline{18,000}$ 2. $27 \times 98 \times 4 = \underline{12,000}$

Mrs. Carter ordered new supplies for Memorial Hospital.

3. About how much will it cost to purchase 48 electronic thermometers?

About \$1,000

4. About how much will it cost to purchase 96 pillows?

About \$2,000

Supplies

Electronic thermometers	\$19 each
Pulse monitors	\$189 each
Pillows	\$17 each
Telephones	\$19 each

Multiplying by 1-Digit Numbers

Mr. McGuire drives his truck 275 miles each day. How far does he drive in 3 days?

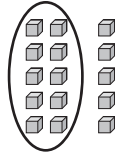
Find 275×3 .

Step 1:

Multiply the ones. Regroup if necessary.

What You Think

3×5 ones = 15 ones
Regroup 15 ones as 1 ten and 5 ones.



What You Write

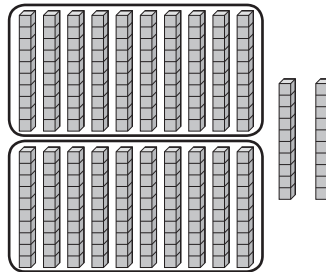
$$\begin{array}{r} 1 \\ 275 \\ \times 3 \\ \hline 5 \end{array}$$

Step 2:

Multiply the tens. Regroup if necessary.

What You Think

3×7 tens = 21 tens
 21 tens + 1 ten = 22 tens
Regroup as 2 hundreds and 2 tens.



What You Write

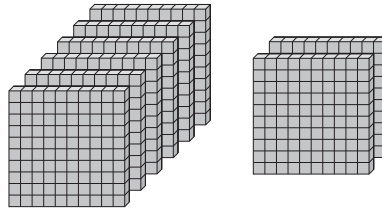
$$\begin{array}{r} 21 \\ 275 \\ \times 3 \\ \hline 25 \end{array}$$

Step 3:

Multiply the hundreds. Regroup if necessary.

What You Think

3×2 hundreds = 6 hundreds
 6 hundreds + 2 hundreds = 8 hundreds
No need to regroup.



What You Write

$$\begin{array}{r} 21 \\ 275 \\ \times 3 \\ \hline 825 \end{array}$$

Mr. McGuire drives 825 miles in 3 days.

Find each product. Estimate to check that your answer is reasonable.

1. $31 \times 7 = \underline{217}$ 2. $29 \times 4 = \underline{116}$ 3. $88 \times 6 = \underline{528}$

4. $25 \times 9 = \underline{225}$ 5. $102 \times 8 = \underline{816}$ 6. $211 \times 7 = \underline{1,477}$

7. $552 \times 3 = \underline{1,656}$ 8. $471 \times 9 = \underline{4,239}$ 9. $73 \times 4 = \underline{292}$

10. $266 \times 8 = \underline{2,128}$ 11. $390 \times 2 = \underline{780}$ 12. $514 \times 6 = \underline{3,084}$

13. **Estimation** Estimate the product of 48 and 7. Do you have an underestimate or overestimate?

Sample answer: $50 \times 7 = 350$; overestimate

Multiplying 2-Digit by 2-Digit Numbers

Find 43×26 .

Step 1:

Multiply by the ones.
Regroup if necessary.

What You Think

6×3 ones = 18 ones
Regroup 18 ones as 1 ten
and 8 ones.

6×4 tens = 24 tens
24 tens + 1 ten = 25 tens
Regroup 25 tens as 2 hundreds
and 5 tens.

What You Write

$$\begin{array}{r} 1 \\ 43 \\ \times 26 \\ \hline 258 \end{array}$$

Step 2:

Multiply by the tens.
Regroup if necessary.

What You Think

20×3 ones = 60 ones
Regroup 60 ones as 6 tens.

20×4 tens = 80 tens
Regroup 80 tens as 8 hundreds.

$$\begin{array}{r} 1 \\ 43 \\ \times 26 \\ \hline 258 \\ \hline 860 \end{array}$$

Step 3:

Add the partial products.

What You Think

$6 \times 43 = 258$
 $20 \times 43 = 860$

$$\begin{array}{r} 1 \\ 43 \\ \times 26 \\ \hline 258 \\ + 860 \\ \hline 1,118 \end{array}$$

← partial products

Find the product.

1. $\begin{array}{r} 38 \\ \times 12 \\ \hline 456 \end{array}$

2. $\begin{array}{r} 64 \\ \times 33 \\ \hline 2,112 \end{array}$

3. $\begin{array}{r} 49 \\ \times 27 \\ \hline 1,323 \end{array}$

4. $\begin{array}{r} 85 \\ \times 15 \\ \hline 1,275 \end{array}$

5. $\begin{array}{r} 26 \\ \times 21 \\ \hline 546 \end{array}$

6. $\begin{array}{r} 73 \\ \times 19 \\ \hline 1,387 \end{array}$

7. $\begin{array}{r} 57 \\ \times 28 \\ \hline 1,596 \end{array}$

8. $\begin{array}{r} 91 \\ \times 86 \\ \hline 7,826 \end{array}$

9. **Number Sense** In the problem 62×45 , what are the partial products?
310 and 2,480

Multiplying Greater Numbers

Find 128×23 .

Estimate: $100 \times 20 = 2,000$

	Step 1 Multiply by the ones. Regroup as needed.	Step 2 Multiply by the tens. Regroup as needed.	Step 3 Add the products.
$\begin{array}{r} 128 \\ \times 23 \\ \hline 384 \\ + 2,560 \\ \hline 2,944 \end{array}$	$\begin{array}{r} 2 \\ 128 \\ \times 3 \\ \hline 384 \end{array}$	$\begin{array}{r} 1 \\ 128 \\ \times 20 \\ \hline 2,560 \end{array}$	

Because the answer is close to the estimate, the answer is reasonable.

Find the product. Estimate to check if your answer is reasonable.

Problem	Multiply by the Ones	Multiply by the Tens	Add the Products
1. $\begin{array}{r} 282 \\ \times 19 \\ \hline 2,538 \\ + 2,820 \\ \hline 5,358 \end{array}$	$\begin{array}{r} 71 \\ 282 \\ \times 9 \\ \hline 2,538 \end{array}$	$\begin{array}{r} 282 \\ \times 10 \\ \hline 2,820 \end{array}$	$\begin{array}{r} 1 \\ 2,538 \\ + 2,820 \\ \hline 5,358 \end{array}$
2. $\begin{array}{r} 538 \\ \times 46 \\ \hline 24,748 \end{array}$			

3. Reasonableness Is 2,750 a reasonable answer for 917×33 ? Explain.

No; if you round 917 to 900 and 33 to 30, the product is 900×30 or 27,000, so 2,750 is not reasonable.

Exponents

You can use exponential notation to write a number that is being multiplied by itself.

There are two parts in exponential notation. The **base** tells you what factor is being multiplied. The **exponent** tells you how many of that factor should be multiplied together. The exponent is *not* a factor.

exponent



$8^2 = 8 \times 8$ The base is 8, so 8 is the factor to be multiplied.



The exponent is 2, so 2 factors of 8 should be multiplied together.

base

You can write 8^2 in two other forms.

In **expanded** form, you write out your factors. Since 8^2 means you multiply two factors of 8, 8^2 in expanded form is 8×8 .

In **standard** form, you write down the product of the factors. Since $8 \times 8 = 64$, 64 is the standard form of 8^2 .

Write in exponential notation.

1. $2 \times 2 \times 2$ 2^3

2. $6 \times 6 \times 6 \times 6 \times 6$ 6^5

Write in expanded form.

3. 1^4 $1 \times 1 \times 1 \times 1$

4. 5^3 $5 \times 5 \times 5$

Write in standard form.

5. $2 \times 2 \times 2 \times 2$ 16

6. 8^3 512

7. A used car lot has 9 lanes for cars and 9 rows for cars in each lane. What is the exponential notation for the number of spaces on the lot? Can the owner fit 79 cars on the lot?

9^2 ; Yes. $9 \times 9 = 81$. There are 81 spaces on the lot.

Problem Solving: Draw a Picture and Write an Equation

A hardware store ordered 9 packs of screws from a supplier. Each pack contains 150 screws. How many screws did the store order?

Read and Understand

What do you know?

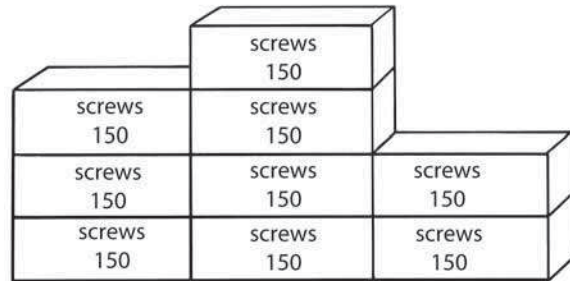
The store ordered nine packs of screws.
Each pack contained 150 screws.

What are you trying to find?

The total number of screws ordered

Plan and Solve

Draw a picture of what you know.



Write an equation.

Let x = the total number of screws.

$$9 \times 150 = x$$

Multiply.

$$\begin{array}{r} 150 \\ \times 9 \\ \hline 1,350 \end{array}$$

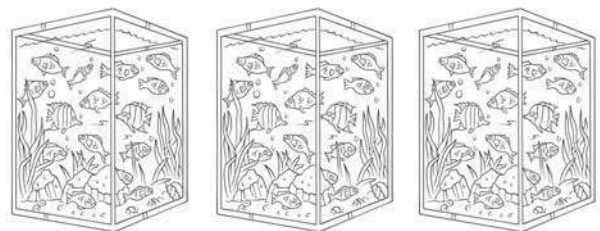
The store ordered 1,350 screws.

Look Back and Check

Is your answer reasonable?

Yes, $150 \times 10 = 1,500$.

A state aquarium has display tanks that each contains 75 fish. Three of these tanks are at the entrance. How many fish are on display at the entrance?



Draw a picture. Write an equation. Solve.

**Sample: $75 \times 3 = x$;
225 fish**



Dividing Multiples of 10 and 100

You can use math facts and patterns to help you divide mentally.

What is $480 \div 6$?

You already know that $48 \div 6 = 8$.

480 has one more zero than 48, so place one more zero in the quotient.

$$480 \div 6 = 80.$$

What is $60,000 \div 6$?

$60 \div 6 = 10$

60,000 has three more zeros than 6, so place three zeros in the quotient.

$$60,000 \div 6 = 10,000.$$

Find each quotient. Use mental math.

1. $32 \div 8 =$

4

2. $320 \div 8 =$

40

3. $3,200 \div 8 =$

400

4. $32,000 \div 8 =$

4,000

5. $56 \div 7 =$

8

6. $560 \div 7 =$

80

7. $5,600 \div 7 =$

800

8. $56,000 \div 7 =$

8,000

9. $15 \div 3 =$

5

10. $150 \div 3 =$

50

11. $1,500 \div 3 =$

500

12. $15,000 \div 3 =$

5,000

13. **Number Sense** Explain how dividing 720 by 9 is like dividing 72 by 9.

You divide 72 by 9. For 720, you add a zero for the ones place.

Arlo has a newspaper delivery job. He wants to wrap each of his newspapers in a plastic bag to protect them from the rain. The newspapers are in bundles.

Arlo's Newspaper Delivery

Number of bundles	12
Number of newspapers per bundle	9

Use mental math to answer the following questions.

14. How many bags will he use for 5 bundles?

45 bags

15. How many bags will he use for 7 bundles?

63 bags

16. How many bags will he use for all 12 bundles?

108 bags

Estimating Quotients

There are several ways to adjust whole numbers to estimate quotients.

Example:

There are 216 students. The school has 8 classrooms.
How many students will be in each classroom?

Estimate $216 \div 8$.

Rounding

You can use rounding to estimate a quotient.

Round 216 to the nearest hundred.

In this case, 216 rounds to 200.

$$200 \div 8 = 25$$

25 students per room is an underestimate, because 216 was rounded down to 200.

Compatible Numbers

You can use compatible numbers to estimate a quotient.

Change 216 to a compatible number for 8.

Compatible numbers for 8 are numbers divisible by 8, such as 160, 240, and 320. Choose 240, because it is the closest compatible number to 216.

$$240 \div 8 = 30$$

30 students per class is an overestimate, because 216 was rounded up to 240.

Multiplication

You can use multiplication to estimate a quotient.

Think: 8 times what number is about 216?

$$8 \times 25 = 200$$

$$8 \times 30 = 240$$

216 is between 200 and 240. So a good estimate is a little more than 25 and a little less than 30 students per classroom.

Estimate each quotient. You may use any method.

1. $411 \div 2$

$$\underline{400 \div 2 = 200}$$

2. $162 \div 4$

$$\underline{160 \div 4 = 40}$$

3. **Number Sense** If you estimate $342 \div 7$ by using $350 \div 7 = 50$, is 50 greater than or less than the exact answer? How did you decide? Is 50 an overestimate or an underestimate?

It is greater, because you rounded 342 up to 350. So, 50 is an overestimate.

Problem Solving: Reasonableness

After you solve a problem, check to see if your answer is reasonable. Also check that you answered the right question.

Example:

74 students are going to a special class dinner where they will be seated 8 to a table. Will 9 tables be enough?

Reasonableness

The answer is close to 9 tables.

$$76 \div 8 = 9 \text{ R}2$$

Answering the right question

All students must have seats, so there must be one more table to hold the remaining 2 students, making 10 tables in all.

Tell whether each answer is reasonable.

1. Kendra wants to paste 500 photographs into an album, 6 photos to a page. She figures that she will need about 100 pages.

Six times 100 is 600, much more space than Kendra needs. Since 500 is between 480 (6×80) and 540 (6×90), she needs between 80 and 90 pages.

2. Hwong has 39 muffins. If each of his guests will eat 2 muffins, Hwong figures that he can serve muffins to 19 guests.

This is reasonable, since $2 \times 19 = 38$.

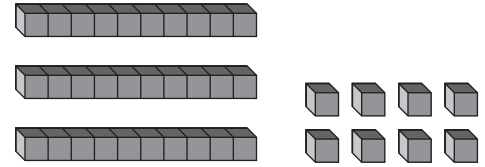
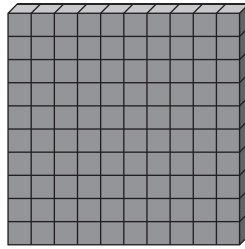
Tell whether each answer answers the right question.

3. Ivan has a piece of lumber 104 inches long. He is sawing it into 12-inch lengths to make fence posts. He says he can get about 9 fence posts out of the board.

Although $104 \div 12$ is closer to 9 than to 8, each post must be a full 12 inches, and therefore any remainder must be thrown away. Ivan can only make 8 posts.

Connecting Models and Symbols

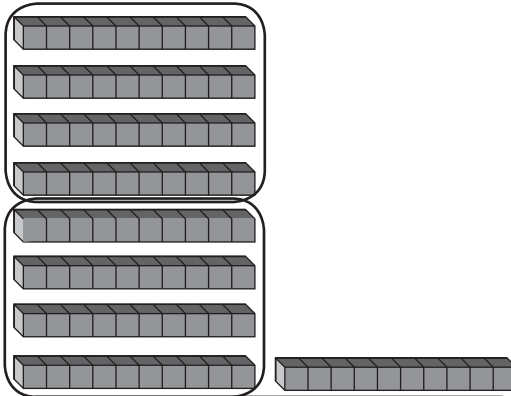
Divide 138 equally into 3 groups.



Step 1:

You can model 138 as 13 groups of 10 plus 8 ones.
 $130 \div 3 = 40$, so each group will get 4 groups of 10.
 $40 \times 3 = 120$
 $130 - 120 = 10$, so there is 1 group of 10 left.

What You Think



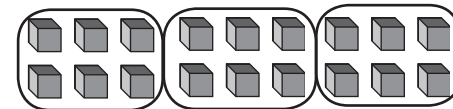
What You Write

$$\begin{array}{r} 4 \\ 3 \overline{)138} \\ \underline{-12} \\ 1 \end{array}$$

Step 2:

There is 1 group of 10 plus 1 group of 8 ones left. You can model 18 as 18 ones.
 $18 \div 3 = 6$, so each group will also get 6 ones. There is nothing left.

What You Think



What You Write

$$\begin{array}{r} 46 \\ 3 \overline{)138} \\ \underline{-12} \\ 18 \\ \underline{-18} \\ 0 \end{array}$$

$138 \div 3 = 46$

Use models to help you divide.

1. $4 \overline{)76}$ **19**

2. $2 \overline{)94}$ **47**

3. $5 \overline{)130}$ **26**

4. $7 \overline{)238}$ **34**

5. $6 \overline{)426}$ **71**

6. $3 \overline{)264}$ **88**

7. **Algebra** If $n \div 3 = 57$, what is the value of n ? **171**

Dividing by 1-Digit Divisors

Find $362 \div 5$.

Step 1: To decide where to place the first digit in the quotient, compare the first digit of the dividend with the divisor.

$3 < 5$, so the first digit in the quotient will not go in the hundreds place.

Now, compare the first two digits of the dividend with the divisor.

$36 > 5$, so the first digit in the quotient will go in the tens place.

Step 2: Divide the tens. Use multiplication facts and compatible numbers.
Think $5 \times ? = 35$.

Write 7 in the tens place of the quotient.
Multiply. $5 \times 7 = 35$

$$\begin{array}{r} 7 \\ 5 \overline{)36} \\ \underline{-35} \\ 1 \end{array}$$

Subtract. $36 - 35 = 1$
Compare. $1 < 5$
Bring down the ones.

Step 3: Divide the ones. Use multiplication facts and compatible numbers.
Think $5 \times ? = 10$.

Write 2 in the ones place of the quotient.
Multiply. $5 \times 2 = 10$

$$\begin{array}{r} 7 \text{ R}2 \\ 5 \overline{)362} \\ \underline{-35} \downarrow \\ 12 \\ \underline{-10} \\ 2 \end{array}$$

Subtract. $12 - 10 = 2$
Compare. $2 < 5$
There are no more digits to bring down, so 2 is the remainder.

Step 4: Check by multiplying.

$$5 \times 72 = 360 + 2 = 362$$

Divide. Check by multiplying.

1. $8 \overline{)863} \text{ }^{107} \text{ R}7$

2. $7 \overline{)249} \text{ }^{35} \text{ R}4$

3. $5 \overline{)365} \text{ }^{73}$

4. $8 \overline{)448} \text{ }^{56}$

5. $2 \overline{)499} \text{ }^{249} \text{ R}1$

6. $6 \overline{)396} \text{ }^{66}$

7. Number Sense How can you tell before you divide 425 by 9 that the first digit of the quotient is in the tens place?

$$4 < 9$$

Zeros in the Quotient

Find $816 \div 4$.

Step 1: Compare the first digit of the dividend with the divisor. $8 > 4$, so the first digit in the quotient will go in the hundreds place.

Divide the hundreds.
Think $4 \times ? = 8$.

Write 2 in the hundreds place of the quotient.
Multiply. $4 \times 2 = 8$

$$\begin{array}{r} 2 \\ 4 \overline{)81} \\ \underline{-8} \downarrow \\ 01 \end{array}$$

Subtract. $8 - 8 = 0$

Compare. $0 < 4$

Bring down the tens.

Step 2: Compare.
 $1 < 4$

You cannot divide the tens, so place 0 in the tens place of the quotient.

Bring down the ones.

$$\begin{array}{r} 20 \\ 4 \overline{)816} \\ \underline{-8} \downarrow \\ 016 \end{array}$$

Step 3: Compare.
 $16 > 4$

Divide the ones. Think
 $4 \times ? = 16$.

Write 4 in the ones place of the quotient.

Multiply. $4 \times 4 = 16$

Subtract. $16 - 16 = 0$

Compare. $0 < 4$

There are no more digits to bring down, so the problem is done.

$$\begin{array}{r} 204 \\ 4 \overline{)816} \\ \underline{-8} \\ 016 \\ \underline{-16} \\ 0 \end{array}$$

Step 4: Check by multiplying.

$$4 \times 204 = 816$$

Find each quotient. Check your answers by multiplying.

1. $8 \overline{)640}$ **80**

2. $3 \overline{)322}$ **107 R1**

3. $4 \overline{)908}$ **227**

4. $15 \overline{)225}$ **15**

5. $6 \overline{)624}$ **104**

6. $6 \overline{)965}$ **160 R5**

7. **Explain It** Is $593 \div 6$ a little less than 10, a little more than 10, a little less than 100, or a little more than 100? Explain.

A little less than 100;

$$\underline{593 \div 6 = 98 \text{ R}5 < 100.}$$

Understanding Factors

You can use the divisibility rules to find the factors of a number.

What are the factors of 92?

Possible Factor	Test of Divisibility	Result of Test	Factors Found
1	Is 92 a whole number?	Yes. Therefore it is divisible by 1.	$92 \div 1 = 92$ 1 and 92 are factors.
2	Is 92 an even number?	Yes. Therefore it is divisible by 2.	$92 \div 2 = 46$ 2 and 46 are factors.
3	Is the sum of the digits divisible by 3?	No. Therefore it is not divisible by 3.	3 is not a factor.
4	Are the last 2 digits divisible by 4?	Yes. Therefore it is divisible by 4.	$92 \div 4 = 23$ 4 and 23 are factors.
5	Is the last digit 5 or 0?	No. Therefore it is not divisible by 5.	5 is not a factor.
6	Is it divisible by both 2 and 3?	No. Therefore it is not divisible by 6.	6 is not a factor.
9	Is the sum of the digits divisible by 9?	No. Therefore it is not divisible by 9.	9 is not a factor.
10	Is the last digit 0?	No. Therefore it is not divisible by 10.	10 is not a factor.

The factors of 92 are 1, 2, 4, 23, 46, and 92.

Find all the factors of each number.

1. 12 1, 2, 3, 4, 6, 12 2. 35 1, 5, 7, 35

3. 45 1, 3, 5, 9, 15, 45 4. 49 1, 7, 49

5. **Number Sense** Is 2,340 divisible by 90? How do you know?

Yes. It is divisible by 9 and 10, because the sum of the digits is divisible by 9, and the last digit is 0.

Prime and Composite Numbers

Numbers such as 2, 3, 5, 7, and 11 are prime numbers. A prime number has *only two* factors, itself and 1. A whole number that has *more than two* factors is called a composite number.

3 is an example of a prime number.
Its only factors are 1 and 3.

○ ○ ○ $1 \times 3 = 3$

8 is a composite number.
Its factors are 1, 2, 4, and 8.

○ ○ ○ ○ ○ ○ ○ ○ $1 \times 8 = 8$

○ ○ ○ ○ $2 \times 4 = 8$
○ ○ ○ ○

To find the prime factorization of a number, you can use a factor tree.

Find the prime factorization of 12.

12 ← Think of two numbers whose product is 12. You can use 2×6 or 3×4 .

2 is prime. → 2 × 6 ← 6 is not prime, so keep dividing.

2 × 2 × 3 ← All the factors are prime, so you can stop dividing.

The prime factorization of 12 is $2 \times 2 \times 3$.

You can also use divisibility rules to tell whether a number is prime or composite. In the example above, 12 is an even number, so it is divisible by 2. Therefore, 12 is a composite number.

Write whether each number is prime or composite.

1. 17 **Prime** _____ 2. 47 **Prime** _____

Find the prime factorization.

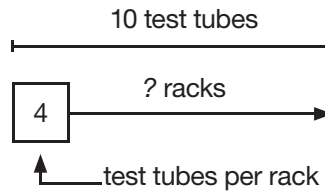
3. 28 **$2 \times 2 \times 7$** _____

4. **Number Sense** The prime factorization of a number is $2 \times 3 \times 3$. What is the number? **18** _____

Problem Solving: Draw a Picture and Write an Equation

Mr. Rodriguez needs to store 10 test tubes in racks that hold 4 test tubes apiece. How many racks does he need?

Draw a picture.



Write an equation.

Let r be the number of test-tube racks needed.

$$10 \div 4 = r$$

Solve the problem.

$$r = 2 \text{ R } 2$$

Think: R 2 means that 2 test tubes are still left. Since they must be in a rack, one more rack is needed.

Write the answer in a complete sentence.

Mr. Rodriguez will need three test-tube racks.

Explain It Mr. Rodriguez could also use racks that hold three test tubes or racks that hold five test tubes. Which racks should he use if he wants each rack completely filled? Explain.

He should use the racks that hold 5 test tubes, because 10 is evenly divisible by 5.

Using Patterns to Divide

You can use basic facts and patterns to divide mentally.

Using basic facts

What is $350 \div 70$?

Think: $350 \div 70$ is the same as 35 tens \div 7 tens.

$$35 \div 7 = 5$$

$$\text{So, } 350 \div 70 = 5.$$

Using patterns

What is $5,400 \div 60$?

$5,400 \div 60$ is the same as $540 \div 6$.

$$54 \div 6 = 9, \text{ so } 540 \div 6 = 90.$$

$$\text{So, } 5,400 \div 60 = 90.$$

Find each quotient. Use mental math.

1. $280 \div 70 = \underline{4}$

2. $320 \div 40 = \underline{8}$

3. $360 \div 60 = \underline{6}$

4. $7,200 \div 80 = \underline{90}$

5. $9,000 \div 30 = \underline{300}$

6. $4,800 \div 80 = \underline{60}$

7. $2,000 \div 40 = \underline{50}$

8. $5,600 \div 70 = \underline{80}$

9. **Number Sense** How is dividing 250 by 50 the same as dividing 2,500 by 500?

Sample answer: $250 \div 50 = 5$ and $2,500 \div 500 = 5$. Both problems use the same basic fact that $25 \div 5 = 5$.

10. **Explain It** Explain how you can mentally determine that $35,000 \div 70 = 500$.

You know that $35 \div 7 = 5$. Using math facts and patterns you also know that $3,500 \div 70 = 50$ and $35,000 \div 70 = 500$.

Estimating Quotients with 2-Digit Divisors

You can use compatible numbers to estimate a quotient.

Find $175 \div 32$.

Step 1: Find compatible numbers for 175 and 32.

32 rounds to 30.

Think: 18 can be divided evenly by 3.

180 is close to 175 and 30 is close to 32.

180 and 30 are compatible numbers.

Step 2: Divide. Use patterns to help you, if possible.

Think: $180 \div 30$ is the same as
18 tens \div 3 tens.

$$18 \div 3 = 6$$

$$\text{So, } 180 \div 30 = 6.$$

Step 3: Check for reasonableness.

$$6 \times 30 = 180$$

So, a good estimate of $175 \div 32$ is 6.

Estimate each quotient using compatible numbers.

1. $298 \div 25$

10

2. $5,391 \div 77$

70

3. $24,303 \div 12$

2,000

4. $276 \div 42$

7

5. $1,347 \div 54$

27

6. $5,564 \div 91$

60

At Elmer Elementary School, fifth-grade students are saving money for a summer trip to Washington, D.C.

7. The money Percy has saved is how many times as great as the money James has saved?

about three times as great

Student	Amount Saved
Percy	\$125
Emily	\$ 80
George	\$202
James	\$ 41
Bertha	\$159

Problem Solving: Multiple-Step Problems

Faye is putting together packets of colored beads to give as gifts. The chart shows the beads she had on hand yesterday. This morning she bought 4 boxes of yellow beads containing 45 beads each. How many packets of 60 beads can she put together?

Trinket Beads	
Color	Number
Red	195
Blue	170
Green	175

Find the hidden question or questions.

- | | |
|-------------------------------------|----------------------------------|
| 1. How many yellow beads are there? | 1. $45 \times 4 = 180$ |
| 2. How many beads are there in all? | 2. $195 + 170 + 175 + 180 = 720$ |

Solve.

$$720 \div 60 = 12$$

Write the answer in a sentence.

Faye will make 12 packets.

Look Back and Check

Is the answer reasonable?

Yes. Since $60 \times 10 = 600$, the answer is reasonable.

1. Faye decides to double the number of yellow beads in the mix. How many packets will she make if she fills each packet with 60 beads?

15

2. **Explain It** Faye plans to fill packets with 60 beads after deciding not to add any yellow beads to the mix. What hidden question or questions would you have to ask? Explain.

The hidden question is “How many beads are in the mix?”

Dividing by Multiples of 10

Find $623 \div 40$.

Step 1: Estimate the quotient using compatible numbers, $600 \div 40 = 15$. Then, divide the tens.

$$\begin{array}{r} 1 \\ 40 \overline{)623} \\ \underline{-40} \\ 22 \end{array}$$

Divide $62 \div 40 = 1$
 Multiply $1 \times 40 = 40$
 Subtract $62 - 40 = 22$
 Compare $22 < 40$

Step 2: Bring down the ones. Then, divide the ones.

$$\begin{array}{r} 15 \\ 40 \overline{)623} \\ \underline{-40} \\ 223 \\ \underline{-200} \\ 23 \end{array}$$

Divide $223 \div 40 = 5$
 Multiply $5 \times 40 = 200$
 Subtract $223 - 200 = 23$

Step 3: Since $23 < 40$, write 23 as the remainder in the quotient.

$$\begin{array}{r} 15 \text{ R}23 \\ 40 \overline{)623} \\ \underline{-40} \\ 223 \\ \underline{-200} \\ 23 \end{array}$$

Compare $23 < 40$

Complete.

$$\begin{array}{r} 4 \text{ R}48 \\ 60 \overline{)288} \\ \underline{-240} \\ 48 \end{array}$$

$$\begin{array}{r} 22 \text{ R}15 \\ 20 \overline{)455} \\ \underline{-40} \\ 55 \\ \underline{-40} \\ 15 \end{array}$$

$$\begin{array}{r} 10 \text{ R}66 \\ 80 \overline{)866} \\ \underline{-80} \\ 66 \end{array}$$

$$\begin{array}{r} 7 \text{ R}23 \\ 30 \overline{)233} \\ \underline{-210} \\ 23 \end{array}$$

$$\begin{array}{r} 9 \text{ R}48 \\ 50 \overline{)498} \\ \underline{-450} \\ 48 \end{array}$$

6. **Reasoning** Celia plans to pack her books in boxes when her family moves. Each box will hold 20 books. Celia has 97 books. How many boxes will she need to pack all her books?

5 boxes

1-Digit Quotients

Find $436 \div 53$.

To find the answer, first estimate the quotient.

Think: $400 \div 50 = 8$ or $450 \div 50 = 9$

Try 9:

$$\begin{array}{r} 9 \\ 53 \overline{)436} \\ -477 \\ \hline \end{array}$$

Write 9 in the ones place.
 Multiply, $9 \times 53 = 477$.
 Subtract, $436 - 477 = -41$.
 This estimate is too high.

Try 8:

$$\begin{array}{r} 8 \\ 53 \overline{)436} \\ -424 \\ \hline 12 \end{array}$$

Write 8 in the ones place.
 Multiply, $8 \times 53 = 424$.
 Subtract, $436 - 424 = 12$.
 Compare, $12 < 53$. Write the remainder in the quotient.

$436 \div 53 = 8 \text{ R}12$

Check:

$8 \times 53 = 424$

$424 + 12 = 436$

Complete.

1. $32 \overline{)245}$ **7 R21**

2. $64 \overline{)332}$ **5 R12**

3. $51 \overline{)489}$ **9 R30**

Divide. Check by multiplying.

4. $49 \overline{)216}$ **4 R20**

5. $79 \overline{)698}$ **8 R66**

6. $25 \overline{)194}$ **7 R19**

7. **Explain It** Explain how you know the answer to the problem below has an error.

$$\begin{array}{r} 2 \text{ R}86 \\ 77 \overline{)240} \\ -154 \\ \hline 86 \end{array}$$

The remainder is greater than the divisor.

2-Digit Quotients

Find $866 \div 34$.

Step 1: Round the divisor to the nearest ten. Look at the first digit in the divisor and the first digit in the dividend. What basic division fact is the best estimate of the quotient of these two numbers?

$$34 \overline{)866} \quad \longrightarrow \quad 30 \overline{)866}$$

$$8 \div 3 = 2 \text{ R}2$$

Step 2: Use this fact to begin the quotient. Write it over the tens place.

$$\begin{array}{r} 2 \\ 34 \overline{)866} \\ \underline{-68} \downarrow \\ 186 \end{array}$$

Multiply, $2 \times 34 = 68$.

Subtract and bring down the next digit in the dividend.

Step 3: What basic division fact is the best estimate of the next division? Use this fact and write it over the ones place.

$$\begin{array}{r} 25 \text{ R}16 \\ 34 \overline{)866} \\ \underline{-68} \\ 186 \\ \underline{-170} \\ 16 \end{array}$$

Multiply, $5 \times 34 = 170$.

Subtract. Compare the remainder with the divisor.

If the remainder is less than the divisor, write it in the quotient.

Check.

$$25 \times 34 = 850$$

$$850 + 16 = 866$$

Complete.

$$1. \quad 39 \overline{)437} \quad 11 \text{ R } \boxed{8}$$

$$2. \quad 24 \overline{)627} \quad \boxed{26} \text{ R}3$$

$$3. \quad 26 \overline{)917} \quad \boxed{35} \text{ R } \boxed{7}$$

Divide. Check by multiplying.

$$4. \quad 13 \overline{)175} \quad \mathbf{13 \text{ R}6}$$

$$5. \quad 44 \overline{)508} \quad \mathbf{11 \text{ R}24}$$

6. **Estimation** April has 95 baseball cards. She wants to organize them on pages that hold 18 cards each. She has 5 pages. Does April have enough pages to organize all her cards?

No, she has room for only 90 cards.

Estimating and Dividing with Greater Numbers

Find $8,037 \div 77$.

You can use a calculator to divide large numbers.

Step 1: Estimate. Round the divisor and the dividend.

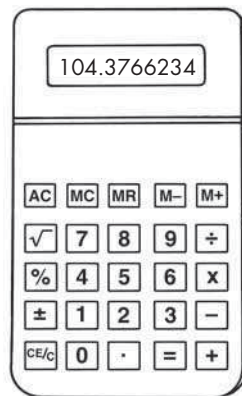
$$8,037 \div 77 \rightarrow$$

$$8,000 \div 80 = 100$$

The quotient should be close to 100.

Step 2: Now, use a calculator to find the quotient.

$$8,037 \div 77$$



Step 3: Round the quotient to the required place. Remember, if the digit is 5 or more, add 1 to the rounding digit. If the digit is less than 5, leave the rounding digit alone.

Round the quotient to the nearest hundredth.

104.3766234 rounded to the nearest hundredth is 104.38.

This is close to the original estimate, so the answer is reasonable.

Estimate first. Then use a calculator to find the quotient. Round to the nearest hundredth if necessary.

$$1. \quad \begin{array}{r} 48.67 \\ 78 \overline{)3,796} \end{array}$$

$$2. \quad \begin{array}{r} 50.75 \\ 51 \overline{)2,588} \end{array}$$

$$3. \quad \begin{array}{r} 103 \\ 38 \overline{)3,914} \end{array}$$

$$4. \quad \begin{array}{r} 202.49 \\ 37 \overline{)7,492} \end{array}$$

$$5. \quad \begin{array}{r} 146.20 \\ 46 \overline{)6,725} \end{array}$$

$$6. \quad \begin{array}{r} 159.85 \\ 62 \overline{)9,911} \end{array}$$

7. **Number Sense** Is $5,309 \div 26$ less than 20, greater than 20 but less than 200, or greater than 200?

Greater than 200

Problem Solving: Missing or Extra Information

Aiko bought 6 red balloons and 11 clear balloons for a party. During the party, 3 clear balloons burst but none of the red balloons did. How many clear balloons did Aiko have after the party?

Read and Understand

What do you know?

Aiko bought 6 red balloons.

Aiko bought 11 clear balloons.

Three clear balloons burst during the party.

No red balloons burst during the party.

What are you trying to find?

The number of clear balloons remaining after the party

Plan and Solve

Draw a picture of what you know.



Solve the problem.

$$11 - 3 = 8$$

Write the answer in a complete sentence.

Aiko had 8 clear balloons after the party.

Look Back and Check

Is your answer correct?

$$\text{Yes, } 8 + 3 = 11$$

Look back at the items listed in “What you know.”

1. What information helped you solve the problem?

the number of clear balloons Aiko bought;
the number of clear balloons that burst

2. What information did **NOT** help you solve the problem?

the number of red balloons Aiko bought;
no red balloons burst

Variables and Expressions

A **variable** is a letter or symbol. It represents an unknown amount that can change.

You can do mathematical operations with variables and numbers.

You can state these operations in word expressions. You can also state them in algebraic expressions.

Operation	+	-	×	÷
Word expression	the sum of r and 3	the difference between r and 3	the product of r and 3	the quotient of r and 3
Algebraic expression	$r + 3$	$r - 3$	$3r$	$r \div 3$
Other ways of saying the word expression	3 added to r 3 more than r	3 subtracted from r 3 less than r	r multiplied by 3 3 times r	r divided by 3 3 equal parts of r

Complete the algebraic expression for each of the following word expressions.

1. the sum of
- b
- and 8

$b \underline{+} 8$

2. the difference between
- m
- and 6

$m \underline{-} 6$

3. the quotient of
- k
- and 16

$k \underline{\div} 16$

4. 7 less than
- z

$z \underline{-} 7$

5. 2 more than
- d

$d \underline{+} 2$

- 6.
- j
- divided by 4

$j \underline{\div} 4$

Circle the letter of the correct word expression for each algebraic expression.

7. $t - 13$

-
- a. 13 subtracted from
- t

- b.
- t
- subtracted from 13

8. $4n$

- a. 4 more than
- n

-
- b. 4 times
- n

9. $11 + s$

-
- a. 11 more than
- s

- b. 11 less than
- s

10. $45a$

-
- a. the product of
- a
- and 45

- b. 45 more than
- a

11. $y \div 6$

- a.
- y
- less 6

-
- b. 6 equal parts of
- y




12. $v - 5$

-
- a. 5 less than
- v



- b.
- v
- subtracted from 5

Patterns and Expressions

Chan makes beaded necklaces for gifts. To make a necklace, he strings beads onto thin ribbon. To tie the necklace, he cuts the ribbon 10 inches longer than the length he wants to bead. The chart shows the length of the ribbon for different beaded lengths.

Word Expression	Number Expression	Algebraic Expression
10 inches longer than 11-inch beaded section 	$11 + 10$	
10 inches longer than 14-inch beaded section 	$14 + 10$	
10 inches longer than z-inch beaded section 		$z + 10$

Zoe is going to use Chan's algebraic expression to make two necklaces. She wants to make one with a 10-inch beaded section. The other necklace will have a 12-inch beaded section. Here is how she evaluated the expression.

Algebraic Expression	Number Expression	Word Expression	Evaluated Expression
$z + 10$	$10 + 10$	10 inches longer than 10-inch beaded section 	20
$z + 10$	$12 + 10$	10 inches longer than 12-inch beaded section 	22

Chan also makes beaded bracelets. The chart shows the length of the beaded part of the bracelet. The second column shows the length of the ribbon he uses to make it.

Beaded Length (in.)	Ribbon Length (in.)
5	$5 + 7$
6	$6 + 7$
8	$8 + 7$

- Write a word expression that describes the pattern of the relationship.

The ribbon length is 7 inches longer than the beaded section.

More Patterns and Expressions

You can use variables to write and evaluate expressions.

Sunny Rent-A-Car rents cars for \$35 per day plus a \$12 fee for cleaning the car when you return it. Write an algebraic expression for the total cost of a car rental. Evaluate the expression for a 2-day car rental; for a 3-day car rental; for a 5-day car rental.

Step 1. The total cost is the cleaning fee plus the cost per day times the number of days. Since the number of days can change, it is your variable. Use d for the number of days.

$$\underline{35d + 12}$$

Step 2. To evaluate the expression, think about counting out the money to pay for the car rental. Each rental will have a stack of \$12 for the cleaning fee. Then, each rental will have a stack of \$35 for each day. A 2-day rental will have 2 stacks of \$35. A 3-day rental will have 3 stacks of \$35. A 5-day rental will have 5 stacks of \$35.

Step 3. To solve for the total cost of the car rentals, multiply the number of stacks by \$35. Then, add the \$12 stack.

2-day rental

$$(2 \times 35) + 12 = 82$$

A 2-day rental will cost \$82.

3-day rental

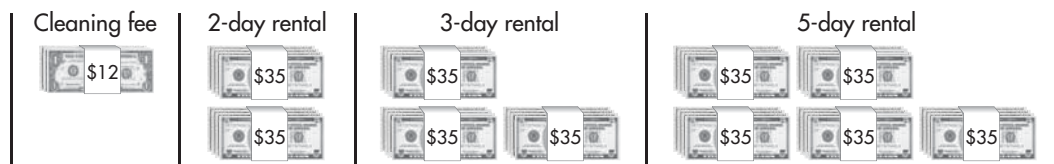
$$(3 \times 35) + 12 = 117$$

A 3-day rental will cost \$117.

5-day rental

$$(5 \times 35) + 12 = 187$$

A 5-day rental will cost \$187.



Write an algebraic expression for each phrase. Let n stand for the number. Evaluate each expression for $n = 4$.

1. 3 plus 4 times a number

$$\underline{3 + 4n; 19}$$

2. 7 times a number, minus 2

$$\underline{7n - 2; 26}$$

3. 6 less than a number times 15

$$\underline{15n - 6; 54}$$

Distributive Property

Hector's rock collection is in 7 cases. Each case holds 28 rocks. How many rocks are in Hector's collection? You can use the Distributive Property to find the product of 7×28 .

Step 1. Split 28 into $20 + 8$.

$$7 \times 28 = 7 \times (20 + 8)$$

Step 2. Multiply 7 times each part of the sum.

$$(7 \times 20) + (7 \times 8)$$

$$140 + 56$$

Step 3. Use addition to find the sum.

$$140 + 56 = 196$$

OR Step 1. Split 28 into $30 - 2$.

$$7 \times 28 = 7 \times (30 - 2)$$

Step 2. Multiply 7 times each part of the difference.

$$(7 \times 30) - (7 \times 2)$$

$$210 - 14$$

Step 3. Use subtraction to find the difference.

$$210 - 14 = 196$$

So, $7 \times 28 = 196$. Hector has 196 rocks in his collection.

Rewrite using the Distributive Property. Then find the product.

1. 3×42 126 2. 39×5 195 3. 6×147 882 4. 19×70 1,330
 5. 54×67 3,618 6. 90×83 7,470 7. 364×26 9,464 8. 45×678 30,510

Algebra For questions 9 through 12, find the value of n .

9. $4 \times 62 = (4 \times n) + (4 \times 2)$ 60

10. $79 \times 20 = (80 \times 20) - (n \times 20)$ 1

11. $53 \times 118 = (53 \times 100) + (n \times 18)$ 53

12. $352 \times 75 = (n \times 75) + (50 \times 75) + (2 \times 75)$ 300

13. Joey's class is collecting food for the school canned food drive. There are 28 children in Joey's class. Each child brought in 15 cans of food. Use the Distributive Property to find out how many cans of food Joey's class collected.

$$(15 \times 30) - (15 \times 2) = 450 - 30 = 420 \text{ cans}$$

Order of Operations

If you do not use the proper order of operations, you will not get the correct answer.

Evaluate $8 \div 2 + 3 \times 6 - (1 \times 5)$.

Step 1. Do the operations inside the parentheses.

$$(1 \times 5) = 5$$

$$8 \div 2 + 3 \times 6 - 5$$

Step 2. Multiply and divide in order from left to right.

$$8 \div 2 = 4 \text{ and } 3 \times 6 = 18$$

$$4 + 18 - 5$$

Step 3. Add and subtract in order from left to right.

$$4 + 18 = 22$$

$$22 - 5 = 17$$

So, $8 \div 2 + 3 \times 6 - (1 \times 5) = 17$

Write which operation should be done first.

1. $6 + 3 \times 2$ multiplication 2. $13 - 1 + 4 \div 2$ division
3. $5 \times (7 - 2) + 1$ subtraction 4. $(19 + 23) - (4 \times 5)$ addition and multiplication

For questions 5 through 8, evaluate the expression for $x = 6$ and $y = 17$.

5. $4x + 5y$ 109 6. $2x + (20 - y)$ 15
7. $x \div 3 + y$ 19 8. $4y \div 2 + (8x + 10)$ 92

9. There are 22 students in Natalie's class. New reading books cost \$7 each. Natalie's class raised \$50 washing cars. If there are 2 teachers in Natalie's class, write and evaluate an expression to show how much more each teacher will have to pay to buy the new reading books.

$$(22 \times 7 - 50) \div 2 = \$52$$

10. **Number Sense** Carlos solved $20 - (2 \times 6) + 8 \div 4 = 29$. Is this the correct answer?

No. Carlos solved the equation from left to right instead of following the order of operations.

Problem Solving: Act It Out and Use Reasoning

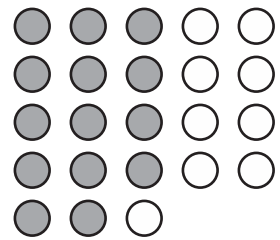
In Mackenzie's class there are 23 students. 14 students have brown hair. 2 times as many students have blonde hair as have red hair. How many students have each color hair?

Draw a diagram to show what you know.

There are 9 students left. You know that two times as many have blonde hair as have red hair. Make a table to try different numbers and see which pair fits the problem.

red		blonde	Do numbers add up to 9?
1	$1 \times 2 =$	2	does not equal 9
2	$2 \times 2 =$	4	does not equal 9
3	$3 \times 2 =$	6	equals 9

23 students total



14 students with brown hair

Since 6 are two times 3 and $6 + 3 = 9$, this is the correct answer.

So, in Mackenzie's class, there are 14 students with brown hair, 6 with blonde hair, and 3 with red hair.

In **1** and **2**, solve. Draw a picture and/or table to help find the answer.

- Jacobson Animal Shelter has half as many cats as dogs. The shelter has 30 dogs. How many total animals does the shelter have?

45 animals

- Summer's mother gave Summer \$20 to share with her 3 brothers equally. How much did each person get?

\$5

Multiplying Decimals by 10, 100, or 1,000

You can use patterns to multiply decimals mentally by 10, 100, and 1,000.

Andrew starts selling his baseball cards for \$0.10 each. After selling 10 cards, he has made \$1.00. After selling 100 cards, he has made \$10.00.



\$0.10



$\$0.10 \times 10 = \1.00



$\$0.10 \times 100 = \10.00

When you multiply by

10

100

1,000

Move the decimal point

1 place to the right

2 places to the right

3 places to the right

If Andrew sold 1,000 cards, how much money would he make? \$100.00

For questions 1 through 4, find the product using mental math.

1. 6.1×10 61

2. 100×37.98 3,798

3. $92.3 \times 1,000$ 92,300

4. 0.418×100 41.8

5. Myla has an antique flower vase that she bought for \$15.75 many years ago. The vase's value is now 1,000 times as great. What is the value of the vase? \$15,750

6. Raul can hit a golf ball 26.4 yards. A.J. can hit a golf ball 10 times as far. How far can A.J. hit the ball? 264 yards

7. **Reasonableness** Is 0.018 a reasonable answer for 1.8×100 ?

No. The answer cannot be a smaller number than the factors unless at least one factor is between 0 and 1. The answer should be 180.

Multiplying a Decimal by a Whole Number

Travis can read a book chapter in 2.6 hours. The book has 18 chapters. How long will it take Travis to read the book?

Step 1. Multiply as with whole numbers.

$$\begin{array}{r} 2.6 \\ \times 18 \\ \hline 208 \\ + 260 \\ \hline 468 \end{array}$$

Step 2. Count the total decimal places in both factors.

$$\begin{array}{r} 2.6 \quad 1 \text{ decimal} \\ \quad \quad \text{place} \\ 18 \quad 0 \text{ decimal} \\ \quad \quad \text{places} \end{array}$$

Step 3. Since there is a total of 1 decimal place in the factors, there is 1 decimal place in the product.

$$46.8$$

It will take Travis 46.8 hours to read the book.

For questions 1 through 3, find the product.

1.
$$\begin{array}{r} 2.3 \\ \times 6 \\ \hline \end{array}$$

13.8

2.
$$\begin{array}{r} 71.7 \\ \times 12 \\ \hline \end{array}$$

860.4

3.
$$\begin{array}{r} 0.894 \\ \times 21 \\ \hline \end{array}$$

18.774

4. Sara is multiplying two factors, one with one decimal place and one with two decimal places. She says that the product should have two decimal places. Is this correct? Explain.

This is incorrect. The factors have a total of three decimal places, so the product should have three decimal places.

5. **Critical Thinking** Light bulbs usually cost \$2. They are on sale for 0.50 of the regular price. What is the sale price? Is this a better price than if the sale price were 0.35 of the regular price?

\$1; No, 0.35 is a smaller fraction than 0.50; the second price would be better.

Estimating the Product of a Decimal and a Whole Number

You can estimate when you are multiplying a decimal by a whole number to check the reasonableness of your product.

Zane needs to buy 27 lb of roast beef for the company party. The roast beef costs \$2.98 per pound. About how much will the roast beef cost?

There are two ways to estimate.

Round both numbers

$$\begin{array}{r} \$2.98 \times 27 \\ \downarrow \quad \downarrow \\ \$3 \times 30 = \$90 \end{array}$$

The roast beef will cost about \$90.

Adjust your factors to compatible numbers you can multiply mentally.

$$\begin{array}{r} \$2.98 \times 27 \\ \downarrow \quad \downarrow \\ \$3 \times 25 = \$75 \end{array}$$

The roast beef will cost about \$75.

Estimate each product. **Sample answers are given.**

1. 0.8×22 **20**

2. 19.3×6 **100**

3. 345×5.79 **1,800**

4. 966×0.46 **500**

Number Sense Use the chart to answer questions 5 through 7.

5. About how much would it cost for Angelina and her 4 sisters to get a shampoo and a haircut?

$$\underline{(5 \times \$8) + (5 \times \$13) =}$$

about \$105

Treatment	Cost
Shampoo	\$7.95
Haircut	\$12.95
Coloring	\$18.25
Perm	\$22.45

6. Could 3 of the sisters get their hair colored for less than \$100?

$$\underline{\text{Yes; } \$20 \times 3 = \$60.}$$

7. Angelina gets 9 haircuts per year. About how much does she spend on haircuts for the year?

$$\underline{10 \times \$13 = \text{about } \$130}$$

Multiplying Two Decimals

Caroline earns \$2.40 per hour for babysitting her brother. She babysat last night for 3.25 hours. How much did she earn?

First, estimate your product so you can check for reasonableness.

$$\$2.40 \times 3.25$$



$\$2 \times 3 = 6$ Caroline earned about \$6.00.

Step 1: Multiply as with whole numbers.

$$\begin{array}{r} 3.25 \\ \times 2.40 \\ \hline 000 \\ 13,000 \\ +65,000 \\ \hline 78,000 \end{array}$$

Step 2: Count the total number of decimal places in both factors.

3.25 2 decimal places

2.40 2 decimal places

So, 4 decimal places total.

Step 3: Place the decimal point in the product with the correct total number of decimal places following it.

$$7.8000 = \$7.80$$

Caroline earned \$7.80 last night. Because \$7.80 is close to your estimate of \$6, your answer is reasonable.

Find each product. Check by estimating.

1. 0.2×4.6 0.92 2. 3.98×7.1 28.258 3. 8.54×0.14 1.1956

4. 0.532×6.4 3.4048 5. 9.3×5.86 54.498 6. 0.37×4.4 1.628

7. **Critical Thinking** Jackie wants to buy a new CD player. It costs \$32.95. She has saved \$26 and has a coupon for 30% off the price. Does Jackie have enough money to buy the CD player?

Yes. The coupon reduces the price of the CD player to \$23.07.

Dividing Decimals by 10, 100, or 1,000

You can use place-value patterns when you divide a decimal by 10, 100, or 1,000.

Sanjai has 27.5 lb of clay. If he uses the clay to make 10 bowls, how much clay will he use for each bowl? What if he makes 100 bowls from the clay? What if he makes 1,000 bowls?

Dividing a number by 10 moves the decimal point one place to the left.

$$27.5 \div 10 = 2.75$$

Dividing a number by 100 moves the decimal point two places to the left.

$$27.5 \div 100 = 0.275$$

Dividing a number by 1,000 moves the decimal point three places to the left.

$$27.5 \div 1,000 = 0.0275$$

Sanjai will use 2.75 lb for each of 10 bowls, 0.275 lb for each of 100 bowls, and 0.0275 lb for each of 1,000 bowls.

Remember: When you divide a number by 10, 100, or 1,000, your quotient will be smaller than that number.

For questions 1 through 6, find the quotient. Use mental math.

1. $16.4 \div 10$ **1.64** 2. $38.92 \div 100$ 3. $297.1 \div 100$ **2.971**

4. $540.9 \div 10$ **54.09** 5. $41.628 \div 1,000$ **0.041628** 6. $0.33 \div 10$ **0.033**

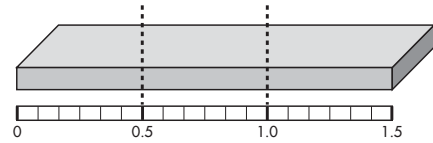
7. The city has a section of land 3,694.7 ft long. The city wants to make 100 equal-sized gardens with this land. How long will each garden be? **36.947 ft**

8. **Reasonableness** Connor divided 143.89 by 100. He said his answer was 14.389. Is this a reasonable answer?

No. Connor moved the decimal point only one place to the left. He should have moved it two places to the left.

Dividing a Decimal by a Whole Number

Jia needs to cut a board into 3 equal pieces to make bookshelves. The board is 1.5 yd long. How long will each bookshelf be?



Laying a measuring tape next to the board, Jia sees that her board can be cut into 3 pieces that are each 0.5 yd long. Jia also writes out her problem to make sure her answer is correct.

Step 1. Write a decimal point in the quotient directly above the decimal point in the dividend.

Step 2. Divide as you would with whole numbers.

$$\begin{array}{r} 0.5 \\ 3 \overline{)1.5} \\ \underline{-15} \\ 0 \end{array}$$

Step 3. Check your quotient by multiplying.

$$0.5 \times 3 = 1.5$$

Since the quotient matches the measurement that Jia saw on her measuring tape, she knows that her answer is correct.

Sometimes you will need to add a zero to the right of the dividend so that you can continue dividing. Example:

$$8.1 \div 18$$

$$\begin{array}{r} 0.45 \\ 18 \overline{)8.10} \\ \underline{-72} \\ 90 \\ \underline{-90} \\ 0 \end{array}$$

For **1** through **6**, find each quotient. Check by multiplying.

1. $14 \overline{)6.3}$ **0.45** 2. $77 \overline{)2.31}$ **0.03** 3. $89 \overline{)2.492}$ **0.028**

4. $123.08 \div 34$ **3.62** 5. $0.57 \div 30$ **0.019** 6. $562.86 \div 59$ **9.54**

7. A family of five people attends a theme park. They purchase 2 adult tickets for \$27.50 each and 3 student tickets for \$12.50 each. If the 5 tickets are purchased with a \$100 bill, how much change do they receive? **\$7.50**

Estimation: Decimals Divided by Whole Numbers

The merry-go-round in the Smithson Town Park took in \$795.60 in the last six months. The ticket taker says that 442 people rode the merry-go-round during that time. About how much does it cost to ride?

You can estimate to find the quotient of $\$795.60 \div 442$ either by rounding or using compatible numbers and mental math.

To round your numbers, cover the digit in the hundreds place with your hand. Look at the number next to your hand. Since $9 > 5$, you round \$795.60 up to \$800. Since $4 < 5$, you round 442 down to 400.



$$800 \div 400 = 2 \quad \text{It costs about \$2 to ride.}$$

Or you can use compatible numbers and mental math.

$$800 \div 400 = 2 \quad \text{It costs about \$2 to ride.}$$

Estimate each quotient. **Sample answers are given.**

1. $81.2 \div 19$ 4 2. $376.44 \div 22$ 20 3. $62.91 \div 16$ 4

4. $763.85 \div 82$ 10 5. $550.8 \div 9$ 60 6. $486.5 \div 3$ 150

7. **Number Sense** Explain how you would know an error had been made if you find the quotient for $231.68 \div 16$ to be 144.8.

Possible answer: by estimating the division as $200 \div 20 = 10$, which is not close to 144.8

8. **Reasonableness** Is $61.5 \div 15$ a little less than 4, a little more than 4, a little less than 40, or a little more than 40?

a little more than 4

Dividing a Decimal by a Decimal

Sebastian is shipping a box to his cousin. The box weighs 23.6 lb. The cost to ship the box is \$88.50. How much does it cost per pound to ship the box?

To divide a decimal by a decimal, you need to change the problem into a simpler type of problem that you already know how to solve.

Step 1: To change this problem, you multiply the divisor by a power of 10 until it becomes a whole number.

$$23.6 \times 10 = 236$$

Step 2: You must also multiply the dividend by the same power of 10. Sometimes, you may have to add zeros as placeholders.

$$88.50 \times 10 = 885.0$$

Step 3: Place a decimal point in the quotient and divide as you would with whole numbers.

$$\begin{array}{r} 3.75 \\ 236 \overline{)885.00} \\ \underline{-708} \\ 1770 \\ \underline{-1652} \\ 1180 \\ \underline{-1180} \\ 0 \end{array}$$

The box costs \$3.75 per pound to ship.

For questions 1 through 6, find each quotient.

1. $0.104 \div 0.08$ **1.3** 2. $5.49 \div 0.9$ **6.1** 3. $50.4 \div 1.2$ **42**

4. $0.427 \div 61$ **0.007** 5. $0.8449 \div 0.71$ **1.19** 6. $9.483 \div 8.7$ **1.09**

7. Miriam needs to buy new notebooks for school. The notebooks cost \$0.98 each including tax. Miriam's mother gave her \$6.45 and told her to buy 8 notebooks. Did her mother give her enough money?

No. She can only buy 6 notebooks with \$6.45.

8. Miriam finds a coupon in the store that reduces the price of notebooks to \$0.75 each including tax. How many notebooks can Miriam buy now?

She can buy 8 notebooks.

Problem Solving: Multiple-Step Problems

A multiple-step problem is a problem where you may need more than one step to find your answer.

Marcie was in a 3-day charity walk. Her friend Gayle said she would give the charity \$1.50 for each mile that Marcie walked. The first day, Marcie walked 26.42 miles. The second day, Marcie walked 32.37 miles. The third day, Marcie walked 28.93 miles. How much money did Gayle give?

Step 1. Read through the problem again and write a list of what you already know.

Marcie walked 26.42, 32.37, and 28.93 miles.

Gayle gave \$1.50 for each mile.

Step 2. Write a list of what you *need* to know.

Total amount Gayle gave

Step 3. Write a list of the steps to solve the problem.

Find the total number of miles Marcie walked.

Find the amount Gayle gave.

Step 4. Solve the problem one step at a time.

$26.42 + 32.37 + 28.93 = 87.72$ *total number of miles Marcie walked*

$87.72 \times \$1.50 = \131.58 *total amount Gayle gave*

Use the information above to answer Exercise 1.

- Marcie's brother Tom was also in the charity walk. He only walked 0.8 as far as Marcie on the first day, 0.7 as far on the second day, and 0.9 as far on the third day. How many miles did Tom walk, rounded to the nearest hundredth of a mile?

69.83 miles


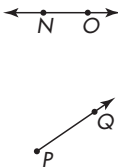
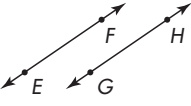
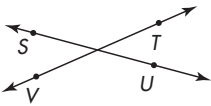
- Diego is buying fruit at the store. Which costs less: 1 pound of each fruit or 4 pounds of peaches?

4 pounds of peaches

Fruit	Cost per pound
Apples	\$0.89
Oranges	\$1.29
Peaches	\$0.99
Grapes	\$1.09

Basic Geometric Ideas

Points and lines are basic geometric ideas. Lines are sometimes described by relationships to other lines.

Draw	Write	Say	Description
	J	point J	J is a point. It shows an exact location in space.
	\overleftrightarrow{NO} \overrightarrow{PQ}	line NO ray PQ	\overleftrightarrow{NO} is a line. It is a straight path of points that goes on forever in two directions. \overrightarrow{PQ} is a ray. It goes on forever in only one direction.
	$\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$	Line EF is parallel to line GH .	\overleftrightarrow{EF} and \overleftrightarrow{GH} are parallel lines. They are the same distance apart and will not cross each other.
	\overleftrightarrow{SU} intersects \overleftrightarrow{VT}	Line SU intersects line VT .	\overleftrightarrow{SU} and \overleftrightarrow{VT} are intersecting lines. They pass through the same point.

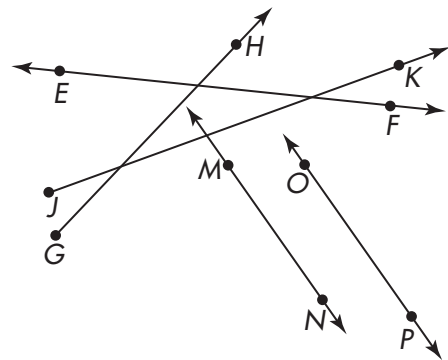
Use the diagram on the right. Name the following.

1. two parallel lines

$\overleftrightarrow{MN}, \overleftrightarrow{OP}$

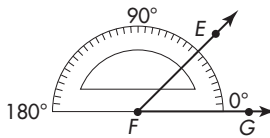
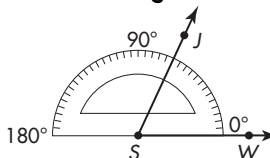
2. two intersecting rays

$\overrightarrow{GH}, \overrightarrow{JK}$

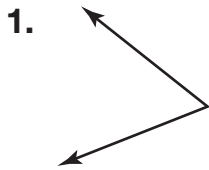


Measuring and Classifying Angles

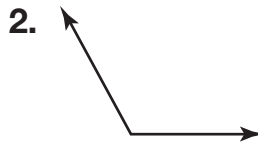
The chart below can help you describe and identify an angle.

<p>Measure $\angle EFG$.</p> 	<p>Remember to place the 0° mark on one side of the angle. $\angle EFG$ measures 45°.</p>	<p>Classifying Angles</p>
<p>Draw an angle of 65°.</p> 	<p>Place the <u>center</u> of the protractor on <u>S</u>. Line up <u>SW</u> with the 0° mark. <u>Place</u> a point at 65°. Label it <u>J</u>. Draw <u>SJ</u>.</p>	
		<p>Right exactly 90°</p>
		<p>Obtuse between 90° and 180°</p>
		<p>Straight exactly 180°</p>

Classify each angle as acute, right, obtuse, or straight. Then measure each angle.



Acute; 60°

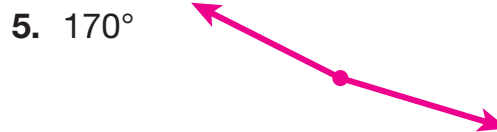
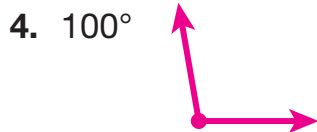


Obtuse; 120°



Straight; 180°

Draw an angle with each measure.

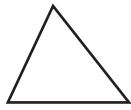


6. **Reasoning** $\angle ABC$ measures less than 180° but more than 90° . Is $\angle ABC$ a right, an acute, an obtuse, or a straight angle?

Obtuse; it has a measure between 90° and 180° .

Polygons

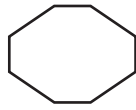
A polygon is a closed plane figure made up of line segments. Common polygons have names that tell the number of sides the polygon has.



Triangle
3 sides



Pentagon
5 sides



Octagon
8 sides



Hexagon
6 sides

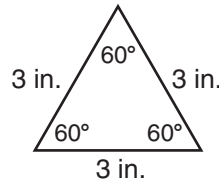


Open Figure



Quadrilateral
4 sides

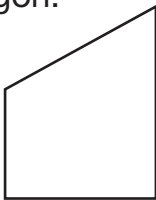
A **regular polygon** has sides of equal length and angles of equal measure.



Each side is 3 in. long.
Each angle is 60°.

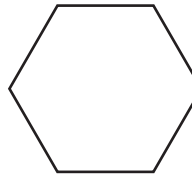
Name each polygon. Then tell if it appears to be a regular polygon.

1.



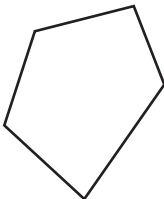
Quadrilateral;
not regular

2.



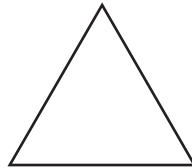
Hexagon;
regular

3.



Pentagon;
not regular

4.



Triangle;
regular

5. **Reasoning** Shakira sorted shapes into two different groups. Use geometric terms to describe how she sorted the shapes.

The shapes in
Group A are all open

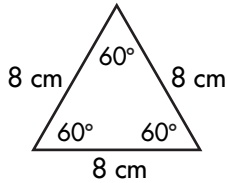
and are not polygons. The shapes in
Group B are closed and are polygons.

Group A	Group B

Triangles

You can classify triangles by the lengths of their sides and the sizes of their angles.

acute
all angles less than 90°

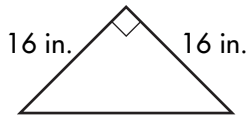


equilateral
all sides the same length

This triangle is both equilateral and acute.

Not all acute triangles are equilateral.

right
one right angle

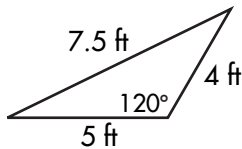


isosceles
two sides the same length

This triangle is both isosceles and right.

Not all right triangles are isosceles.

obtuse
one obtuse angle



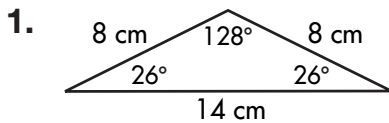
scalene
no sides the same length

This triangle is both scalene and obtuse.

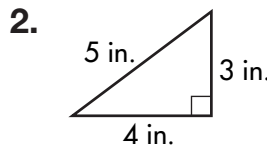
Not all obtuse triangles are scalene.

Remember that the sum of the measures of the angles of a triangle is 180° .

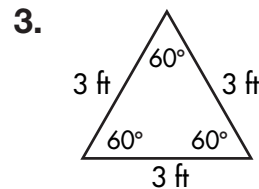
Classify each triangle by its sides and then by its angles.



Isosceles;
obtuse



Scalene;
right



Equilateral;
acute

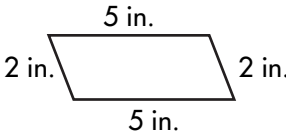
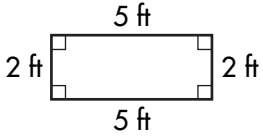
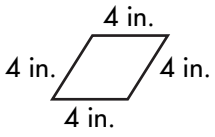
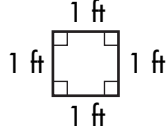
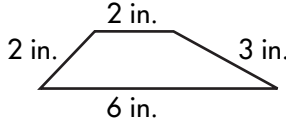
The measures of two angles of a triangle are given. Find the measure of the third angle.

4. $40^\circ, 100^\circ, \underline{40^\circ}$

5. $14^\circ, 98^\circ, \underline{68^\circ}$

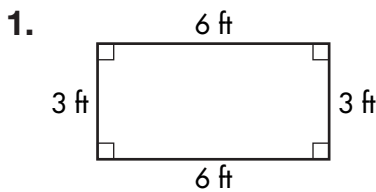
6. $38^\circ, 38^\circ, \underline{104^\circ}$

Quadrilaterals

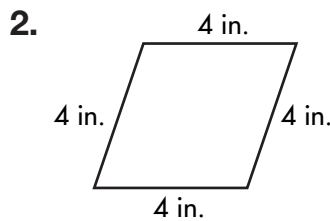
Quadrilateral	Definition	Example
Parallelogram	A quadrilateral with both pairs of opposite sides parallel and equal in length	
Rectangle	A parallelogram with four right angles	
Rhombus	A parallelogram with all sides the same length	
Square	A rectangle with all sides the same length	
Trapezoid	A quadrilateral with only one pair of parallel sides	

Remember that the sum of the measures of the angles of a quadrilateral is 360° .

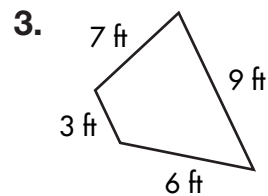
Classify each quadrilateral. Be as specific as possible.



Rectangle



Rhombus



Trapezoid

The measures of three angles of a quadrilateral are given. Find the measure of the fourth angle.

4. $65^\circ, 150^\circ, 89^\circ, \underline{56^\circ}$

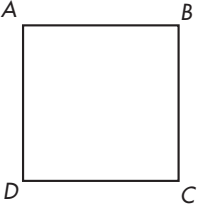
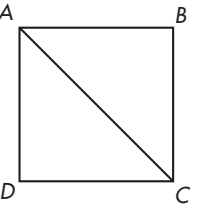
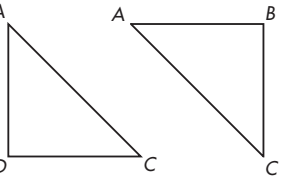
5. $100^\circ, 80^\circ, 100^\circ, \underline{80^\circ}$

6. $82^\circ, 78^\circ, 90^\circ, \underline{110^\circ}$

Problem Solving: Make and Test Generalizations

Here is a generalization to be tested: any square can be cut in half through a diagonal. The result is always two isosceles triangles, each with a 90° angle.

Test one example of this generalization:

<p>Draw a square, $ABCD$.</p> 	<p>Draw a diagonal, AC.</p> 	<p>Inspect the triangles, ABC and CDA.</p> 
--	--	--

Triangle ABC :

- $AB = BC$ *All sides of a square are equal length.*
- Angle $B = 90^\circ$ *All angles of a square are 90° .*

Triangle CDA :

- $CD = DA$ *All sides of a square are equal length.*
- Angle $D = 90^\circ$ *All angles of a square are 90° .*

Conclusion: Each triangle has two equal sides and contains a right angle. The generalization is true for the square $ABCD$.

Repeat for more squares. If for each square the conclusion is the same, the generalization appears to be correct.

Explain It Show that the triangles ABC and CDA are congruent.

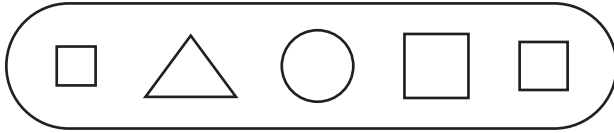
Sample answer: $AB = CD$ and $BC = DA$

(All sides of a square are equal length.)

$AC = AC$. Two triangles are congruent if all three sides have matching lengths.

Meanings of Fractions

What fraction of the set of shapes are squares?



Step 1: Find the denominator.

How many shapes are there in the set?

There are 5 shapes in the set.

The denominator is the total number of shapes. So, the denominator is 5.

Step 2: Find the numerator.

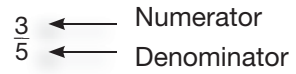
How many squares are there in the set?

There are 3 squares in the set.

The numerator is the number of squares in the set. So, the numerator is 3.

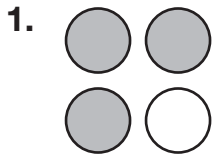
Step 3: Write the fraction.

Write the numerator over the denominator.

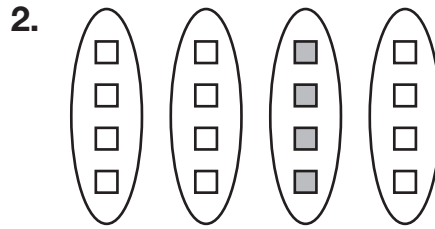


$\frac{3}{5}$ of the set are squares.

Write the fraction that names the shaded part.



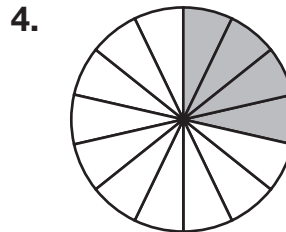
$\frac{3}{4}$



$\frac{4}{16}$



$\frac{5}{8}$



$\frac{4}{14}$

5. **Number Sense** If $\frac{1}{5}$ of a region is not shaded, what part is shaded?

$\frac{4}{5}$

6. Alex has 7 dimes and 3 nickels. What fraction of the coins are dimes?

$\frac{7}{10}$

Fractions and Division

Fractions can represent division. You can write a division expression as a fraction. For example:

Write a fraction for $5 \div 7$.

The first number in the division expression is the numerator of the fraction. The second number in the division expression is the denominator of the fraction.

$$\begin{array}{l} \xrightarrow{\hspace{1.5cm}} 5 \text{ Numerator} \\ 5 \div 7 \longrightarrow \frac{5}{7} \text{ Denominator} \end{array}$$

So, $5 \div 7 = \frac{5}{7}$.

Give each answer as a fraction.

1. $3 \div 10$

$$\frac{3}{10}$$

2. $7 \div 12$

$$\frac{7}{12}$$

3. $2 \div 3$

$$\frac{2}{3}$$

4. $8 \div 9$

$$\frac{8}{9}$$

5. $2 \div 5$

$$\frac{2}{5}$$

6. $1 \div 6$

$$\frac{1}{6}$$

7. $6 \div 10$

$$\frac{6}{10}$$

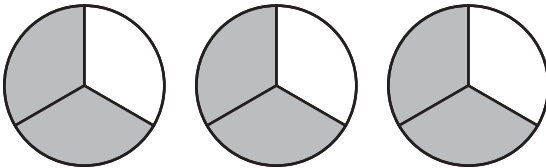
8. $9 \div 13$

$$\frac{9}{13}$$

9. $14 \div 16$

$$\frac{14}{16}$$

Reasoning Three congruent circles are each divided into three equal parts. Use these three circles for **10** through **12**.



10. What part of a whole circle is shown by the white, or unshaded, area of one circle?

$$\frac{1}{3}$$

11. What part of a whole circle is shown by the white, or unshaded, area of two circles?

$$\frac{2}{3}$$

12. What part of a whole circle is shown by the white, or unshaded, area of three circles?

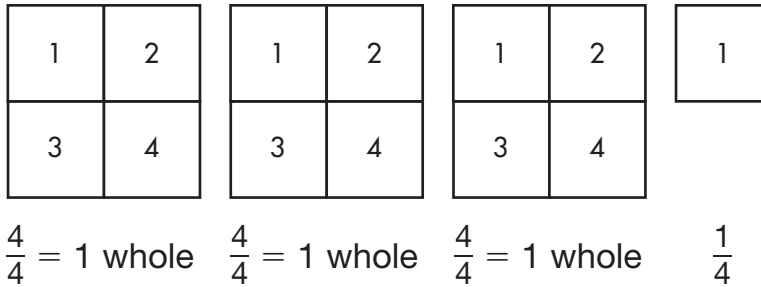
$$\frac{3}{3} \text{ or } 1$$

Mixed Numbers and Improper Fractions

Example 1: Write $\frac{13}{4}$ as a mixed number.

To write an improper fraction as a mixed number, draw a model.

Draw squares divided into 4 equal parts until you have 13 parts.



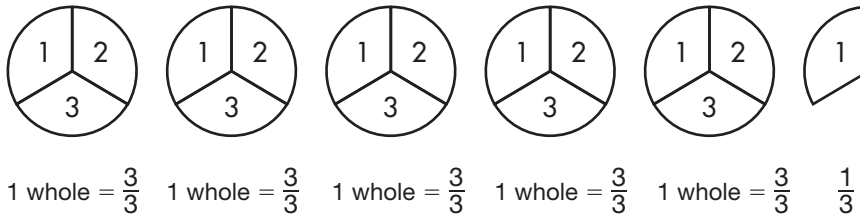
The 13 parts make up 3 wholes and $\frac{1}{4}$ of a whole.

$$\frac{13}{4} = 3\frac{1}{4}$$

Example 2: Write $5\frac{1}{3}$ as an improper fraction.

To write a mixed number as an improper fraction, draw a model.

Draw circles divided into 3 equal parts to represent $5\frac{1}{3}$.



$$\frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{16}{3}$$

Write each improper fraction as a mixed number.

1. $\frac{8}{3}$ $2\frac{2}{3}$ _____

2. $\frac{10}{7}$ $1\frac{3}{7}$ _____

3. $\frac{5}{2}$ $2\frac{1}{2}$ _____

Write each mixed number as an improper fraction.

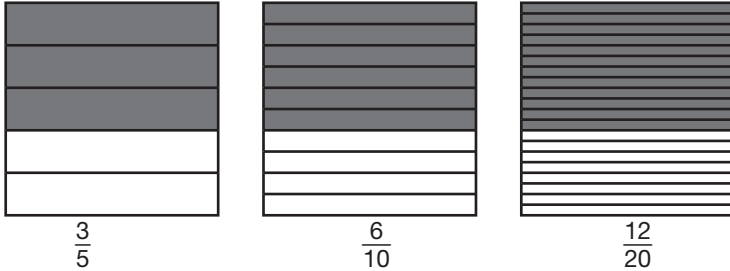
4. $1\frac{2}{5}$ $\frac{7}{5}$ _____

5. $4\frac{6}{7}$ $\frac{34}{7}$ _____

6. $2\frac{5}{8}$ $\frac{21}{8}$ _____

Equivalent Fractions

The fractions shown below are equivalent.
They all describe the same part of a whole.



To find equivalent fractions, multiply or divide the numerator and denominator by the same number.

$$\frac{3}{5} \times \frac{2}{2} = \frac{6}{10} \quad \frac{6}{10} \times \frac{2}{2} = \frac{12}{20} \quad \frac{12}{20} \div \frac{2}{2} = \frac{6}{10}$$

$$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5} \quad \frac{12}{20} \div \frac{4}{4} = \frac{3}{5}$$

Name two equivalent fractions for each fraction. **Sample answers given**

1. $\frac{1}{3}$ $\frac{2}{6}$, $\frac{3}{9}$

2. $\frac{2}{12}$ $\frac{1}{6}$, $\frac{4}{24}$

3. $\frac{4}{20}$ $\frac{2}{10}$, $\frac{1}{5}$

4. $\frac{2}{16}$ $\frac{1}{8}$, $\frac{4}{32}$

Find the missing number to make the fractions equivalent.

5. $\frac{4}{7} = \frac{8}{\square}$

6. $\frac{\square}{18} = \frac{4}{6}$

7. $\frac{3}{4} = \frac{\square}{12}$

8. $\frac{15}{\square} = \frac{3}{4}$

14

12

9

20

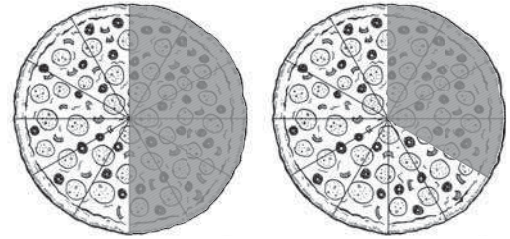
9. **Number Sense** Are $\frac{3}{4}$ and $\frac{12}{16}$ equivalent fractions? Explain.

Yes: If you multiply the numerator and denominator of $\frac{3}{4}$ by 4, you get $\frac{12}{16}$.

Comparing and Ordering Fractions and Mixed Numbers

You can compare fractions by finding a common denominator.

Samantha and her brother Jacob went out for pizza. Samantha ate $\frac{1}{2}$ of her pizza. Jacob ate $\frac{4}{12}$ of his pizza. Who ate more pizza?



Samantha's Pizza

Jacob's pizza

Because these fractions have different denominators, you need to find a common denominator. Then you can compare them.

Step 1. Write multiples of the two denominators until you get a common multiple.

$$2: 2, 4, 6, 8, 10, \textcircled{12}$$

$$12: \textcircled{12}, 24, 36, 48, 60 \quad \text{Use 12 as the common denominator.}$$

Step 2. Since you multiply 2×6 to get 12, you must multiply 1×6 .

$$\frac{1}{2} = \frac{6}{12}$$

Step 3. Compare the fractions with common denominators.

$$\frac{6}{12} > \frac{4}{12}$$

So, Samantha ate more pizza.

Remember: If you don't know the multiples of the denominators, you can multiply the denominators together to get a common denominator.

Compare. Write $>$, $<$, or $=$ for each \bigcirc .

$$1. \frac{2}{3} \textcircled{>} \frac{1}{6}$$

$$2. \frac{3}{4} \textcircled{>} \frac{1}{2}$$

$$3. \frac{5}{6} \textcircled{<} \frac{21}{24}$$

Order the numbers from least to greatest.

$$4. \frac{4}{5}, \frac{3}{5}, \frac{3}{4} \quad \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \quad \frac{3}{5}, \frac{3}{4}, \frac{4}{5}$$

$$5. 1\frac{5}{6}, 1\frac{3}{6}, 1\frac{2}{12} \quad \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \quad 1\frac{2}{12}, 1\frac{3}{6}, 1\frac{5}{6}$$

6. **Geometry** Sofia baked three kinds of pie. Sofia's Mom told her to bring $\frac{8}{16}$ of the apple pie, $\frac{4}{8}$ of the pecan pie, and $\frac{3}{6}$ of the pumpkin pie to school to share with her friends. Draw the pies and show which pie will have the greatest amount brought to school.

All three pies will have equal amounts brought to school, because $\frac{8}{16} = \frac{4}{8} = \frac{3}{6}$.

Common Factors and Greatest Common Factor

The greatest common factor (GCF) of two numbers is the greatest number that is a factor of both.

Find the greatest common factor of 12 and 18.

You can use arrays to find the factors of 12.

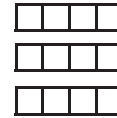
$1 \times 12 = 12$



$2 \times 6 = 12$



$3 \times 4 = 12$



Factors of 12 are 1, 2, 3, 4, 6, and 12.

You can use arrays to find the factors of 18.

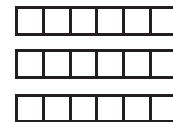
$1 \times 18 = 18$



$2 \times 9 = 18$



$3 \times 6 = 18$



Factors of 18 are 1, 2, 3, 6, 9, and 18.

You can see that the common factors of 12 and 18 are 2, 3, and 6. The greatest common factor of 12 and 18 is 6.

Find the GCF of each pair of numbers.

1. 9, 27 9

2. 25, 40 5

3. 7, 36 1

4. 40, 48 8

5. **Number Sense** Can the GCF of 18 and 36 be greater than 18? Explain.

No. No number greater than 18 can be a factor of 18.

Fractions in Simplest Form

There are two different ways to write a fraction in simplest form.

Write $\frac{20}{24}$ in simplest form.

Divide by Common Factors

- Divide by common factors until the only common factor is 1.
- You can start by dividing by 2, since both numbers are even.

$$\frac{20 \div 2}{24 \div 2} = \frac{10}{12}$$

But both 10 and 12 are also even, so they can be divided by 2.

$$\frac{10 \div 2}{12 \div 2} = \frac{5}{6}$$

- Since 5 and 6 do not have any common factors, $\frac{5}{6}$ is the simplest form.

Divide by the GCF

- First find the GCF of 20 and 24.

$$20: \textcircled{1} \textcircled{2} \textcircled{4} \quad 5, \quad 10, \quad 20$$

$$24: \textcircled{1} \textcircled{2} \quad 3, \quad \textcircled{4} \quad 6, \quad 8, \quad 12, \quad 24$$

- The common factors of 20 and 24 are 1, 2, and 4. The GCF of 20 and 24 is 4.
- Divide both the numerator and the denominator by 4.

$$\frac{20 \div 4}{24 \div 4} = \frac{5}{6}$$

$\frac{20}{24}$ written in simplest form is $\frac{5}{6}$.

Write each fraction in simplest form.

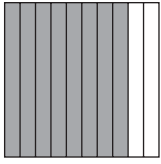
- | | | | | | |
|--------------------------|---------------|--------------------------|---------------|---------------------------|-----------------|
| 1. $\frac{16}{20}$ _____ | $\frac{4}{5}$ | 2. $\frac{8}{16}$ _____ | $\frac{1}{2}$ | 3. $\frac{5}{10}$ _____ | $\frac{1}{2}$ |
| 4. $\frac{8}{32}$ _____ | $\frac{1}{4}$ | 5. $\frac{18}{42}$ _____ | $\frac{3}{7}$ | 6. $\frac{15}{100}$ _____ | $\frac{3}{20}$ |
| 7. $\frac{18}{21}$ _____ | $\frac{6}{7}$ | 8. $\frac{24}{40}$ _____ | $\frac{3}{5}$ | 9. $\frac{55}{70}$ _____ | $\frac{11}{14}$ |

10. **Number Sense** Explain how you can tell that $\frac{31}{33}$ is in simplest form.

Sample answer: 31 is a prime number; 31 and 33 do not have any common factors except 1, so $\frac{31}{33}$ is in simplest form.

Tenths and Hundredths

Fractions can also be named using decimals.



8 out of 10 sections are shaded.

The fraction is $\frac{8}{10}$.

The word name is eight tenths.

The decimal is 0.8.

Remember: the first place to the right of the decimal is tenths.

Write $\frac{2}{5}$ as a decimal.

Sometimes a fraction can be rewritten as an equivalent fraction that has a denominator of 10 or 100.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{4}{10} = 0.4$$

$$\text{So, } \frac{2}{5} = 0.4.$$

Write $3\frac{3}{5}$ as a decimal.

First write the whole number.

3

Write the fraction as an equivalent fraction with a denominator of 10.

Change the fraction to a decimal.

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$$

Write the decimal next to the whole number

3.6

$$\text{So, } 3\frac{3}{5} = 3.6.$$

Write 0.07 as a fraction.

The word name for 0.07 is seven hundredths.

“Seven” is the numerator, and “hundredths” is the denominator.

$$\text{So, } 0.07 = \frac{7}{100}.$$

Remember: the second place to the right of the decimal is hundredths.

Write each fraction or mixed number as a decimal.

1. $\frac{1}{5}$ **0.2**

2. $\frac{6}{25}$ **0.24**

3. $2\frac{3}{4}$ **2.75**

4. $3\frac{9}{10}$ **3.9**

Write each decimal as a fraction or mixed number.

5. 1.25 **$1\frac{25}{100}$**

6. 3.29 **$3\frac{29}{100}$**

7. 0.65 **$\frac{65}{100}$**

8. 5.6 **$5\frac{6}{10}$**

9. **Number Sense** Dan says $\frac{3}{5}$ is the same as 3.5. Is he correct? Explain.

No. 3.5 is the same as $3\frac{5}{10}$ or $3\frac{1}{2}$; $\frac{3}{5}$ is the same as 0.6.

Thousandths

Example 1: Write 0.025 as a fraction.

Ones	.	Tenths	Hundredths	Thousandths
0	.	0	2	5

You can use a place-value chart to write a decimal as a fraction. Look at the place-value chart above. The place farthest to the right that contains a digit tells you the denominator of the fraction. In this case, it is thousandths. The number written in the place-value chart tells you the numerator of the fraction. Here, it is 25.

$$0.025 = \frac{25}{1,000}$$

Example 2: Write $\frac{11}{1,000}$ as a decimal.

Ones	.	Tenths	Hundredths	Thousandths
	.			

You can also use a place-value chart to write a fraction as a decimal. The denominator tells you the last decimal place in your number. Here, it is thousandths. The numerator tells you the decimal itself. Write a 1 in the hundredths place and a 1 in the thousandths place. Fill in the other places with a 0.

$$\frac{11}{1,000} = 0.011$$

Write each decimal as a fraction.

1. 0.002 $\frac{2}{1,000}$

2. 0.037 $\frac{37}{1,000}$

3. 0.099 $\frac{99}{1,000}$

Write each fraction as a decimal.

4. $\frac{5}{1,000}$ **0.005**

5. $\frac{76}{1,000}$ **0.076**

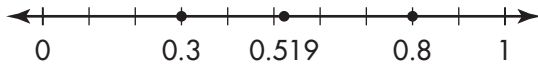
6. $\frac{40}{1,000}$ **0.040**

7. **Explain It** Matt reasoned that he can write $\frac{9}{1,000}$ as 0.9. Is he correct? Explain your answer.

No: $\frac{9}{1,000}$ is read as “nine thousandths,” which is the decimal 0.009. 0.9 is read as “nine tenths,” which is the fraction $\frac{9}{10}$.

Fractions and Decimals on the Number Line

Show 0.8, $\frac{6}{20}$, and 0.519 on the same number line.



Step 1: Starting at 0, count 8 tenths to the right. This point is 0.8 or $\frac{8}{10}$.

Step 2: Change $\frac{6}{20}$ to a decimal. $\frac{6}{20}$ can be thought of as $6 \div 20$.
 $6 \div 20 = 0.3$

Starting at 0, count 3 tenths to the right. This point is $\frac{6}{20}$ or 0.3.

Step 3: Estimate the location of 0.519.

You know that 0.5 is the same as 0.500. You also know that 0.6 is the same as 0.600. So, 0.519 is between 0.5 and 0.6.

You know that $0.519 < 0.550$. So, 0.519 is between 0.500 and 0.550 and closer to 0.500 than to 0.550.

Show each set of numbers on the same number line.

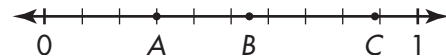


Name the fraction and decimal for each point.

4. Point A 0.3 or $\frac{3}{10}$

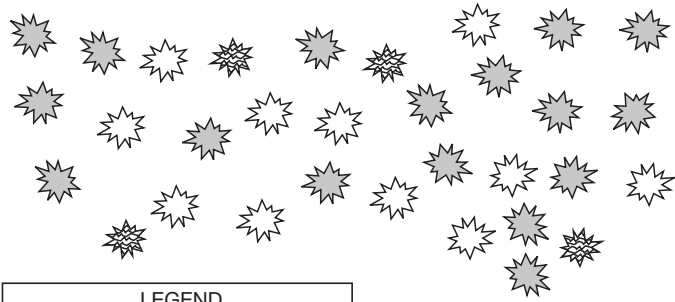
5. Point B 0.55 or $\frac{55}{100}$




6. Point C 0.89 or $\frac{89}{100}$







































Problem Solving: Writing to Explain

An environmental scientist is studying an old apple orchard. The orchard is shown on the right. Some of the trees are infected with mold. Other trees are infested with beetles. Some trees are normal.



LEGEND	
	normal apple tree
	apple tree infected with mold
	apple tree infested with beetles

The scientist knows that pictures and symbols can be used to write a good math explanation. So she decides to organize her findings in the chart on the right.

Use this chart to estimate the fractional part of the orchard that is infected with mold, using a benchmark fraction that is close to the actual amount.

A little more than half the grid is covered by trees that are infected with mold.

Use this chart to estimate the fractional part of the orchard that is infested with beetles. Explain how you decided.

One of the eight columns is filled

with trees infested with beetles.

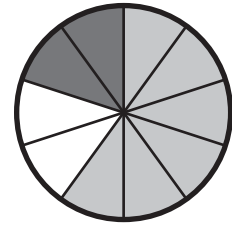
This column represents $\frac{1}{8}$ of the orchard.

Adding and Subtracting Fractions with Like Denominators

Aisha cut her birthday cake into 10 slices. She and her friends ate 6 slices. Her parents ate 2 slices. What fraction of cake was eaten?

$$\frac{6}{10} = \text{amount of cake Aisha and her friends ate.}$$

$$\frac{2}{10} = \text{amount of cake Aisha's parents ate.}$$



When two fractions have the same denominator, their sum or difference will also have the same denominator.

Step 1. Add the numerators.

$$6 + 2 = 8$$

Step 2. Write the sum over the denominator.

$$\frac{8}{10}$$

Step 3. Simplify the fraction if possible. $\frac{8}{10} = \frac{4}{5}$ So, $\frac{4}{5}$ of the cake was eaten.

You can do the same when subtracting fractions with like denominators.

How much more cake did Aisha and her friends eat than Aisha's parents?

$$\frac{6}{10} - \frac{2}{10} =$$

Step 1. Subtract the numerators.

$$6 - 2 = 4$$

Step 2. Place the difference over the denominator and simplify if possible.

$$\frac{4}{10} = \frac{2}{5}$$

Find the sum or difference. Simplify your answer.

$$\begin{array}{r} 1. \quad \frac{3}{8} \\ + \frac{3}{8} \\ \hline \frac{3}{4} \end{array}$$

$$\begin{array}{r} 2. \quad \frac{11}{12} \\ - \frac{5}{12} \\ \hline \frac{1}{2} \end{array}$$

$$\begin{array}{r} 3. \quad \frac{9}{10} \\ + \frac{4}{10} \\ \hline 1\frac{3}{10} \end{array}$$

$$\begin{array}{r} 4. \quad \frac{7}{9} \\ - \frac{2}{9} \\ \hline \frac{5}{9} \end{array}$$

5. Joachim has 15 medals he won at swim meets. Of the medals, 6 are first place, 4 are second place, 2 are third place, and 3 are fourth place. What fraction of Joachim's medals are first or second place?

$$\frac{2}{3}$$

6. **Algebra** Mr. Lucero's cornbread recipe calls for 3 cups of cornmeal. He put in $1\frac{2}{3}$ cups from one open bag and $\frac{2}{3}$ cup from another open bag. How many cups of cornmeal does Mr. Lucero need to add when he opens a third bag?

$$\frac{2}{3} \text{ cups}$$

Common Multiples and Least Common Multiple

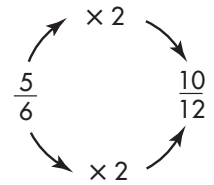
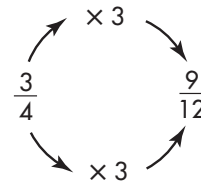
To find the least common denominator (LCD) of two fractions with different denominators, you need to find the least common multiple (LCM).

Find the least common multiple and LCD of $\frac{3}{4}$ and $\frac{5}{6}$.

Step 1. Find the least common multiple of 4 and 6.

4 XXXX	6 XXXXXX	Are there the same number of X's in each column? Since the 1st column has fewer, add another set of 4 X's.
4 XXXX XXXX	6 XXXXXX	Are there the same number of X's in each column? Since the 2nd column has fewer, add another set of 6 X's.
4 XXXX XXXX	6 XXXXXX XXXXXX	Are there the same number of X's in each column? Since the 1st column has fewer, add another set of 4 X's.
4 XXXX XXXX XXXX	6 XXXXXX XXXXXX	Are there the same number of X's in each column? Yes. Since the columns are equal, the number of X's is the LCM. The LCM = 12.

Step 2. Make the least common multiple the least common denominator of both fractions.



Find the least common multiple of each number pair.

1. 2 and 3 **6** 2. 6 and 9 **18** 3. 5 and 6 **30** 4. 8 and 3 **24**

5. **Reasonableness** Can the LCD of $\frac{4}{9}$ and $\frac{13}{17}$ be less than 17? Explain.

No. The LCD for two fractions must be the LCM of the denominators. The LCM of two numbers cannot be less than either of the numbers.

Adding Fractions with Unlike Denominators

Danisha ate $\frac{2}{3}$ cup of yogurt at breakfast. She ate $\frac{1}{4}$ cup of yogurt at lunch. How much yogurt did she eat today?

You can add fractions with unlike denominators.

Step 1: Find the least common denominator of the two fractions.

multiples of 3: 3, 6, 9, **12**, 15

multiples of 4: 4, 8, **12**, 16, 20

$$\frac{2}{3} = \frac{8}{12} \text{ and } \frac{1}{4} = \frac{3}{12}$$

Step 2: Once you have equivalent fractions with the same denominator, add the numerators.

$$8 + 3 = 11$$

Step 3: Place the sum over the common denominator and simplify your fraction if possible.

Danisha ate $\frac{11}{12}$ cup of yogurt today.

For questions 1 through 5, find the sum. Simplify if possible.

$$\begin{array}{r} 1. \quad \frac{4}{5} \\ + \frac{1}{2} \\ \hline 1\frac{3}{10} \end{array}$$

$$\begin{array}{r} 2. \quad \frac{8}{9} \\ + \frac{5}{6} \\ \hline 1\frac{13}{18} \end{array}$$

$$\begin{array}{r} 3. \quad \frac{4}{9} \\ + \frac{3}{4} \\ \hline 1\frac{7}{36} \end{array}$$

$$4. \quad \frac{1}{2} + \frac{1}{6} + \frac{3}{4} = \underline{1\frac{5}{12}} \quad 5. \quad \frac{2}{3} + \frac{1}{9} + \frac{5}{6} = \underline{1\frac{11}{18}}$$

6. Kevin and some friends ordered several pizzas and cut them into different numbers of slices. Kevin ate $\frac{1}{6}$ of one pizza, $\frac{1}{4}$ of another, $\frac{5}{12}$ of another, and $\frac{1}{3}$ of another. Did Kevin eat the equivalent of a whole pizza?

yes: $1\frac{1}{6}$ pizzas

7. Cathy spent $\frac{3}{10}$ of an hour on her math homework, $\frac{2}{5}$ of an hour on her science homework, and $\frac{3}{4}$ of an hour on her reading homework. How long did Cathy work on homework?

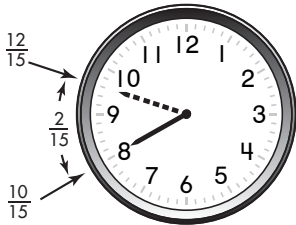
$1\frac{9}{20}$ hours

Subtracting Fractions with Unlike Denominators

You can subtract fractions with unlike denominators by using the least common multiple (LCM) and the least common denominator (LCD).

Beth wants to exercise for $\frac{4}{5}$ hour. So far, she has exercised for $\frac{2}{3}$ hour. What fraction of an hour does she have left to go?

Step 1: Find the LCM of 5 and 3.

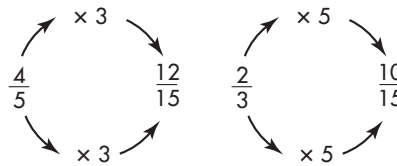


multiples of 5: 5, 10, **15**, 20

multiples of 3: 3, 6, 9, 12, **15**

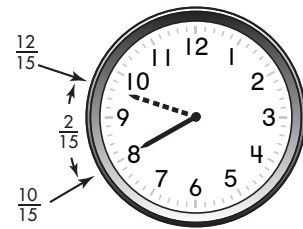
Since 15 is the LCM, it is also your LCD.

Step 2: Using your LCD, write the equivalent fractions.



Step 3: Subtract the numerators.

Place the difference over the LCD. Simplify if possible.



$$12 - 10 = 2 \quad \frac{2}{15} \text{ hour left}$$

In **1** through **5**, find each difference. Simplify if possible.

$$\begin{array}{r} 1. \quad \frac{3}{4} \\ - \frac{2}{5} \\ \hline \frac{7}{20} \end{array}$$

$$\begin{array}{r} 2. \quad \frac{7}{10} \\ - \frac{1}{5} \\ \hline \frac{1}{2} \end{array}$$

$$3. \quad \frac{7}{12} - \frac{1}{4} = \underline{\frac{1}{3}}$$

$$4. \quad \frac{5}{6} - \frac{3}{8} = \underline{\frac{11}{24}}$$

$$5. \quad \frac{23}{24} - \frac{7}{8} = \underline{\frac{1}{12}}$$

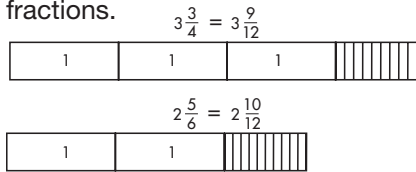
6. Natasha had $\frac{7}{8}$ gallon of paint. Her brother Ivan took $\frac{1}{4}$ gallon to paint his model boat. Natasha needs at least $\frac{1}{2}$ gallon to paint her bookshelf. Did Ivan leave her enough paint?

Yes. She has $\frac{5}{8}$ gallon left.

Adding Mixed Numbers

Randy talks on the telephone for $2\frac{5}{6}$ hours, and then surfs the Internet for $3\frac{3}{4}$ hours. How many hours did he spend on the two activities?

Step 1. Write equivalent fractions with the least common denominator. You can use fraction strips to show the equivalent fractions.



Step 2. Add the fractions. Then add the whole numbers.

$$\frac{9}{12} + \frac{10}{12} = \frac{19}{12} \qquad 3 + 2 = 5$$

$$\text{So, } 3\frac{3}{4} + 2\frac{5}{6} = 5\frac{19}{12}$$

Step 3. Simplify the sum if possible.

$$5\frac{19}{12} = 6\frac{7}{12} \text{ hours}$$

In **1** through **6**, find each sum. Simplify if possible.

$$1. \quad \begin{array}{r} 2\frac{5}{6} \\ + 3\frac{1}{4} \\ \hline \end{array}$$

$$6\frac{1}{12}$$

$$4. \quad 10\frac{1}{3} + \frac{7}{9} = \underline{11\frac{1}{9}}$$

$$2. \quad \begin{array}{r} 1\frac{3}{8} \\ + 6\frac{3}{4} \\ \hline \end{array}$$

$$8\frac{1}{8}$$

$$5. \quad 3\frac{1}{4} + 6\frac{2}{3} = \underline{9\frac{11}{12}}$$

$$3. \quad \begin{array}{r} 5\frac{2}{5} \\ + 4\frac{1}{2} \\ \hline \end{array}$$

$$9\frac{9}{10}$$

$$6. \quad 1\frac{5}{7} + 3\frac{1}{2} = \underline{5\frac{3}{14}}$$

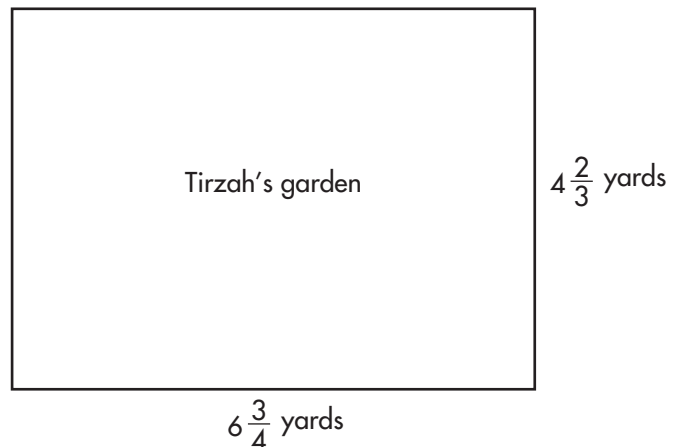
- 7. Geometry** Tirzah wants to put a fence around her garden. She has 22 yards of fence material. Does she have enough to go all the way around the garden?

No. She needs

$22\frac{5}{6}$ yards to

go around the

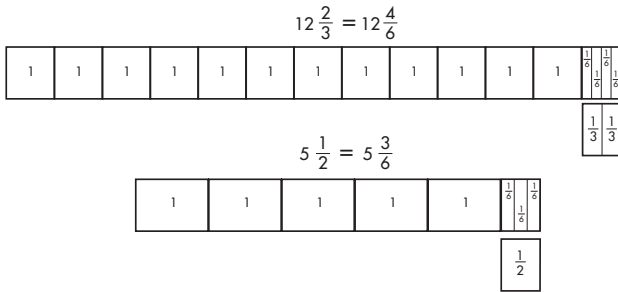
garden.



Subtracting Mixed Numbers

The Plainville Zoo has had elephants for $12\frac{2}{3}$ years. The zoo has had zebras for $5\frac{1}{2}$ years. How many years longer has the zoo had elephants?

Step 1. Write equivalent fractions with the least common denominator. You can use fraction strips.



Step 2. Subtract the fractions. Then subtract the whole numbers. Simplify the difference if possible.

$$\frac{4}{6} - \frac{3}{6} = \frac{1}{6} \qquad 12 - 5 = 7$$

So, $12\frac{2}{3} - 5\frac{1}{2} = 7\frac{1}{6}$ years.

Tip: Sometimes you may have to rename a fraction so you can subtract.

$$\begin{array}{r}
 6 \longrightarrow \text{rename} \longrightarrow 5\frac{8}{8} \\
 - 2\frac{3}{8} \\
 \hline
 3\frac{5}{8}
 \end{array}$$

For questions 1 through 4, find the difference. Simplify if possible. Remember: You may have to rename a fraction in order to subtract.

1.
$$\begin{array}{r} 4\frac{3}{5} \\ - 2\frac{1}{3} \\ \hline 2\frac{4}{15} \end{array}$$

2.
$$\begin{array}{r} 5\frac{6}{7} \\ - 1\frac{1}{2} \\ \hline 4\frac{5}{14} \end{array}$$

3.
$$\begin{array}{r} 3 \\ - 1\frac{3}{4} \\ \hline 1\frac{1}{4} \end{array}$$

4.
$$\begin{array}{r} 6\frac{5}{6} \\ - 5\frac{1}{2} \\ \hline 1\frac{1}{3} \end{array}$$

5. **Number Sense** Rename the number 7 so that you would be able to find the difference of $7 - 3\frac{5}{12}$.

$$6\frac{12}{12}$$

6. Robyn ran $5\frac{3}{4}$ miles last week. She ran $4\frac{1}{10}$ miles this week. How many more miles did she run last week?

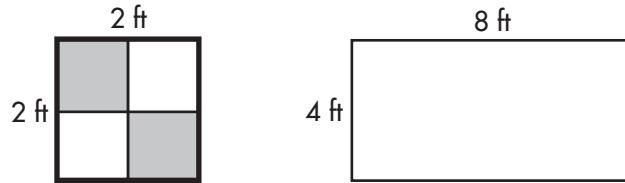
$$1\frac{13}{20}$$

Problem Solving: Try, Check, and Revise

Can you use similar square tiles to cover a floor without cutting the tiles?

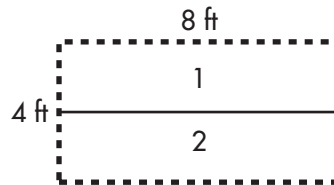
Try

Look at the square tile and the floor.



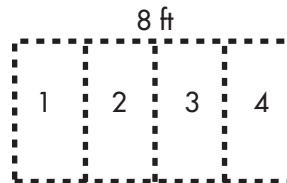
Ask yourself:

1. Can I divide the floor into a whole number of rectangles that are 2 feet wide?



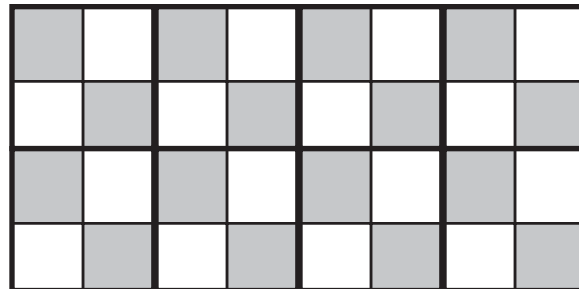
AND

2. Can I divide the floor into a whole number of rectangles that are 2 feet long?



If the answers are both yes, you can use the tiles to cover the floor without cutting them.

Check.



Answer the following questions remembering that you cannot cut the tiles.

1. Can you completely cover a 4 ft by 8 ft area with 3 ft by 3 ft tiles?
2. Can you completely cover a 3 ft by 6 ft area with 3 ft by 3 ft tiles?
3. Can you completely cover a 3 ft by 6 ft area with 2 ft by 2 ft tiles?

No

Yes

No

Multiplying Fractions and Whole Numbers

You can find the product of a fraction and a whole number.

Tran needs $\frac{2}{3}$ yard of fabric to sew a pair of shorts. How many yards of fabric will Tran need to sew 6 pairs of shorts?

Step 1: Multiply the numerator by the whole number.

$$2 \times 6 = 12$$

Step 2: Place the product over the denominator. Simplify if possible.

$$\frac{12}{3} = 4 \text{ yards of fabric}$$

Remember: In word problems, “of” means “multiply.”

Example: $\frac{3}{5}$ of 15 = $\frac{3}{5} \times 15$

In **1** through **4**, find each product. Simplify if possible.

1. $\frac{1}{3} \times 60 = \underline{20}$

2. $\frac{3}{4}$ of 32 = 24

3. $\frac{7}{8} \times 40 = \underline{35}$

4. $\frac{2}{7}$ of 35 = 10

For Exercises **5** through **7**, use the table to the right.

5. What is $\frac{1}{7}$ the speed of a cheetah? 10 mi/h

6. What is $\frac{1}{5}$ the speed of a cat? 6 mi/h

7. What is $\frac{1}{5}$ the speed of a jackal? 7 mi/h

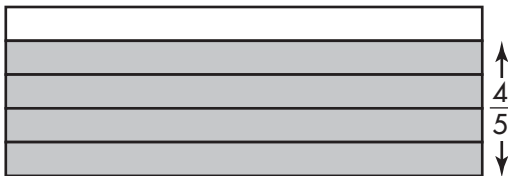
Animal	Speed (in mi/h)
Cat	30
Cheetah	70
Jackal	35

Multiplying Two Fractions

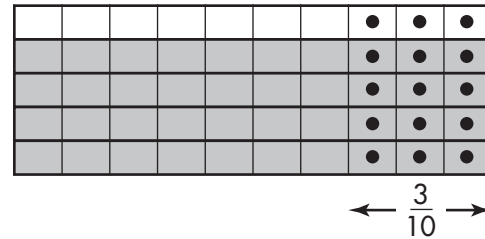
Musa and Karen are riding a bike path that is $\frac{4}{5}$ mile long. Karen's bike got a flat tire $\frac{3}{10}$ of the way down the path and she had to stop. How many miles did Karen ride?

You can find the product of two fractions by drawing a diagram.

Step 1. Draw a diagram using shading to represent $\frac{4}{5}$.



Step 2. Draw lines vertically using dots to represent $\frac{3}{10}$.



Step 3. Count the parts of the diagram that are shaded and dotted. This is the product numerator.

12

Step 4. Count the total number of parts of the diagram. This is the product denominator.

50

Step 5. Simplify if possible.

$$\frac{12}{50} = \frac{6}{25}$$

Another way to find the product:

Step 1. Multiply the numerators: $4 \times 3 = 12$.

Step 2. Multiply the denominators: $5 \times 10 = 50$.

Step 3. Simplify if possible: $\frac{12}{50} = \frac{6}{25}$.

In **1** through **6**, find the product. Simplify if possible.

1. $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

2. $\frac{5}{8} \times \frac{1}{4} = \frac{5}{32}$

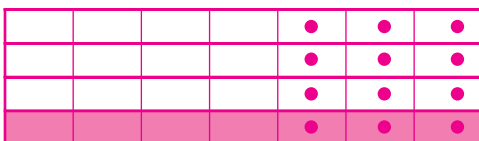
3. $\frac{5}{6} \times \frac{3}{10} = \frac{1}{4}$

4. $\frac{1}{2} \times 6 = 3$

5. $14 \times \frac{3}{7} = 6$

6. $\frac{3}{5} \times \frac{1}{2} \times \frac{6}{7} = \frac{9}{35}$

7. Draw a picture Using a diagram, show $\frac{3}{7} \times \frac{1}{4}$.



Multiplying Mixed Numbers

You can find the product of two mixed numbers.

Millwood City is constructing a new highway through town. The construction crew can complete $5\frac{3}{5}$ miles of road each month. How many miles will they complete in $6\frac{1}{2}$ months?

Step 1. Round the mixed numbers to whole numbers so you can make an estimate.

$$5\frac{3}{5} \times 6\frac{1}{2}$$

$$6 \times 7 = 42$$

So, they can complete about 42 miles.

Step 2. Write the mixed numbers as improper fractions.

$$5\frac{3}{5} \times 6\frac{1}{2} = \frac{28}{5} \times \frac{13}{2}$$

Step 3. Multiply the numerators and the denominators. Simplify the product if possible. Remember to look for common factors.

$$\frac{\overset{14}{\cancel{28}}}{5} \times \frac{13}{\underset{1}{\cancel{2}}} = \frac{182}{5} = 36\frac{2}{5}$$

Step 4. Compare your product to your estimate to check for reasonableness.

$36\frac{2}{5}$ is close to 42, so this answer is reasonable.

The construction crew will complete $36\frac{2}{5}$ miles of highway in $6\frac{1}{2}$ months.

For **1** through **6**, estimate a product. Then solve for each actual product. Simplify if possible.

1. $1\frac{3}{4} \times 2\frac{1}{2} = \underline{4\frac{3}{8}}$	2. $1\frac{1}{5} \times 1\frac{2}{3} = \underline{2}$	3. $2 \times 2\frac{1}{4} = \underline{4\frac{1}{2}}$
4. $1\frac{2}{5} \times 2\frac{1}{4} = \underline{3\frac{3}{20}}$	5. $2\frac{1}{2} \times 10 = \underline{25}$	6. $1\frac{2}{3} \times \frac{1}{5} = \underline{\frac{1}{3}}$

7. Using the example above, the new highway will be a total of 54 miles long. Will the highway be finished in 8 months?

No. They will have completed only $44\frac{4}{5}$ miles.

8. **Reasonableness** Sayed gave an answer of $6\frac{6}{7}$ for the problem $4\frac{2}{7} \times 1\frac{3}{5}$. Using estimates, is this a reasonable answer?

Yes: $4 \times 2 = 8$, and 8 is close to $6\frac{6}{7}$.

Relating Division to Multiplication of Fractions

How can you divide by a fraction?

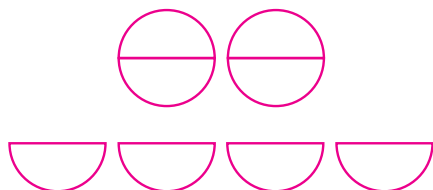
Dividing a whole number by a fraction

$2 \div \frac{1}{3}$	Think: How can I divide two into one-thirds?
<ol style="list-style-type: none"> 1. Two is the sum of one plus one. 2. Each one is the sum of three one-thirds. 3. Count the number of one-thirds. <p>Check To divide a whole number by a fraction, multiply the whole number by the reciprocal of the fraction.</p>	$2 = 1 + 1$ <div style="text-align: center;"> </div> 6 $2 \div \frac{1}{3} = 2 \times \frac{3}{1} = \frac{2}{1} \times \frac{3}{1} = \frac{6}{1} = 6$

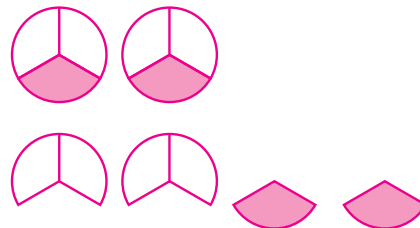
$3 \div \frac{3}{4}$	Think: How can I divide three into three-fourths?
<ol style="list-style-type: none"> 1. Three is the sum of one plus one plus one. 2. Each one is the sum of one three-fourths and one one-fourth. 3. Count the number of three-fourths. <p>Check Multiply the whole number by the reciprocal of the fraction.</p>	$3 = 1 + 1 + 1$ <div style="text-align: center;"> </div> $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ 4 $3 \div \frac{3}{4} = 3 \times \frac{4}{3} = \frac{3}{1} \times \frac{4}{3} = \frac{12}{3} = 4$

Draw a picture that shows each division and write the answer.

1. $2 \div \frac{1}{2}$ **4**



2. $2 \div \frac{2}{3}$ **3**



Problem Solving: Draw a Picture and Write an Equation

Travis earned 3 stickers for each song he played in his piano lesson. He received a total of 24 stickers. How many songs did he play?

You can solve a problem like this by drawing a picture and writing an equation.

Step 1. Write out what you already know.
Travis earned 3 stickers for each song he played. Travis had 24 stickers at the end of the lesson.

Step 2. Draw a picture to show what you know.

Step 3. Write out what you are trying to find.
How many songs did Travis play?

Step 4. Write an equation from your drawing.
Since you are dividing Travis's total stickers into groups of 3 (stickers earned per song), this is a division problem.

$$24 \div 3 = s \quad s = \text{number of songs Travis played}$$

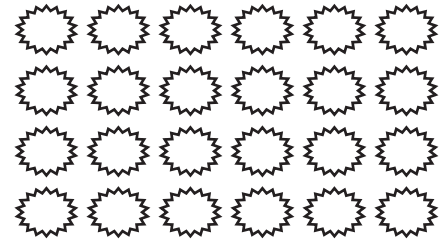
Step 5. Solve the equation.

$$24 \div 3 = 8 \quad s = 8$$

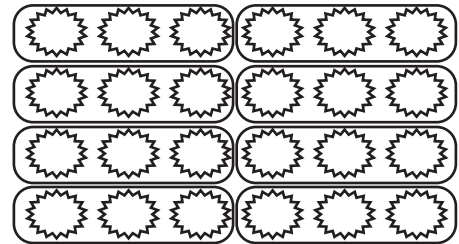
So, Travis played 8 songs during his lesson.

Step 6. Check your answer by working backward.

$$8 \times 3 = 24: \text{ your answer is correct.}$$



Travis's total stickers

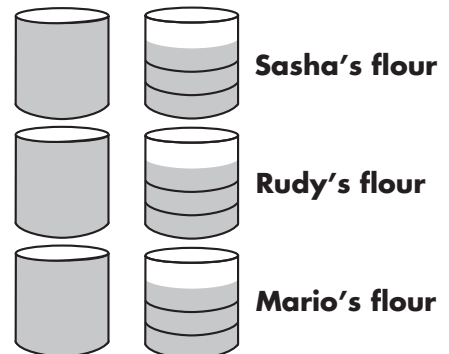


**groups of 3 stickers
Travis earned per song**

Draw a picture, write an equation, and solve.

- Sasha, Rudy, and Mario each have $1\frac{3}{4}$ cups of flour. Can they make a recipe for bread that needs 5 cups of flour?

$3 \times 1\frac{3}{4} = c; c = 5\frac{1}{4}$ cups of flour; yes

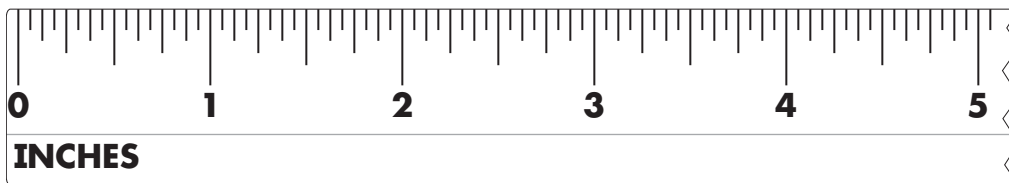


Using Customary Units of Length

One customary unit of length is the inch. On most rulers it is divided into fractions to help you measure more precisely.

How to measure length

Measure the length of the pen to the nearest inch, nearest $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, and $\frac{1}{8}$ inch.



Step 1: Measure to the nearest inch.

The length is more than 4 in., but less than 5 in.

It is closer to 5 in.

The pen's length is closest to 5 in.

Step 2: Measure to the nearest $\frac{1}{2}$ inch.

The length is more than $4\frac{1}{2}$ in., but less than $4\frac{2}{2}$ (5) in.

It is closer to $4\frac{1}{2}$ in.

The pen's length is closest to $4\frac{1}{2}$ in.

Step 3: Measure to the nearest $\frac{1}{4}$ inch.

The length is more than $4\frac{2}{4}$ ($4\frac{1}{2}$) in., but less than $4\frac{3}{4}$ in.

It is closer to $4\frac{1}{2}$ in.

The pen's length is closest to $4\frac{1}{2}$ ($4\frac{2}{4}$) in.

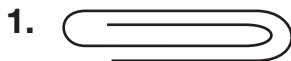
Step 4: Measure to the nearest $\frac{1}{8}$ inch.

The length is more than $4\frac{4}{8}$ ($4\frac{1}{2}$) in., but less than $4\frac{5}{8}$ in.

It is closer to $4\frac{5}{8}$ in.

The pen's length is closest to $4\frac{5}{8}$ in.

Measure to the nearest inch, $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, and $\frac{1}{8}$ inch.



1 in., 1 in., $1\frac{1}{4}$ in., and $1\frac{1}{4}$ ($1\frac{2}{8}$) in.

Use your ruler to draw a line segment of each length.

2. $\frac{7}{8}$ inch _____

3. $2\frac{1}{4}$ inch _____

Using Metric Units of Length

Measurements in the metric system are based on the meter.

Example A

Which metric unit of length would be most appropriate to measure the length of a bumblebee?

A bumblebee is very small, so the millimeter is the most appropriate unit of length.

Example B

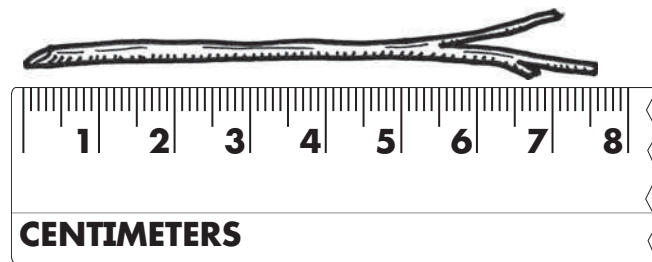
Write mm, cm, m, or km to complete the following sentence.

A chair is about 1 _____ tall and a child's hand is about 8 _____ wide.

A chair is about 1 m tall, and a child's hand is about 8 cm wide.

Example C

What is the length of the twig to the nearest centimeter and to the nearest millimeter?



Step 1: Measure to the closest centimeter.

The length of the twig is more than 7 cm but less than 8 cm.

The twig's length is closest to 8 cm.

Step 2: Measure to the closest millimeter.

The length of the twig is more than 70 mm but less than 80 mm.

The length of the twig is more than 75 mm but less than 76 mm.

The twig's length is closest to 76 mm.

Which unit would be most appropriate for each measurement?

Write mm, cm, m, or km.

1. distance between two cities km 2. width of a room m

3. Measure the line segment below to the nearest centimeter and to the nearest millimeter.

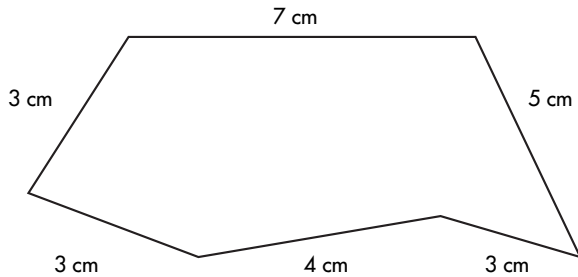
3 cm; 31 mm

Perimeter

The perimeter is the distance around the outside of a polygon.
You can find the perimeter in two different ways.

Add the lengths of the sides:

Find the perimeter of the figure.



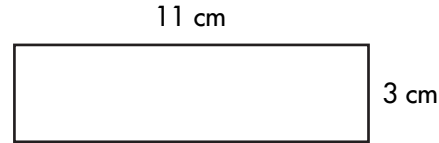
To find the perimeter, add up the sides.

$$3 + 3 + 7 + 5 + 3 + 4 = 25$$

So, the perimeter of the figure is 25 cm.

Use a formula:

Find the perimeter of the rectangle.



$$\text{Perimeter} = (2 \times \text{length}) + (2 \times \text{width})$$

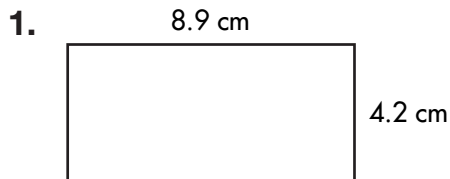
$$P = (2 \times l) + (2 \times w)$$

$$P = (2 \times 11) + (2 \times 3)$$

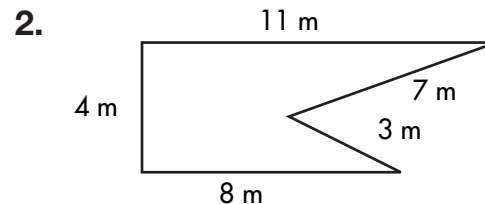
$$P = 22 + 6 = 28 \text{ cm}$$

So, the perimeter of the rectangle is 28 cm.

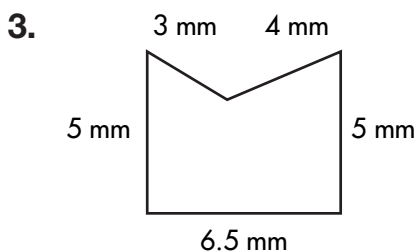
Find the perimeter of each figure.



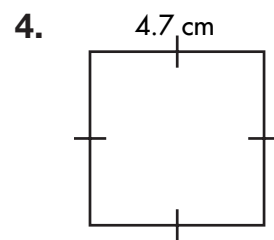
26.2 cm



33 m



23.5 mm



18.8 cm

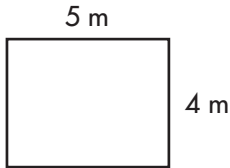
5. **Number Sense** The perimeter of a square is 24 in.
What is the length of each side?

6 in.

Area of Squares and Rectangles

You can use formulas to find the areas of rectangles and squares.

Find the area of the rectangle.



Use this formula for rectangles:

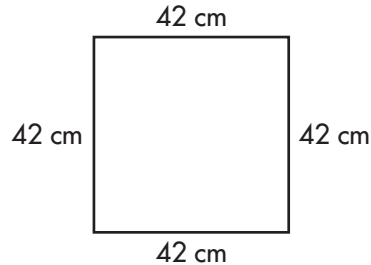
$$\text{Area} = \text{length} \times \text{width}$$

$$A = l \times w$$

$$A = 5 \text{ m} \times 4 \text{ m}$$

$$A = 20 \text{ square meters} = 20 \text{ m}^2$$

Find the area of the square.



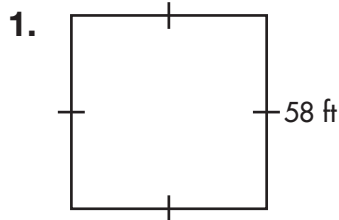
Use this formula for squares:

$$\text{Area} = \text{side} \times \text{side}$$

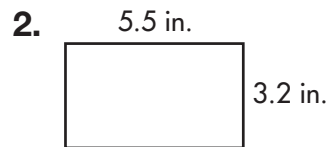
$$A = 42 \text{ cm} \times 42 \text{ cm}$$

$$A = 1,764 \text{ square centimeters} \\ = 1,764 \text{ cm}^2$$

Find the area of each figure.



$$\underline{3,364 \text{ ft}^2}$$



$$\underline{17.6 \text{ in}^2}$$

3. a square with a side of 12.4 m

$$\underline{153.76 \text{ m}^2}$$

4. a rectangle with a length of 9.7 cm and a width of 7.3 cm

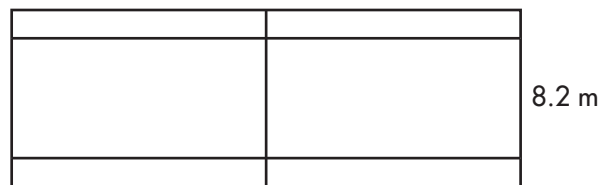
$$\underline{70.81 \text{ cm}^2}$$

5. **Number Sense** If the area of a square is 81 in^2 , what is the length of one side?

$$\underline{9 \text{ in.}}$$

6. What is the area of the tennis court?

$$\underline{195.16 \text{ m}^2}$$

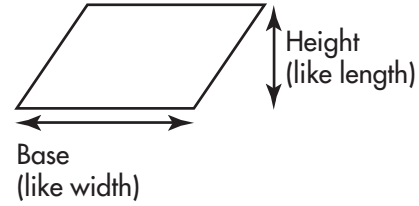


23.8 m

8.2 m

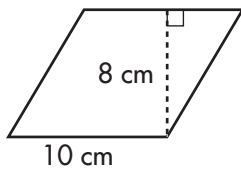
Area of Parallelograms

The formula used to find the area of a parallelogram is similar to the one you used to find the area of a rectangle. Instead of using $\text{length} \times \text{width}$, use $\text{base} \times \text{height}$.



How to find the area of a parallelogram:

Find the area of the parallelogram.



Area = base \times height

$$A = b \times h$$

$$A = 10 \text{ cm} \times 8 \text{ cm}$$

$$A = 80 \text{ cm}^2$$

How to find the missing measurement of a parallelogram:

$$\text{Area} = 55 \text{ cm}^2 \quad \text{base} = 11 \text{ cm} \\ \text{height} = ? \text{ cm}$$

Remember, $\text{area} = \text{base} \times \text{height}$.

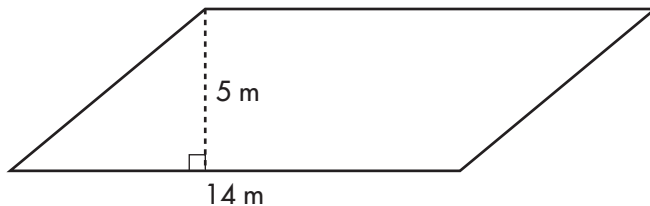
$$55 \text{ cm}^2 = 11 \text{ cm} \times ? \text{ cm}$$

$$55 \text{ cm}^2 = 11 \text{ cm} \times 5 \text{ cm}$$

So, the height of the parallelogram is 5 cm.

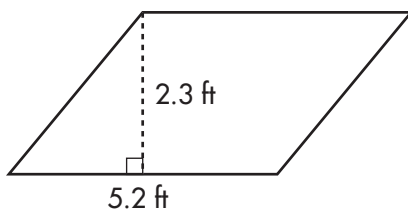
Find the area of each parallelogram.

1.



$$\underline{70 \text{ m}^2}$$

2.



$$\underline{11.96 \text{ ft}^2}$$

Find the missing measurement for each parallelogram.

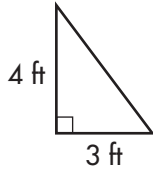
3. $A = 72 \text{ in}^2$, $b = 9 \text{ in.}$, $h = \underline{8 \text{ in.}}$

4. $A = 238 \text{ ft}^2$, $b = \underline{17 \text{ ft}}$, $h = 14 \text{ ft}$

Area of Triangles

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

How to find the area of a triangle:



$$A = \frac{1}{2} \times b \times h$$

$$A = \frac{1}{2} \times 3 \text{ ft} \times 4 \text{ ft}$$

$$A = \frac{1}{2} \times 12 \text{ ft}^2$$

$$A = 6 \text{ ft}^2$$

How to find the missing measurement of a triangle:

Area = 100 cm^2 base = 40 cm
height = ? cm

Remember, Area = $\frac{1}{2} \times \text{base} \times \text{height}$.

$$100 \text{ cm}^2 = \frac{1}{2} \times 40 \text{ cm} \times ? \text{ cm}$$

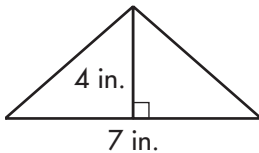
$$100 \text{ cm}^2 = 20 \text{ cm} \times ? \text{ cm}$$

$$100 \text{ cm}^2 = 20 \text{ cm} \times 5 \text{ cm}$$

So, the height of the triangle is 5 cm.

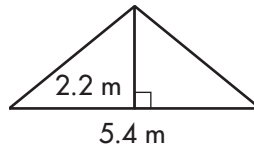
Find the area of each triangle.

1.



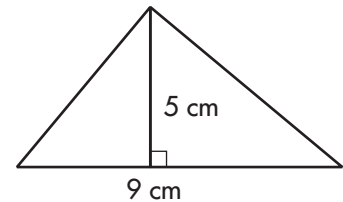
$$\underline{14 \text{ in}^2}$$

2.



$$\underline{5.94 \text{ m}^2}$$

3.



$$\underline{22.5 \text{ cm}^2}$$

Find the missing measurement for each triangle.

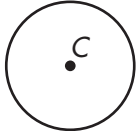
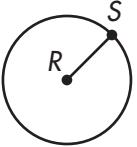
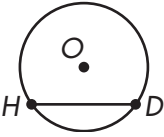
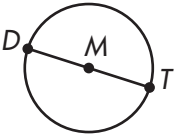
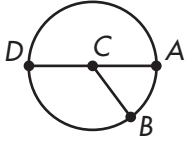
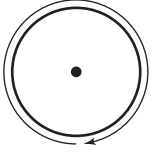
4. $A = 16 \text{ in}^2$, $b = 8 \text{ in.}$, $h = \underline{4 \text{ in.}}$

5. $A = 20 \text{ m}^2$, $b = \underline{10 \text{ m}}$, $h = 4 \text{ m}$

6. $A = \underline{24.32 \text{ ft}^2}$, $b = 6.4 \text{ ft}$, $h = 7.6 \text{ ft}$

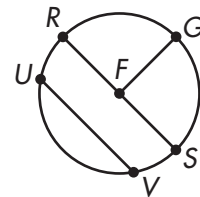
7. $A = 14 \text{ cm}^2$, $b = 2 \text{ cm}$, $h = \underline{14 \text{ cm}}$

Circles and Circumference

<p>All the points on a circle are the same distance from the center. The center of circle C is point C.</p>		<p>A radius goes from the center to any point on the circle. \overline{RS} is a radius of circle R.</p>	
<p>A chord has both endpoints on a circle. \overline{HD} is a chord of circle O.</p>		<p>A diameter is a chord that passes through the center of a circle. \overline{DT} is a diameter of circle M.</p>	
<p>A central angle is an angle whose vertex is the center. $\angle ACB$ is a central angle.</p>		<p>The circumference is the distance around a circle. The circumference equals the circle's diameter multiplied by π (3.1416). $C = \pi D$</p>	

Use the terms at the top of the page to identify each figure in circle F .

1. point F **Center** _____
2. $\angle GFS$ **Central angle** _____
3. \overline{RS} **Diameter** _____
4. \overline{UV} **Chord** _____
5. \overline{GF} **Radius** _____



6. **Estimation** If a circle has a circumference of three feet, estimate its diameter to the nearest foot.

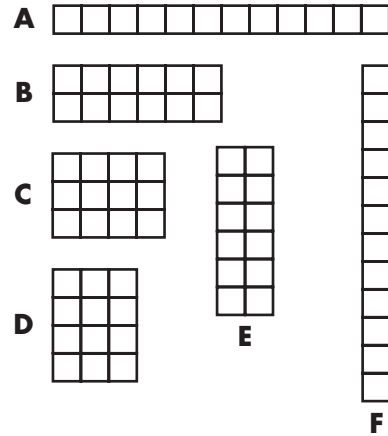
1 foot

Problem Solving: Draw a Picture and Make an Organized List

In a garden, a landscaper has to make a small patio using twelve 1-foot-square tiles. The patio must have the smallest possible perimeter. What should be the dimensions of the patio?

Draw a picture.

1. Construct pictures of rectangles using the twelve tiles.
2. List the perimeter of each figure in a table.
3. Count tile edges or calculate to find the perimeter.



Solve.

Both the 3×4 -tile and the 4×3 -tile patios have the same perimeter, 14 feet. This value is the smallest perimeter in the table. The landscaper can use either design.

Rectangle	Length in feet	Width in feet	Perimeter in feet
A	12	1	26
B	6	2	16
C	4	3	14
D	3	4	14
E	2	6	16
F	1	12	26

The landscaper must build another patio with the smallest perimeter possible using nine 1-foot-square tiles. What should be the dimensions?

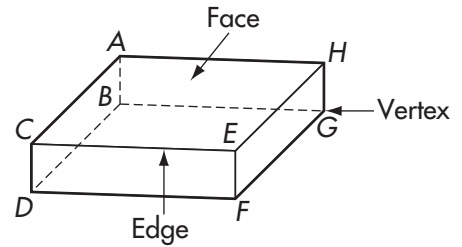
3 feet \times 3 feet

Solids

The solid's vertices are: $A, B, C, D, E, F, G,$ and $H.$

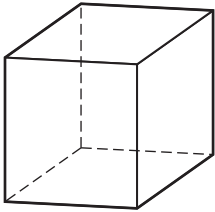
The solid's edges are: $\overline{AC}, \overline{AB}, \overline{CD}, \overline{DB}, \overline{AH}, \overline{BG},$
 $\overline{HG}, \overline{HE}, \overline{GF}, \overline{EF}, \overline{CE},$ and $\overline{DF}.$

The solid's faces are: $ACEH, BDFG, ABCD,$
 $EFGH, CDEF,$ and $ABHG.$

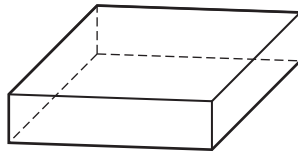


Here are some common solid figures:

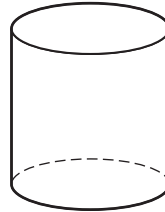
Cube



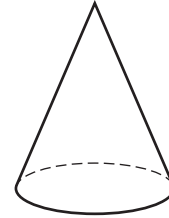
Rectangular Prism



Cylinder



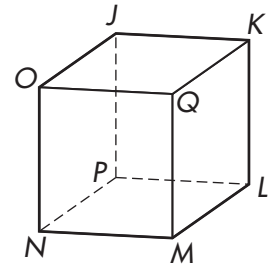
Cone



Use the solid at the right for 1 through 3.

1. Name the edges.

$\overline{OJ}, \overline{JK}, \overline{KQ}, \overline{OQ}, \overline{ON}, \overline{NM},$
 $\overline{MQ}, \overline{KL}, \overline{LM}, \overline{JP}, \overline{NP}, \overline{LP}$



2. Name the faces.

$NMQO, KLMQ, JKLP, JONP, JKQO, LMNP$

3. Name the vertices.

J, K, L, M, N, O, P, Q

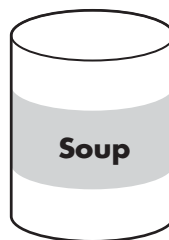
What solid figure does each object resemble?

- 4.



Rectangular prism

- 5.



Cylinder

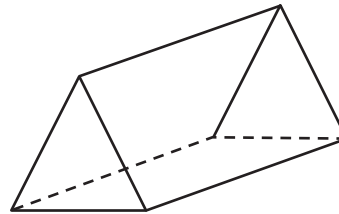
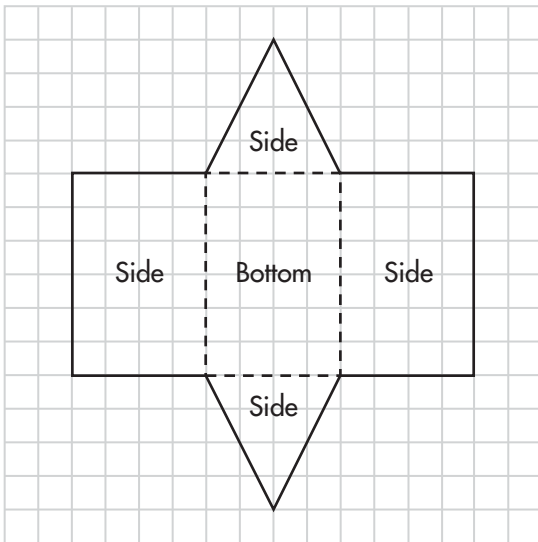
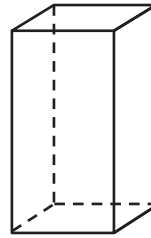
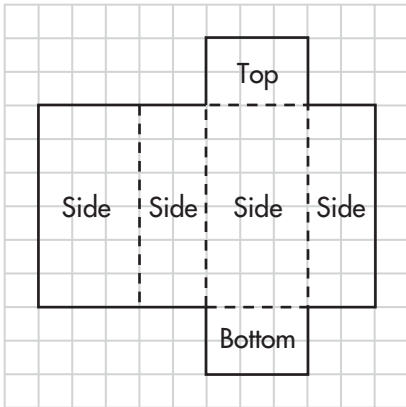
- 6.



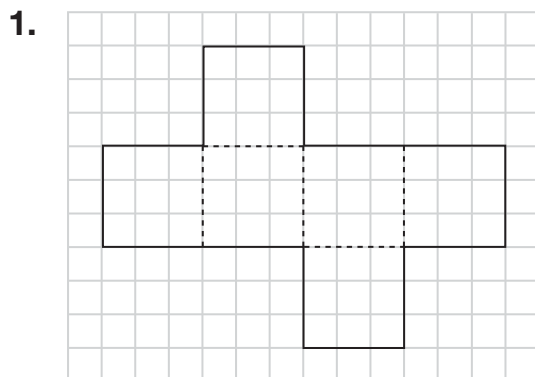
Cone

Relating Shapes and Solids

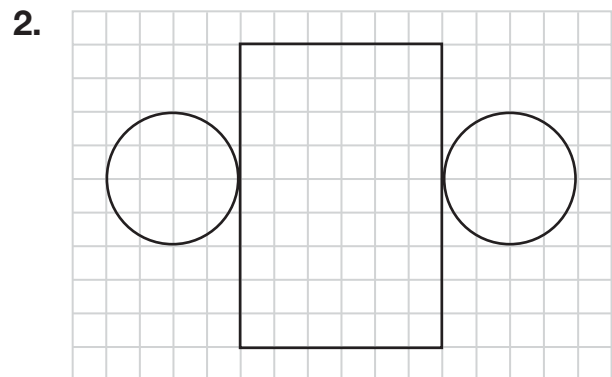
If each net on the left were folded, it would form the figure on the right.



What does each net represent?



Cube

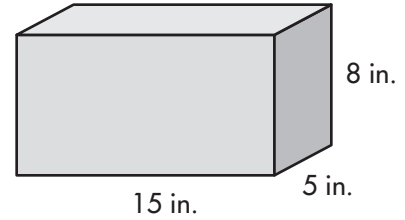


Cylinder

Surface Area

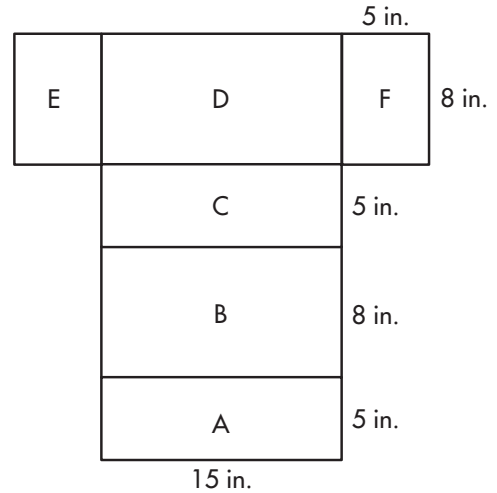
The **surface area** of a rectangular prism is the sum of the areas of all of its faces.

Danica has a wood block. She wants to paint it. In order to know how much paint to buy, Danica needs to find the surface area of the block.



If you could “unfold” this block, it would look like this:

You can find the area of each face by multiplying the length times the width.



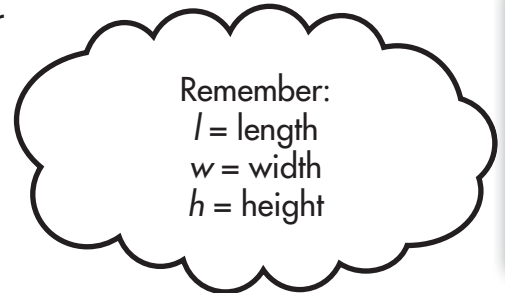
- area of A = $15 \times 5 = 75$
- area of B = $15 \times 8 = 120$
- area of C = $15 \times 5 = 75$
- area of D = $15 \times 8 = 120$
- area of E = $8 \times 5 = 40$
- area of F = $8 \times 5 = 40$

Adding all the areas together will give the total surface area.

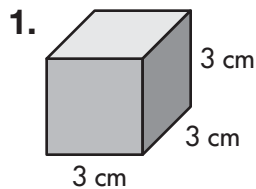
$$75 + 120 + 75 + 120 + 40 + 40 = 470 \text{ in}^2$$

Another way to find the surface area of a rectangular prism is to use the following formula.

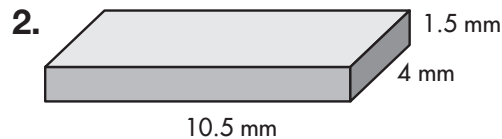
$$\begin{aligned} \text{surface area} &= 2(l \times w) + 2(l \times h) + 2(w \times h) \\ \text{surface area} &= 2(15 \times 5) + 2(15 \times 8) + 2(5 \times 8) \\ \text{surface area} &= 150 + 240 + 80 \\ \text{surface area} &= 470 \text{ in}^2 \end{aligned}$$



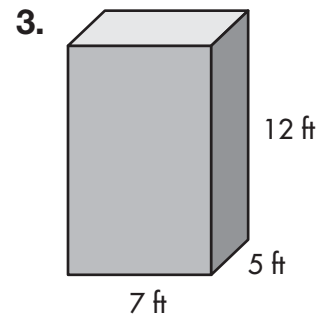
In **1** through **3**, find the surface area of each figure.



54 cm²



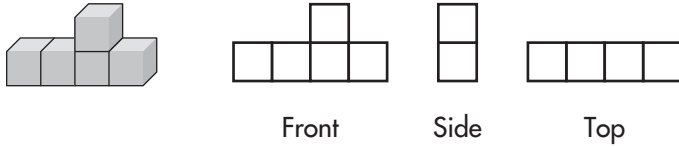
127.5 mm²



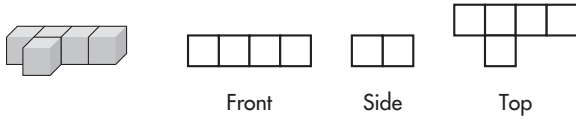
358 ft²

Views of Solids

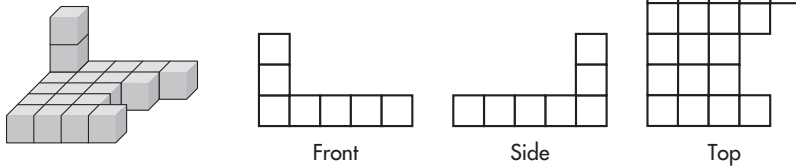
Here is how each figure on the left would look like from the front, side, and top. In each figure, the number of cubes hidden by the view is given beneath it.



There are no hidden cubes.

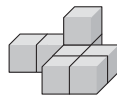


There are no hidden cubes.



There is one hidden cube.

Look at the figure. Label its front, side, and top views.



1.
Side

2.
Front

3.
Top

How many cubes are hidden in each figure?

4.
None

5.
One

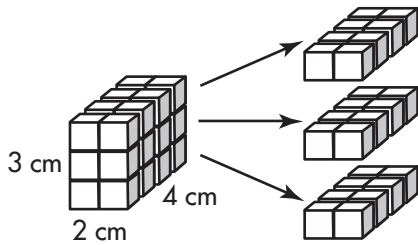
6.
None

Volume

Volume is a measure of the space inside a solid figure. It is measured in cubic units. A **cubic unit** is the volume of a cube which has edges that are 1 unit.

How to find the volume of a rectangular prism

Counting unit cubes



Count the cubes in each layer: 8 cubes.

Multiply by the number of layers.

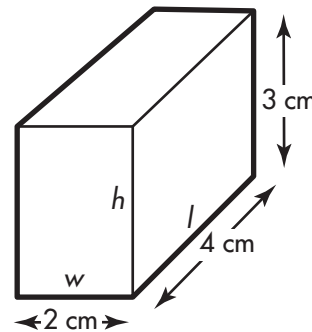
$$8 \text{ cubes} \times 3 = 24 \text{ cubes}$$

The volume of each cube is 1 cm^3 .

The volume of the prism is 24 cm^3 .

Using a formula

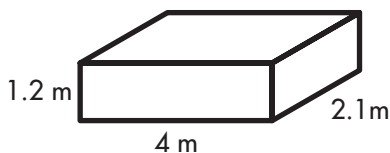
You know the length, l , the width, w , and the height, h . Calculate the volume, V , using the formula $V = l \times w \times h$.



$$V = 2 \text{ cm} \times 4 \text{ cm} \times 3 \text{ cm}$$

$$V = 24 \text{ cm}^3$$

- Find the volume of the rectangular prism using a formula.



$$10.08 \text{ m}^3$$

Irregular Shapes and Solids

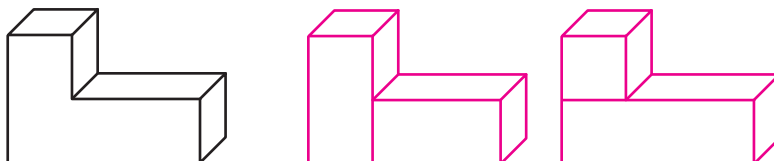
To find the area of an irregular shape, separate it into familiar shapes. Find the area of each shape and add the areas.

<p>1. To find the area of the figure at the right, separate the shape into two rectangles. (See the dotted line in the figure.)</p>							
<p>2. Use the formula $A = \ell \times w$ to find the area of each rectangle; then add the areas.</p>	<table style="width: 100%; border: none;"> <tr> <td style="text-align: center; border-bottom: 1px solid black;"><u>Area of Rectangle A</u></td> <td style="text-align: center; border-bottom: 1px solid black;"><u>Area of Rectangle B</u></td> </tr> <tr> <td style="text-align: center;">$A = 10 \times 5 = 50$</td> <td style="text-align: center;">$A = 6 \times 3 = 18$</td> </tr> <tr> <td colspan="2" style="text-align: center; padding-top: 10px;">The area of the shape is $50 + 18 = 68 \text{ cm}^2$.</td> </tr> </table>	<u>Area of Rectangle A</u>	<u>Area of Rectangle B</u>	$A = 10 \times 5 = 50$	$A = 6 \times 3 = 18$	The area of the shape is $50 + 18 = 68 \text{ cm}^2$.	
<u>Area of Rectangle A</u>	<u>Area of Rectangle B</u>						
$A = 10 \times 5 = 50$	$A = 6 \times 3 = 18$						
The area of the shape is $50 + 18 = 68 \text{ cm}^2$.							

To find the volume of an irregular solid, separate it into familiar shapes. Find the volume of each shape and add the volumes.

<p>1. To find the volume of the figure at the right, separate the shape into two rectangular prisms. (See the dotted line in the figure.)</p>							
<p>2. Use the formula $V = \ell \times w \times h$ to find the volume of each prism; then add the volumes.</p>	<table style="width: 100%; border: none;"> <tr> <td style="text-align: center; border-bottom: 1px solid black;"><u>Volume of Prism A</u></td> <td style="text-align: center; border-bottom: 1px solid black;"><u>Volume of Prism B</u></td> </tr> <tr> <td style="text-align: center;">$A = 1 \times 4 \times 7 = 28$</td> <td style="text-align: center;">$A = 2 \times 4 \times 2 = 16$</td> </tr> <tr> <td colspan="2" style="text-align: center; padding-top: 10px;">The volume of the solid is $28 + 16 = 44 \text{ ft}^3$.</td> </tr> </table>	<u>Volume of Prism A</u>	<u>Volume of Prism B</u>	$A = 1 \times 4 \times 7 = 28$	$A = 2 \times 4 \times 2 = 16$	The volume of the solid is $28 + 16 = 44 \text{ ft}^3$.	
<u>Volume of Prism A</u>	<u>Volume of Prism B</u>						
$A = 1 \times 4 \times 7 = 28$	$A = 2 \times 4 \times 2 = 16$						
The volume of the solid is $28 + 16 = 44 \text{ ft}^3$.							

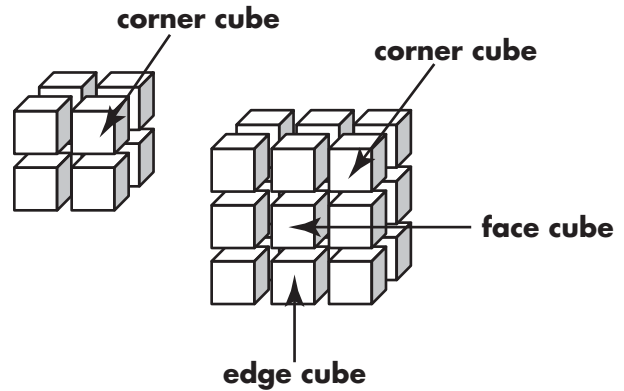
1. Show two ways of dividing the given solid into two regular solids.



Problem Solving: Use Objects and Solve a Simpler Problem

At a math fair, Willie saw a puzzle about a giant cube made of identical white smaller cubes. The giant cube was $4 \times 4 \times 4$. It contained 64 smaller cubes. Each of the six faces of the giant cube was painted red. The puzzle asked, "If the giant cube were taken apart, how many smaller cubes would have only one face painted red?" Here is how Willie tried to solve the puzzle.

1. Construct cubes using 8 and 27 smaller cubes. Imagine painting each of the larger cubes.
2. Classify the smaller cubes. *Think: Where are the cubes located in the larger cube? How are they painted differently from each other?*
3. Make a table.



Location	Cube A		Cube B	
	Number	Painted Faces	Number	Painted Faces
Corner	8	3	8	3
Edge	none		12	2
Face	none		6	1
Center	none		1	0

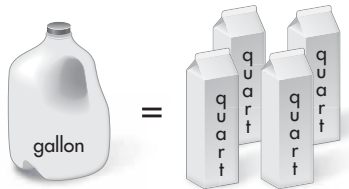
Willie organized the data about the 64 smaller cubes in the giant cube. Use the table above to complete the table below. One set of data has already been completed.

Small cubes		
Painted Surfaces	Location in giant cube	Number
3	corner (<i>Think: Same as a $3 \times 3 \times 3$.</i>)	8
2	edge (<i>Think: One more than a $3 \times 3 \times 3$ on each edge.</i>)	24
1	face (<i>Think: Three more than a $3 \times 3 \times 3$ on each face.</i>)	24
0	center (<i>Think: The center is now $2 \times 2 \times 2$.</i>)	8

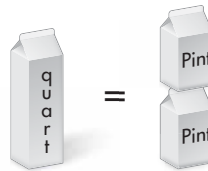
Customary Units of Capacity

Capacity is the measure of an amount of liquid. Common units of capacity are the gallon (gal), quart (qt), pint (pt), and cup (c).

The chart below shows how capacity units are related.



$$1 \text{ gallon} = 4 \text{ quarts}$$



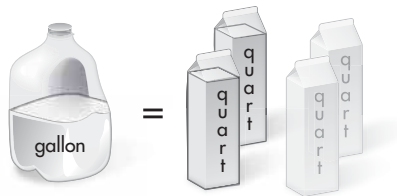
$$1 \text{ quart} = 2 \text{ pints}$$



$$1 \text{ pint} = 2 \text{ cups}$$

How to estimate volume in different units:

What is the volume estimated in gallons? in quarts?



Step 1: Think: The gallon container looks about half full.

Step 2: Estimate.

Step 3: Remember how gallons and quarts are related.

Step 4: Relate $\frac{1}{2}$ gallon to quarts.

Step 5: Simplify.

Step 6: State your estimate in quarts.

The volume of the liquid is about half the capacity of the container.

The volume of the liquid is about $\frac{1}{2}$ gallon.

$$1 \text{ gal} = 4 \text{ qt}$$

$$\frac{1}{2} \text{ gal} = \frac{1}{2} (4 \text{ qt})$$

$$\frac{1}{2} (4 \text{ qt}) = 2 \text{ qt}$$

The volume of the liquid is about 2 quarts.

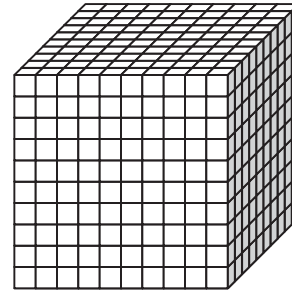
1. Number Sense How many cups are in $1\frac{1}{2}$ quarts of lemonade?

6

Metric Units of Capacity

Two common metric units of capacity are the liter (L) and the milliliter (mL).

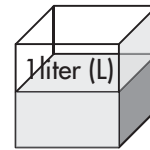
$$1 \text{ cm} \begin{array}{c} \square \\ 1 \text{ cm} \end{array} 1 \text{ cm} = \begin{array}{c} \square \\ 1 \text{ milliliter (mL)} \end{array}$$



1 liter (L) = 1,000 mL

How to estimate volume in liters and milliliters:

What is the volume estimated in liters? in milliliters?



Step 1: Think.

The liter container looks about half full.

Step 2: Estimate.

The volume of the liquid is about $\frac{1}{2}$ liter.

Step 3: Relate liters to milliliters.

$$1 \text{ L} = 1,000 \text{ mL}$$

Step 4: Relate $\frac{1}{2}$ liter to milliliters.

$$\frac{1}{2} \text{ L} = \frac{1}{2} (1,000 \text{ mL}) = 500 \text{ mL}$$

Complete.

1. 2,400 mL = 2.4 L

2. 0.9 L = 900 mL

3. 334 mL = 0.334 L

4. 0.23 L = 230 mL

Units of Weight and Mass

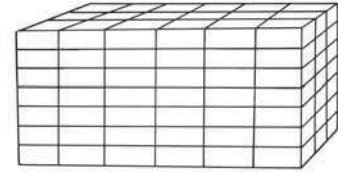
Weight is a measure of how heavy or light something is.



rose: about 1 oz

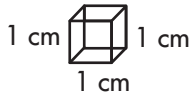


man's shoe: about 1 lb



500 bricks: about 1 ton

Mass is a measure of the quantity of matter, or stuff, in an object.



mL of water: about 1 mg



dollar bill: about 1 g



liter of milk: about 1 kg

How Weight Units are Related	How Mass Units are Related
16 oz = 1 lb	1,000 mg = 1 g
2,000 lb = 1 T	1,000 g = 1 kg

Which customary weight unit would be best for measuring each object?

1. elephant

ton

2. hot dog bun

ounce

3. loaf of bread

pound

Which metric mass unit would be best for measuring the amount of stuff in each object?

4. dust particle

milligram

5. plastic fork

gram

6. dictionary

kilogram

7. **Estimation** Estimate the weight in pounds of 162 oz of hot dogs.

10 lb

Converting Customary Units

1 gal = 128 fl oz															
1 qt = 32 fl oz				1 qt = 32 fl oz				1 qt = 32 fl oz				1 qt = 32 fl oz			
1 pt (16 fl oz)		1 pt (16 fl oz)		1 pt (16 fl oz)		1 pt (16 fl oz)		1 pt (16 fl oz)		1 pt (16 fl oz)		1 pt (16 fl oz)		1 pt (16 fl oz)	
1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c	1 c

Converting a capacity measurement from a smaller unit to a larger unit:

<p style="text-align: center;">4 pints to ____ quarts</p> <p>Think: Quarts are larger than pints. If I convert pints to quarts, there will be a smaller number of quarts than there were pints. Operation: divide.</p>	<p>You know 2 pt = 1 qt.</p> <p>Find $4 \div 2$.</p> <p>4 pt = 2 qt</p>
--	--

1 yd = 36 in.																																						
1 ft = 12 in.												1 ft = 12 in.												1 ft = 12 in.														
1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.	1 in.

Converting a length measurement from a larger unit to a smaller unit:

<p style="text-align: center;">2 feet = ____ inches</p> <p>Think: Inches are smaller than feet. If I convert feet to inches, there will be a larger number of inches. Operation: multiply.</p>	<p>You know 1 ft = 12 in.</p> <p>Find 2×12.</p> <p>2 ft = 24 in.</p>
--	--

Complete.

1. 16 oz = 1 lb
2. 2,000 lb = 1 T
3. 8 gal = 32 qt
4. 10 c = 5 pt
5. 6 ft = 2 yd
6. 3 mi = 15,840 ft
7. 48 oz = 3 lb
8. 6,000 lb = 3 T
9. **Estimation** A vat has a capacity of 624 fl oz. Estimate its capacity in cups. 80 c

Converting Metric Units

Converting a capacity measurement from a smaller metric unit to a larger metric unit:

2,000 milliliters = _____ liters	You know 1,000 mL = 1 L.
Think: Liters are larger than milliliters. If I convert milliliters to liters, there will be a smaller number of liters. Operation: divide.	Find $2,000 \div 1,000$. 2,000 mL = 2 L

Converting a length measurement from a larger metric unit to a smaller metric unit:

2 kilometers = _____ meters	You know 1 km = 1,000 m.
Think: Meters are shorter than kilometers. If I convert kilometers to meters, there will be a larger number of meters. Operation: multiply.	Find $2 \times 1,000$. 2 km = 2,000 m

Complete.

1. 1 kg = 1,000 g

2. 1 g = 1,000 mg

3. 3 L = 3,000 mL

4. 8,000 mL = 8 L

5. 5 m = 500 cm

6. 14,000 mm = 14 m

7. 72 g = 72,000 mg

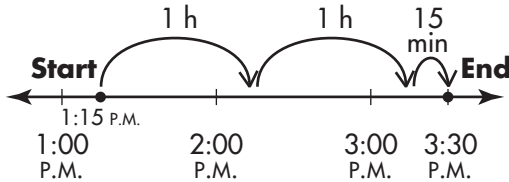
8. 8,000 g = 8 kg

Elapsed Time

How to find elapsed time:

Iris left her house at 1:15 P.M. and arrived at her grandparents' house at 3:30 P.M. How long did the trip take?

Use a number line to count up.



The trip took 2 h 15 min.

How to use elapsed time to find when an event began or ended:

Omar and his brothers played floor hockey for 1 hour and 9 minutes. They finished playing at 6:30 P.M. At what time did they begin playing?

You can subtract to find the start time.

End Time – Elapsed Time = Start Time

$$\begin{array}{r} 6 \text{ h } 30 \text{ min} \\ - 1 \text{ h } 9 \text{ min} \\ \hline 5 \text{ h } 21 \text{ min} \end{array}$$

So, they began playing at 5:21 P.M.

Find each elapsed time.

1. 8:13 P.M. to 10:00 P.M.

1 h 47 min

2. 1:24 P.M. to 4:47 P.M.

3 h 23 min

3. 3:35 P.M. to 6:09 P.M.

2 h 34 min

4. 9:55 P.M. to 11:42 P.M.

1 h 47 min

Find each start time or end time using the given elapsed time.

5. Start: 5:49 A.M.

Elapsed: 5 h 20 min

End: 11:09 A.M.

6. End: 8:27 P.M.

Elapsed: 4 h 13 min

Start: 4:14 P.M.

Add or subtract.

7.
$$\begin{array}{r} 6 \text{ h } 31 \text{ min} \\ + 7 \text{ h } 16 \text{ min} \\ \hline \end{array}$$

13 h 47 min

8.
$$\begin{array}{r} 7 \text{ h } 12 \text{ min} \\ - 3 \text{ h } 30 \text{ min} \\ \hline \end{array}$$

3 h 42 min

9.
$$\begin{array}{r} 3 \text{ h } 5 \text{ min} \\ + 8 \text{ h } 55 \text{ min} \\ \hline \end{array}$$

12 h

10.
$$\begin{array}{r} 9 \text{ h } 5 \text{ min} \\ - 8 \text{ h } 22 \text{ min} \\ \hline \end{array}$$

43 min

Elapsed Time in Other Units

A ship left Houston, Texas, at 7:10 A.M. on Tuesday. It arrived at Progreso, Mexico, at 10:15 P.M. on Wednesday. How long did the voyage take?

Method 1

1. Represent each time on a 24-hour clock. See the chart below.

12 Hour (P.M.)	1:00	2:00	3:00	4:00	5:00	6:00	7:00	8:00	9:00	10:00	11:00	12:00
24 Hour	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00	21:00	22:00	23:00	00:00

2. Find the time from departing Houston until midnight of Tuesday.

$$\begin{array}{r} 24 \text{ h } 00 \text{ min} \\ - 7:10 \text{ min} \\ \hline 16 \text{ h } 50 \text{ min} \end{array}$$

3. Find the time the boat sailed on Wednesday until it reached Progreso.

$$\begin{array}{r} 0 \text{ h } 00 \text{ min} \\ + 22 \text{ h } 15 \text{ min} \\ \hline 22 \text{ h } 15 \text{ min} \end{array}$$

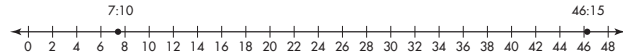
4. Add the two times.

$$\begin{array}{r} 16 \text{ h } 50 \text{ min} \\ + 22 \text{ h } 15 \text{ min} \\ \hline 38 \text{ h } 65 \text{ min} \end{array}$$

$$\begin{array}{r} 38 \text{ h } 65 \text{ min} \\ + 1 \text{ h } - 60 \text{ min} \\ \hline 39 \text{ h } 5 \text{ min} \end{array}$$

Method 2

1. Represent each time on a 48-hour number line. See the line below.



2. Subtract the departing time from the arrival time.

$$\begin{array}{r} 46 \text{ h } 15 \text{ min} \\ - 7 \text{ h } 10 \text{ min} \\ \hline 39 \text{ h } 5 \text{ min} \end{array}$$

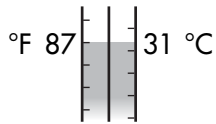
Use a 12-hour clock to represent time. Complete the following addition to find the sailing time from Houston to Progreso, Mexico.

$$4 \text{ h } 50 \text{ min} + \underline{12 \text{ h } 00 \text{ min}} + 12 \text{ h } 00 \text{ min} + \underline{10 \text{ h } 15 \text{ min}}$$

Temperature Change

How to read temperature:

Miami's average high temperature in June is 87°F . What is the average high temperature in degrees Celsius?



Step 1: Find 87° on the Fahrenheit scale.

Step 2: Read across to find the temperature on the Celsius scale.

31°C is the same as 87°F .

How to find changes in temperatures:

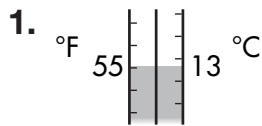
Find the change in temperature from 57°F to 18°F .

Step 1: Find the difference between 57°F and 18°F by subtracting.
 $57^{\circ}\text{F} - 18^{\circ}\text{F} = 39^{\circ}\text{F}$

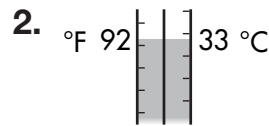
Step 2: Tell if the difference is an increase or decrease. Since the second temperature is less than the first temperature, there was a decrease.

So, the change is a decrease of 39°F .

Write each temperature in Celsius and Fahrenheit.



55°F , **13°C**



92°F , **33°C**

Find each change in temperature.

3. 34°F to 89°F

55°F increase

4. 11°C to 26°C

15°C increase

5. 86°F to 54°F

32°F decrease

6. 30°C to 8°C

22°C decrease

7. 3°F to 81°F

78°F increase

8. 12°C to 5°C

7°C decrease

9. **Number Sense** Yesterday's high temperature was 76°F . The difference between the high and low temperature was 33°F . What was yesterday's low temperature?

43°F

Problem Solving: Make a Table

A bus company schedules three buses from Elm to Coretown. Starting at 8:05 A.M., the first bus departs Elm with a bus following every 45 minutes. The trip takes 70 minutes. What time is the third bus scheduled to arrive in Coretown?

Construct a table of what you know.

Bus	Departure Time (A.M.)	Travel Time	Arrival Time
1	8:05	70 min	8:05 + 70 min
2	8:05 + 45 min	70 min	8:05 + 45 min + 70 min
3	8:50 + 45 min + 45 min	70 min	8:05 + 45 min + 45 min + 70 min

Simplify the arrival time of the third bus.

$$8:05 + 45 \text{ min} + 45 \text{ min} + 70 \text{ min}$$

$$= 8:05 + 160 \text{ min}$$

$$8 \text{ h} \quad 05 \text{ min}$$

$$+ 160 \text{ min}$$

$$8 \text{ h} \quad \underline{165 \text{ min}}$$

$$+ 2 \text{ h} - 120 \text{ min}$$

$$\underline{10 \text{ h}} \quad \underline{45 \text{ min}}$$

$$10:45 \text{ A.M.}$$

Arrival time of third bus in Coretown:

Look back and check:
is your answer reasonable?

$$\text{Yes, } \frac{3}{4} \text{ h} + \frac{3}{4} \text{ h} + 1\frac{1}{4} \text{ h is } 2\frac{3}{4} \text{ h;}$$

$$8:00 \text{ A.M.} + 2\frac{3}{4} \text{ h is } 10:45 \text{ A.M.}$$

State the following times in standard form.

- Arrival time of first bus at Coretown
- Departure time of second bus from Elm

9:15 A.M.

8:50 A.M.

Solving Addition and Subtraction Equations

Paige gave 6 peaches to Simon. Paige now has 8 peaches. How many peaches did Paige have to start?

The number of peaches Paige now has equals the number of peaches she had at the start minus the number she gave to Simon.

$$\text{So, } p - 6 = 8 \quad p = \text{number of peaches at the start}$$

To solve this equation, you need to get p alone. You can do this by using an inverse operation. An inverse operation is an opposite—addition and subtraction, for example, are inverse operations.

To get p alone, add 6 to both sides of the equation.

Tip: If you add or subtract on one side of the equation, you must do the same on the other side.

$$p - 6 + 6 = 8 + 6$$

$$p = 14 \quad \text{So, Paige started out with 14 peaches.}$$

For questions 1 through 4, solve each equation.

1. $a + 3 = 5$ **$a = 2$**

2. $y - 14 = 9$ **$y = 23$**

3. $8 + p = 15$ **$p = 7$**

4. $52 = c + 18$ **$c = 34$**

5. Mary's sister is 4 years older than Mary. Her sister is 12. Use the equation $m + 4 = 12$ to find Mary's age.

$m = 8$: Mary is eight years old.

6. **Explain It** Lyndon solved $x - 18 = 4$. He said the answer was $x = 4$. Explain why this is correct or incorrect.

This is incorrect because he didn't add 18 to both sides of the equation. The answer should be $x = 22$.

Solving Multiplication and Division Equations

Dan's computer prints at the rate of 3 pages per minute. How many pages can he print in 15 minutes?

The number of minutes is equal to the number of pages divided by the rate the pages are printed.

$$15 = d \div 3 \quad d = \text{number of pages}$$

To solve this equation, you can do an inverse operation. An inverse operation is an opposite—multiplication and division are inverse operations.

To get d alone, multiply 3 times both sides of the equation.

Remember: If you multiply or divide one side of an equation, you must do the same to the other side.

$$15 \times 3 = d \div 3 \times 3$$

$$45 = d$$

Dan can print 45 pages in 15 minutes.

In 1 through 6, solve each equation.

1. $12a = 48$ **$a = 4$**

2. $t \div 9 = 7$ **$t = 63$**

3. $250 = 25x$ **$x = 10$**

4. $75 = 3c$ **$c = 25$**

5. $p \div 33 = 3$ **$p = 99$**

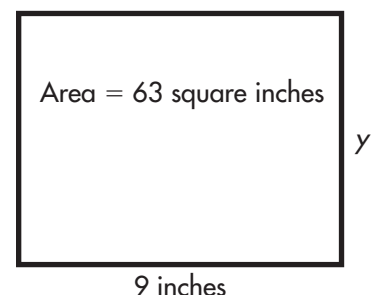
6. $4 = y \div 16$ **$y = 64$**

7. **Number Sense** A carton of eggs is on sale for \$0.49. Use the equation $\$0.49x = \2.45 to find out how many cartons you can purchase for \$2.45.

$x = 5$ cartons

8. **Geometry** Shanika has a piece of cardboard with an area of 63 square inches; one side is 9 inches long. Write an equation and solve for the value of the other side.

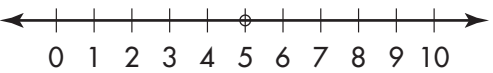
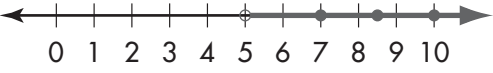
$9y = 63, y = 7$ inches

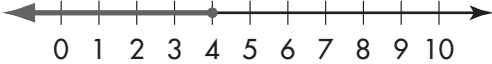


Inequalities and the Number Line

How can you graph an inequality?

An inequality is a mathematical sentence that contains one of the symbols $>$, $<$, \geq , or \leq . A solution of an inequality is any number that makes the inequality true.

Graph the inequality $x > 5$.	
1. Draw a number line 0–10 with an open circle around 5. The open circle shows that 5 itself is not a solution.	
2. Find and graph three solutions of $x > 5$ on the number line. Draw an arrow from the open circle toward the graphed solutions. The arrow shows that all numbers greater than 5 are solutions to the inequality.	

Determine the inequality represented by the following graph.	
	
1. The closed circle represents the number 4. One value of x can be 4.	$x = 4$
2. The arrow represents all numbers less than 4. So x can have any value less than 4.	$x < 4$
3. The graph shows that x can have a value equal to or less than 4.	$x \leq 4$

Graph each inequality.

1. $x > 6$

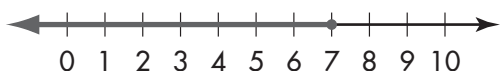


2. $x \leq 10$



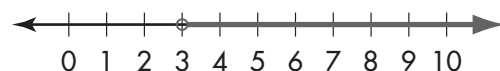
Determine the inequality represented by each graph.

3.



$x \leq 7$

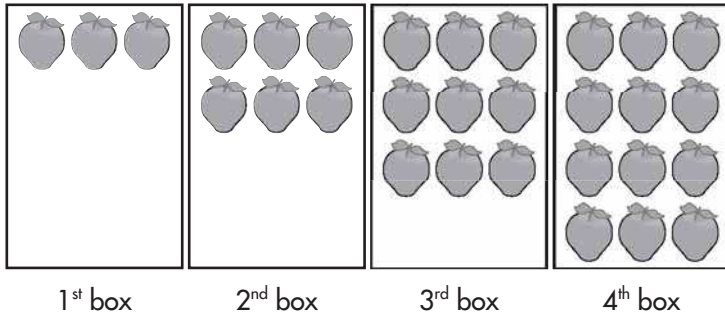
4.



$x > 3$

Patterns and Equations

You can find a rule for a pattern and use that rule to fill in a table.



How many apples will be in the 8th box? Which box will have 39 apples?

Step 1: Draw a table to show the information.

Box number (<i>b</i>)	Number of apples (<i>a</i>)
1	3
2	6
3	9
4	12

Step 2: Find a relationship between the two columns that is the same for each pair. This relationship is the rule for the pattern.

For these two columns the relationship is to multiply the box number by 3.

Step 3: Write an equation to express the rule.

number of apples = 3 times the box number

$$a = 3b$$

Step 4: Use this equation to find the missing parts of the table.

$$a = 3 \times 8 \qquad 39 = 3b$$

$$a = 24, 24 \text{ apples} \qquad 39 \div 3 = b$$

$$\text{in the 8}^{\text{th}} \text{ box} \qquad 13 = b, 13^{\text{th}} \text{ box will have}$$

$$39 \text{ apples}$$

For questions 1 through 4, write an equation for each table and solve for the missing values for *x* and *y*.

1.

<i>x</i>	<i>y</i>
0	5
3	8
7	12
10	15

$$y = x + 5$$

2.

<i>x</i>	<i>y</i>
16	64
9	36
6	24
4	16

$$y = 4x$$

3.

<i>x</i>	<i>y</i>
2	-4
4	-2
8	2
16	10

$$y = x - 6$$

4.

<i>x</i>	<i>y</i>
30	3
40	4
50	5
80	8

$$y = \frac{x}{10}$$

Problem Solving: Draw a Picture and Write an Equation

Salim collects football cards and baseball cards. He has 38 baseball cards. If he has 25 football cards, how many sports cards does Salim have?

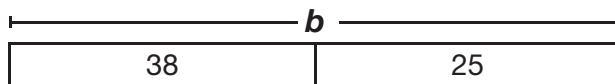
Step 1: Write down what you already know.

Salim has 25 football cards.
He has 38 baseball cards.

Step 2: What are you trying to find?

How many sports cards Salim has.

Step 3: Draw a diagram to show the information. Choose a variable to represent what you are trying to find.



Step 4: Write an equation and solve.

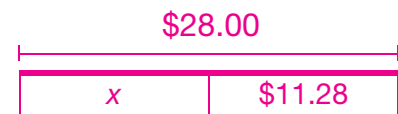
$$38 + 25 = b$$

$$b = 63$$

Salim has 63 sports cards.

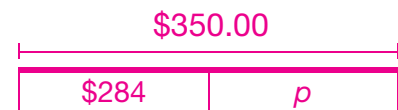
For questions 1 through 3, draw a picture or diagram, write an equation, and solve the equation to answer the question.

1. Hallie spent \$28 buying two figurines at the flea market. If one figurine costs \$11.28, how much did Hallie pay for the other one?



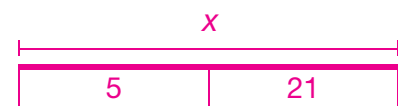
$$\underline{\$28.00 = x + \$11.28, x = \$16.72}$$

2. Cody is selling popcorn for his Cub Scout pack. His goal is to sell \$350. So far, Cody has sold \$284. How much more does Cody need to sell to meet his goal?



$$\underline{p + \$284 = \$350, p = \$66}$$

3. In Briana's class there are 5 fewer students than there were last year. There are 21 students in Briana's class. How many students were there last year?



$$\underline{x - 5 = 21, x = 26 \text{ students last year}}$$

Understanding Ratios

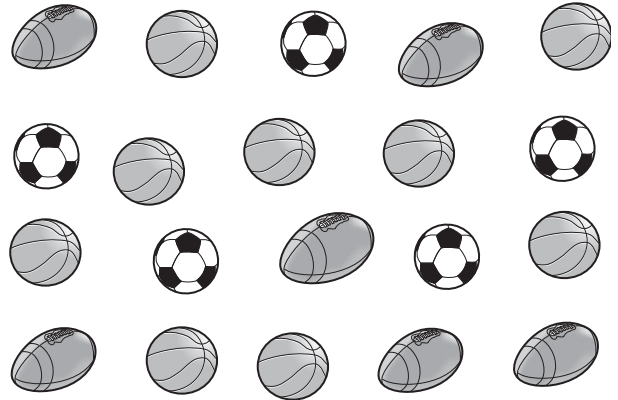
Coach Sanders has 9 basketballs, 6 footballs, and 5 soccer balls in her equipment bin. What is the ratio of basketballs to footballs? What is the ratio of soccer balls to total balls?

Using ratios, you can compare parts of a whole to the whole, some parts to other parts, or the whole to its parts.

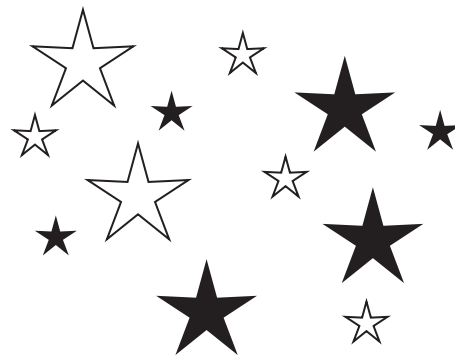
number of basketballs	number of footballs
9	6

Remember There are three different ways to write the same ratio.

9 to 6	9:6	$\frac{9}{6}$
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In **1** through **4**, write each ratio. Give each answer in the three different ratio forms.



1. small stars to large stars

7 to 5, 7:5, $\frac{7}{5}$

2. filled stars to total stars

6 to 12, 6:12, $\frac{6}{12}$

3. large unfilled stars to small unfilled stars

2 to 4, 2:4, $\frac{2}{4}$

4. small filled stars to total stars

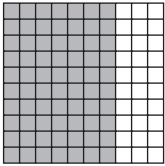
3 to 12, 3:12, $\frac{3}{12}$

Understanding Percent

A percent is a special kind of ratio that shows the relationship between a number and 100. Percent actually means *per hundred*.

The Hortonville Hoopsters have won 70 games of the last 100 games they have played. What percentage of the games has the team won?

Using a 100-square grid can help.

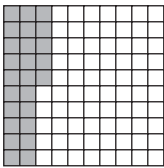


There are 70 squares shaded so the ratio is 70 to 100 or $\frac{70}{100}$.

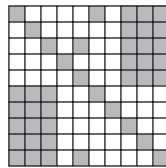
$$\frac{70}{100} = 70\%$$

Remember The % sign is the same thing as saying “out of 100.”

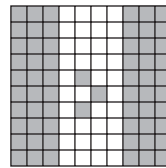
In **1** through **4**, write the ratio and percent of the shaded section on each grid.



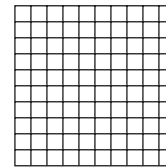
1. $\frac{25}{100}$, **25%**



2. $\frac{43}{100}$, **43%**



3. $\frac{63}{100}$, **63%**



4. $\frac{0}{100}$, **0%**

Number Sense

5. What is “eighty-two percent” written as a fraction in simplest form? $\frac{41}{50}$

6. Mystery Bookstore has 100 books on its shelves. Forty-one are hardbacks and 59 are paperbacks. What percentage of hardbacks and what percentage of paperbacks are there?

41% hardbacks, 59% paperbacks

Percents, Fractions, and Decimals

At Slider Park, 40% of the children playing were boys. What is this percent expressed as a fraction and a decimal?

You can change percents into fractions and decimals just as you can change fractions and decimals into percents.

To change a percent into a fraction, drop the percent sign and place the number over 100. Simplify the fraction if possible.

$$40\% = \frac{40}{100} = \frac{2}{5}$$

To change a fraction to a percent, divide the numerator by the denominator. Make sure your quotient goes to the hundredths place. If your quotient has too few digits, add zeros as placeholders. If your quotient has too many places, round to the hundredths. Then move the decimal point two places to the right and add the percent sign.

$$\frac{2}{5} = 0.40 = 40\%$$

To change a percent to a decimal, drop the percent sign and move the decimal point two places to the left.

$$40\% = 0.40$$

In **1** through **3**, write each percent as a decimal and a fraction in simplest form.

1. 36% 0.36, $\frac{9}{25}$ 2. 13% 0.13, $\frac{13}{100}$ 3. 59% 0.59, $\frac{59}{100}$

4. **Reasonableness** Pilar wrote that $3\% = 0.30$. Is this a reasonable answer?

No, she forgot to put a zero as a placeholder in the tenths place. It should be 0.03.

5. On the Pennyville Pigskins football team, two quarterbacks are competing for the starting job. Jake completes 87% of his passes. Brett only has 13% of his passes fall incomplete.

Who is the better quarterback?

They are equal. If 87% of the passes are complete, then 13% of the passes are incomplete.

Finding Percent of a Whole Number

Stefani wants to buy a skirt that costs \$28. She received a coupon in the mail for a 20% discount from the store. How much will she save?

To find the percent of a whole number, multiply the whole number by the decimal equivalent of the percent.

$$\begin{array}{r} 28 \\ \times 0.20 \\ \hline \$5.60 \text{ discount} \end{array}$$

In 1 through 6, find the percent of each number.

1. 17% of 82 **13.94** 2. 3% of 115 **3.45** 3. 42% of 600 **252**
 4. 50% of 800 **400** 5. 73% of 280 **204.4** 6. 39% of 22 **8.58**

7. In the problem above, Stefani is able to save \$6 each week. With the discount, how many weeks will it take for Stefani to save enough to buy the skirt?

4 weeks

8. **Geometry** Kirk ordered a pizza with 10 slices. How many slices would amount to 30% of the pizza?

3 slices

9. Jackson High School's band sells programs at the State Fair. They receive 15% of the total amount sold. The band sold \$3,500 worth of programs. How much money did they receive?

\$525

10. **Mental Math** Using mental math, find 50% of 500. How did you get your answer?

250: 50% is the same as $\frac{1}{2}$, and $\frac{1}{2}$ of 500 is 250.

Problem Solving: Make a Table and Look for a Pattern

Sometimes when you are trying to solve a problem, it can help to make a table and look for a pattern.

Nasser has 5 hours every day between when school ends and when he goes to bed. He spent 3 hours last night studying. If he continues like that every night, what percent of his time will be spent studying?

Step 1. Make a table with the information you already know.

Hours spent studying	3			
Total hours	5			

Step 2. Fill in the table using equal ratios until you get a comparison with 100.

Remember You find equal ratios by multiplying or dividing the top and bottom of the ratio by the same number.

Hours spent studying	3	12	30	48	60
Total hours	5	20	50	80	100

So, Nasser spends 60% of his time studying.

In 1 and 2, find the percent by completing the table.

1. won 7 out of 10 games 70%

Games won	7	14	35	70
Total games	10	20	50	100

2. ran 13 miles out of 20 65%

Miles run	13	6.5	32.5	65
Total miles	20	10	50	100

3. Denise gets 450 minutes each month to use on her cell phone. Last month, she used 360 minutes. What percent of her minutes did Denise use? 80%

Understanding Integers

You can use integers to represent word descriptions.

Remember: An integer is a whole number or its opposite (+7 and -7 are integers). 0 is its own opposite.

Use a positive integer to represent word descriptions that show an increase.

up 3 floors $+3$

45 degrees above zero $+45$

5 steps forward $+5$

Use a negative integer to represent word descriptions that show a decrease.

300 feet below sea level -300

cut off 10 inches -10

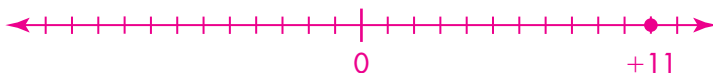
2 steps backward -2

In **1** through **8**, write an integer for each word description.

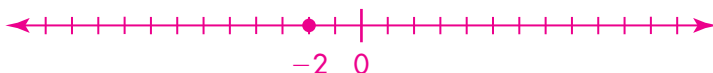
- | | |
|---|--|
| 1. win 5 games <u>$+5$</u> | 2. earned \$3 <u>$+3$</u> |
| 3. lose 100 points <u>-100</u> | 4. grew 20 inches <u>$+20$</u> |
| 5. spent \$14 <u>$-14$</u> | 6. 2,200 feet above sea level <u>$+2,200$</u> |
| 7. go down 8 floors <u>-8</u> | 8. 15 minutes before test time <u>-15</u> |

Draw a Picture Draw a number line to show the following:

9. $+11$



10. -2



Ordered Pairs

You can locate points on a grid using ordered pairs.

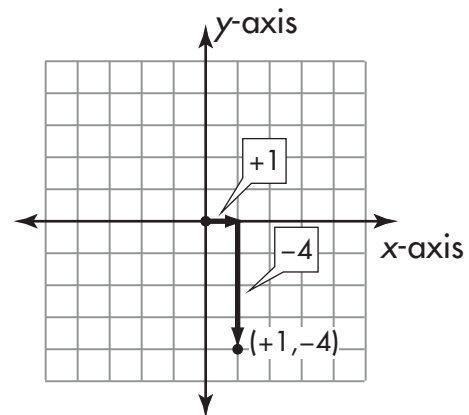
Draw a point on the grid for $(+1, -4)$.

On a grid, the axis that goes across is called the x -axis. The axis that goes up and down is called the y -axis. The point where the x -axis and the y -axis cross is called the origin.

To locate a point on a grid using an ordered pair, start at the origin.

The first number in the ordered pair is called the x -coordinate. It tells how far to move to the right (if positive) or to the left (if negative). Since $+1$ is positive, move 1 unit to the right.

The second number in the ordered pair is the y -coordinate. It tells how far to move up (if positive) or down (if negative). Since -4 is negative, move 4 units down to the point $(+1, -4)$.



Write the ordered pair for each point.

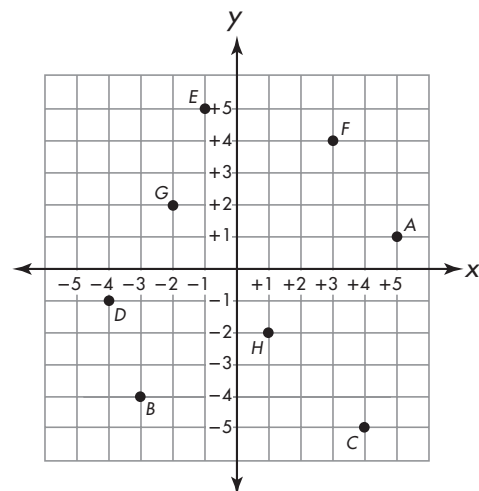
1. B **$(-3, -4)$** 2. D **$(-4, -1)$**

Name the point for each ordered pair.

3. $(+3, +4)$ **F** 4. $(-2, +2)$ **G**

5. What is the ordered pair for the origin?

$(0, 0)$

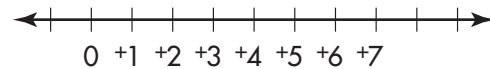
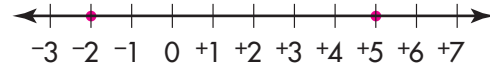


Distances on Number Lines and the Coordinate Plane

How do you find a distance on a number line?

Finding the distance from -2 to 5 on a number line:

1. Plot the integers on a number line.
2. Draw a second number line beneath the first. Beneath the -2 on the top number line, start labeling the lower number line $0, +1, +2$. Stop labeling when you reach $+5$ on the top line. Count the units on the lower line.

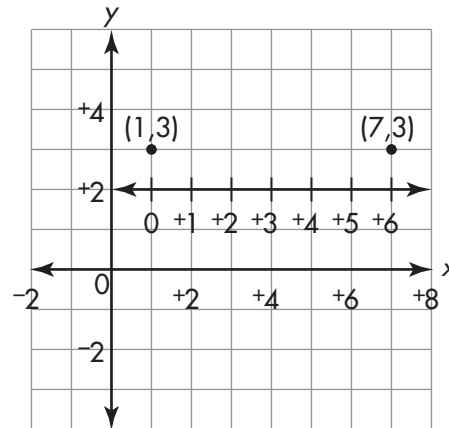


The distance between -2 and 5 is 7 .

How do you find a distance on a coordinate grid?

Finding the distance between $(1, 3)$ and $(7, 3)$ on a grid:

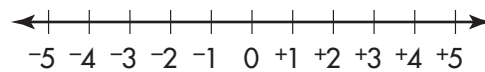
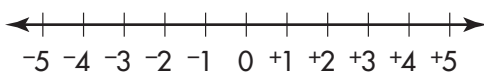
1. Plot the points named $(1, 3)$ and $(7, 3)$ on a coordinate grid.
2. Because the y -coordinate of each named point is the same, draw a matching number line parallel to the x -axis. Next to the point named $(1, 3)$, start labeling the number line $0, +1, +2$, until you reach the point $(7, 3)$. Count the units on the number line. The distance between $(1, 3)$ and $(7, 3)$ is 6 .



Find the distance between each pair of integers.

1. $-3, +4$ 7

2. $-2, +3$ 5



Find the distance between each pair of points.

3. $(-1, 6)$ and $(3, 6)$ 4

4. $(3, -2)$ and $(3, 5)$ 7

Graphing Equations

An equation can predict data. The data can be plotted on a coordinate grid. If the graph is a straight line, the equation is called a **linear equation**.

How to graph an equation:

Step 1

Use the equation to calculate ordered pairs. Put at least 3 ordered pairs into a table.

Step 2

Choose what to plot on the horizontal axis and what to plot on the vertical axis.

Choose the starting point and ending point for each axis.

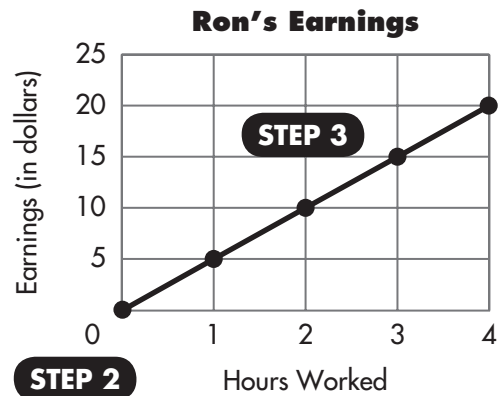
Choose an interval for each axis. Label and number the axes.

Step 3

Graph the data by using the coordinates for each set of data as a point. Connect all the points in a straight line. Title your graph.

STEP 1

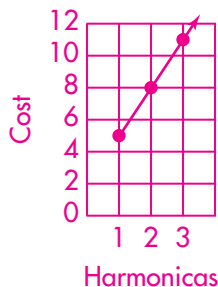
Hours	Earnings
0	0
1	5
2	10
3	15
4	20



Graph the equation below to show the cost, c , of buying harmonicas, h . Harmonicas are available online for \$3 each, plus a single shipping charge of \$2.

$$c = 3h + 2$$

h	c
1	5
2	8
3	11



Data from Surveys

In a survey, each student was asked this question. How many trees do you have in your yard? Here are the responses: 1, 2, 3, 1, 4, 4, 1, 2, 1, 2, 4.

How to display data collected from surveys:

The data can be displayed in a frequency table or a line plot.

Frequency Table	Line Plot
<ol style="list-style-type: none"> Count the number of times each different response was made. 1: ////; 2: ///; 3: /; 4: /// 	<ol style="list-style-type: none"> Count the number of times each different response was made. 1: ////; 2: ///; 3: /; 4: ///
<ol style="list-style-type: none"> Construct a frequency table. The table lists each response and its frequency. (The frequency of a response is how many times it was made.) 	<ol style="list-style-type: none"> Construct a line plot. The plot lists each response along a horizontal line. The frequencies are stacked as x's above each response.
<ol style="list-style-type: none"> Give the frequency table a title that clearly explains what information is in the table. 	<ol style="list-style-type: none"> Give the line plot a title that clearly explains what information is in the plot.

Survey Question:
How many trees are in your yard?

Number of Trees	Number
1	4
2	3
3	1
4	3

Survey Question:
How many trees are in your yard?

- How many students responded to the tree survey? 11
- Which number of trees had the greatest responses? 1
- Explain It** Describe how you might pick a sample of 50 minivan owners that represent the minivan owners of your state.

Sample response: Minivan owners living in four different regions of the state might be asked to respond.

Bar Graphs and Picture Graphs

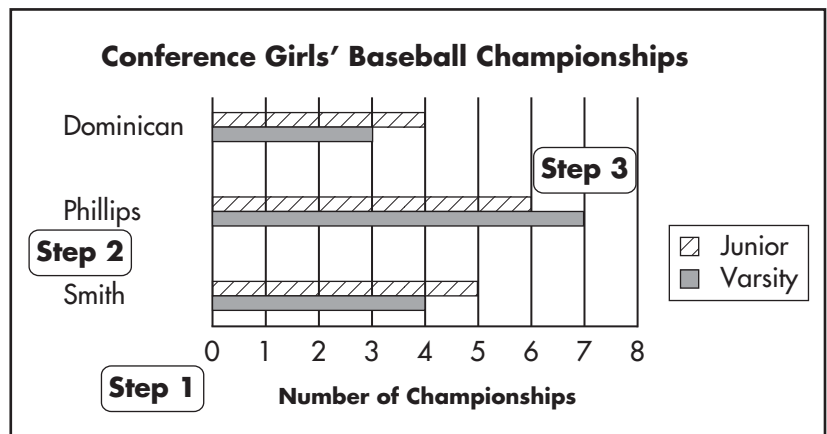
A bar graph uses rectangles to show data. A double bar graph uses two different-colored or shaded bars to show two similar sets of data. A picture graph uses symbols to show data. You can use each to compare data.

Look at the information about the baseball teams in the frequency table at right. Display this information as a double bar graph.

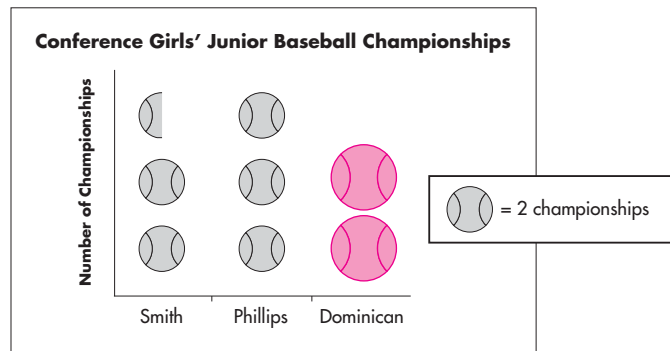
Conference Baseball Championships

High School	Junior	Varsity
Dominican	4	3
Phillips	6	7
Smith	5	4

1. Decide on a scale and its intervals.
2. Draw the graph. Label the axes.
3. Graph the data by drawing bars of the correct length and title the graph.



Look at the information about the junior teams in the frequency table above. Use this information to complete the picture graph with the values for Dominican High School.



Line Graphs

How to make a line graph:

Step 1

Put the data into a table so that each set of data has two values.

Step 2

Choose what to plot on the horizontal axis and what to plot on the vertical axis.

Choose the starting point and ending point for each axis. (The starting and ending points must include the smallest and largest data values.)

Choose an interval for each axis. Label and number the axes.

Step 3

Graph the data by using the coordinates for each set of data as a point. Draw lines to connect the points.

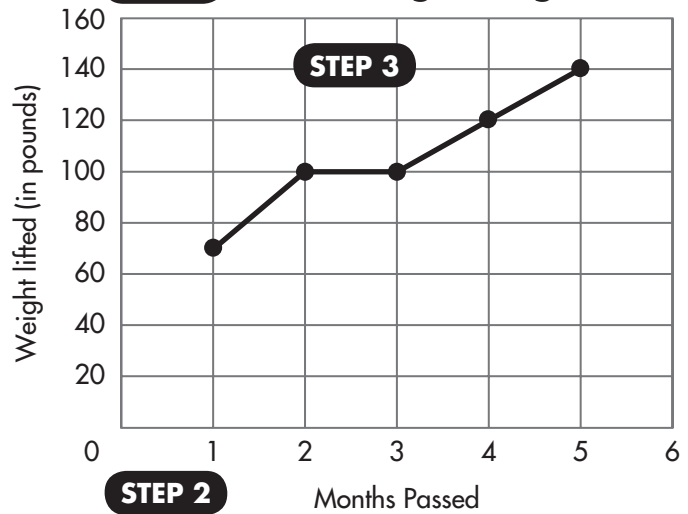
Step 4

Title your graph.

STEP 1

Months passed	Weight lifted (pounds)
1	70
2	100
3	100
4	120
5	140

STEP 4 Dave's Weightlifting



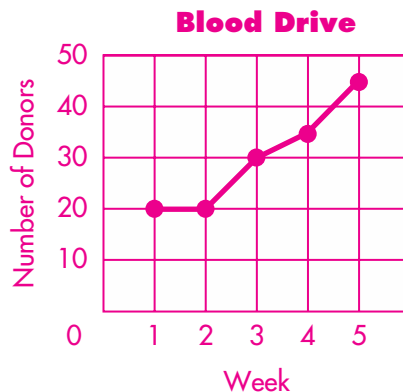
STEP 2

Make a line graph of the data.

Since the data for the x-axis are from 1 to 5, the scale can go from 0 to **5**.

Since the data for the y-axis are from 20 to 45, the scale can go from 0 to **50**.

Blood Drive	
Week	Number of donors
1	20
2	20
3	30
4	35
5	45



Stem-and-Leaf Plots

A stem-and-leaf plot is a convenient way to organize data.

A school records the number of students absent each day. The records for a two-week period are shown in the stem-and-leaf plot on the right.	Number of Students Absent Each Day	
	Stem	Leaf
	1	2 5 6 9
	2	1 4 7 8
	3	0 3
	KEY: 1 2 = 12	

How do you read a stem-and-leaf plot?

1. Identify the stem for each data value.	The stems in this plot represent the tens digits 10, 20, and 30.
2. Identify the leaf for each data value.	The leaves in this plot represent the ones digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
3. Identify the key.	The key shows that each stem-leaf combination represents a two-digit number.

Read the following and complete the following questions.

A small card shop records the number of birthday cards sold each day. The record is displayed as the stem-and-leaf plot shown on the right.	Number of Birthday Cards Sold Each Day	
	Stem	Leaf
	2	1 5 6 7 8 9 9
	3	0 3
	4	2 2 3 4 9
	KEY: 2 1 = 21	

- Using the key, what is the value of 3|0? **30**
- How many days are represented by the plot? **14**
- What is the least number of cards sold in one day? **21**
- What is the most number of cards sold in one day? **49**

Histograms

A histogram is a type of bar graph that shows frequency of data with equal intervals. The equal intervals are shown on the horizontal axis.

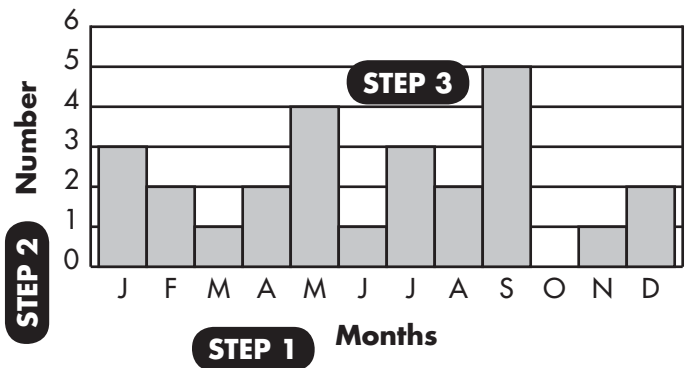
Mr. Sato asked each student in his class which month they were born in. He put the data into a table. Make a histogram to determine the two most common birthday months.

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
Frequency	3	2	1	2	4	1	3	2	5	0	1	2

Step 1: You want to divide the data by month, so lay out the months along the horizontal axis.

Step 2: The number of students born in each month is the frequency data, which goes on the vertical axis.

Step 3: Use the data from the table to fill in the histogram.



September and May are the most common birthday months.

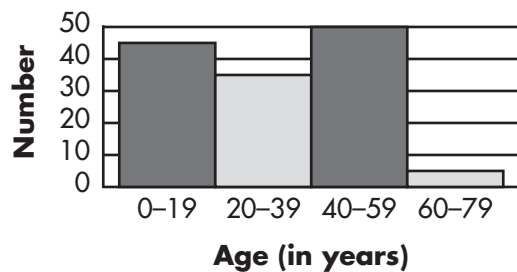
- How many people ages 0–19 and 40–59 combined attended the tournament?

95 people

- What is the total number of people who attended the tournament?

135 people

Spectators at School Tournament



Circle Graphs

A circle graph shows 100% of a data set. Each wedge in the circle represents part of the whole amount.

One hundred people were asked if they owned a pet and if they did, what kind of pet. The table at the right shows the data.

From this table, you can create a circle graph.

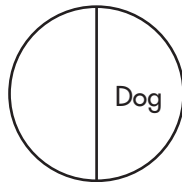
Pet	
Dog	50
Cat	25
Bird	10
Other	10
No pet	5

Each category is part of the whole. You can find out how big a wedge to use for each category by changing the ratios into fractions.

50 people out of 100 had a dog.

$$\frac{50}{100} \text{ is } \frac{1}{2}.$$

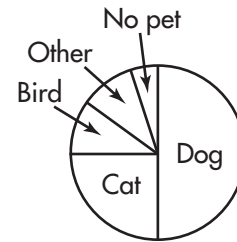
So, the wedge for dog will be $\frac{1}{2}$ of the circle.



To finish the graph, do the same with the rest of the table.

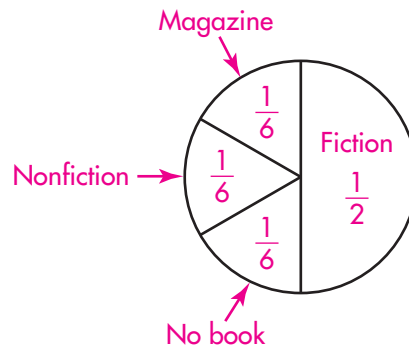
$$\text{cat} = \frac{25}{100} = \frac{1}{4}, \text{ bird} = \frac{10}{100} = \frac{1}{10},$$

$$\text{other} = \frac{10}{100} = \frac{1}{10}, \text{ no pet} = \frac{5}{100} = \frac{1}{20}$$



- The table below shows what kind of books people said they had checked out at the library. Using the data, copy the circle graph and label each section with the correct type of book and a fraction in simplest form.

Type of book	
Fiction	60
Magazine	20
Nonfiction	20
No book	20



- How many total people were asked about their book choice?

120

Mean

The mean is the sum of all the values in a set divided by the number of items in the set. The mean is also called the average.

How to find the mean of a set of data:

Eduardo surveyed 7 of his friends to find out how many books they read during the month. The frequency table shows the data. What is the average number of books read by Eduardo's friends?

Book Reading	
Friend	Number of books read
Jean	2
Raul	3
Sally	8
Jonathan	5
Haley	6
Kristen	3
Owen	1

1. Add the number of books read by each friend.

$$2 + 3 + 8 + 5 + 6 + 3 + 1 = 28$$

2. Divide the sum by the number of friends.

$$\frac{28}{7} = 4$$

3. Use the average to answer the question.

Eduardo's friends read an average of 4 books during the month.

1. Find the mean of this set of data: 241, 563, 829, 755. _____

597

2. This frequency table shows the number of silver medals won by American athletes in Summer Olympic Games between 1972 and 2000. What is the mean of this set of data?

US Silver Medals Summer Olympics Games	
Year	Medals
2000	24
1996	32
1992	34
1988	31
1984	61
1980	0
1976	35
1972	31

31

3. **Estimation** What is the approximate average of these three numbers: 9, 18, and 31? **about 20**

4. **Explain It** Explain how you would find the mean of this set of data: 4, 3, 5.

Sample answer: Add the numbers in the set. Then divide by three.

Median, Mode, and Range

The median, mode, and range are each numbers that describe a set of data.

Here is Eduardo's survey of how many books his friends read last month.

What are the median, mode, and range of Eduardo's survey?

Book Reading	
Friend	Number of books read
Jean	2
Raul	3
Sally	8
Jonathan	5
Haley	6
Kristen	3
Owen	1

Median: The median is the middle number in a set of data. To find it:

1. Arrange the data in order from least to greatest.
2. Locate the middle number.

1, 2, 3, 3, 5, 6, 8



middle number = 3

The median number of books read is 3.

Mode: The mode is the data value that occurs most often. To find it:

1. List the data. 1, 2, 3, 3, 5, 6, 8
2. Find the number that occurs most. 3

The mode of the books read by Eduardo's friends is 3 books.

Range: The range is the difference between the greatest and least values. To find it:

1. Identify the greatest and least values. 8 and 1
2. Subtract the least from the greatest value. $8 - 1 = 7$

The range of the books read by Eduardo's friends is 7 books.

1. Find the median of this data set: 12, 18, 25, 32, 67.

25

2. Find the mode of this data set: 123, 345, 654, 123, 452, 185.

123

3. Find the range of this data set: 24, 32, 38, 31, 61, 35, 31.

37

Problem Solving: Make a Graph

Choosing which graph to use depends on what kind of data set you are looking at.

Bar graphs and **circle graphs** are good graphs to use when you are comparing data that can be counted. **Line graphs** are good at showing changes in the data over time. **Histograms** divide the data into intervals and record how many data items fall within each interval.

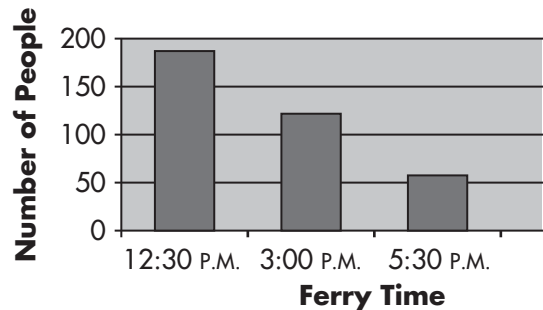
There are three ferries that run from Port Shoals to Tiger Island. 187 people were on the 12:30 P.M. ferry, 122 people were on the 3:00 P.M. ferry, and 58 people were on the 5:30 P.M. ferry. Make a graph for this data set and analyze the data.

This data set is comparing data that can be counted, so a bar graph or a circle graph would be a good graph choice. Make a bar graph of the data.

What can you say about the data from this graph?

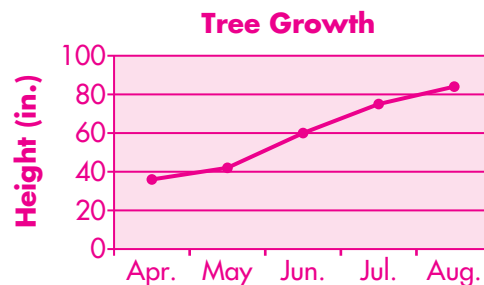
The 12:30 P.M. ferry was the most popular.

The 5:30 P.M. ferry was the least popular.



- Each month, Martina measures the height of a tree she planted. Make a line graph to show the tree's growth.

Month	Height (in.)
April	36
May	42
June	60
July	75
August	84

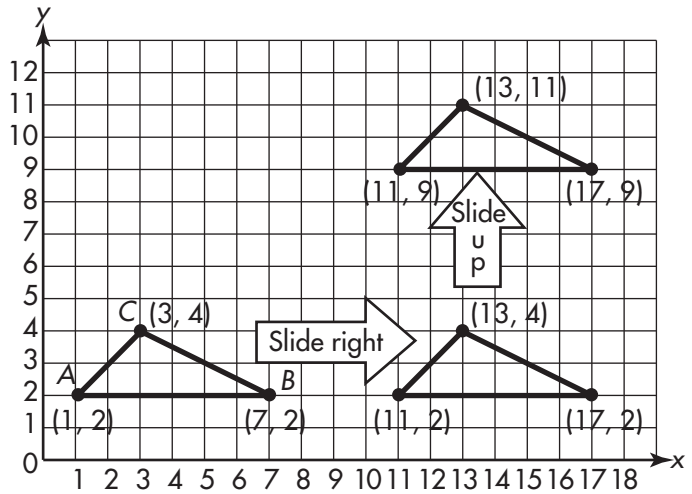


Translations

A translation is the motion of a figure up, down, left, or right.

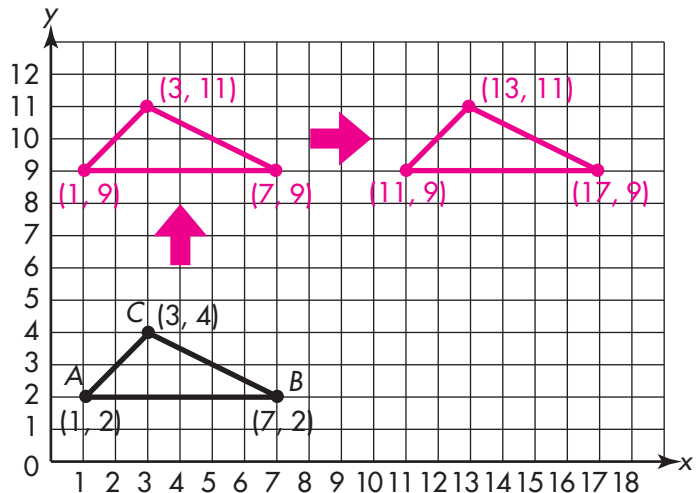
How to translate a figure:

Translate the triangle to the right 10 units and up 7 units.



Part 1	Part 2
Translate the triangle to the right 10 units.	Translate the triangle up 7 units.
Step 1: Increase the x -coordinates of each vertex of the triangle by 10. (Do not change the y -coordinate of each vertex.)	Step 4: Increase the y -coordinates of each vertex of the triangle at its new location. (Do not change the x -coordinate of each vertex.)
Step 2: Draw the triangle at its new location.	Step 5: Draw the triangle at its final location.
Step 3: Label each vertex with its ordered pair.	Step 6: Label each vertex with its ordered pair.

- Translate the triangle shown here up 7 units and to the right 10 units.



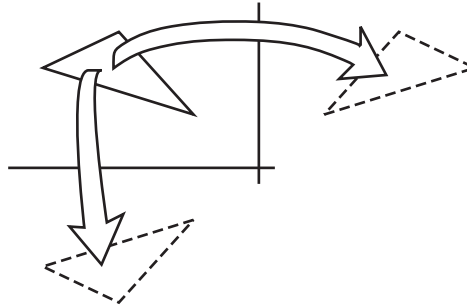
- Number sense** Vertex A of a figure is located at $(18, 6)$. The rhombus is translated down 5 units and to the left 3 units. What is the new location of vertex A ?

(15, 1)

Reflections

A **reflection** is the mirror image of a figure. The reflection appears as if the image has been flipped.

How to recognize a reflection:



Reflection across a horizontal line	Reflection across a vertical line
The image looks like the figure flipped top to bottom.	The image looks like the figure flipped left to right.

Place a checkmark beneath the correct image for each reflection.

1. Reflection across a vertical line 2. Reflection across a horizontal line



3. Reflection across a vertical line



4. Reflection across a horizontal line



Rotations

A **rotation** is a turn that moves a figure about a point.

How to recognize a rotation:

<p>Rotation of the triangle around point A</p> <p>Each vertex rotates the same amount and in the same direction around A.</p>	<p>Rotation of the triangle around point D</p> <p>Each vertex rotates the same amount and in the same direction around D.</p>

1. Place a checkmark beneath the image that is **NOT** a rotation of the figure around K.

<p>_____</p>	<p>_____</p>	<p>_____ ✓</p>	<p>_____</p>	<p>_____</p>

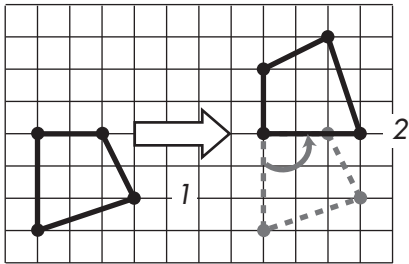
2. Place a checkmark beneath the image that is **NOT** a rotation of the figure around R.

<p>_____</p>	<p>_____</p>	<p>_____</p>	<p>_____</p>	<p>_____ ✓</p>

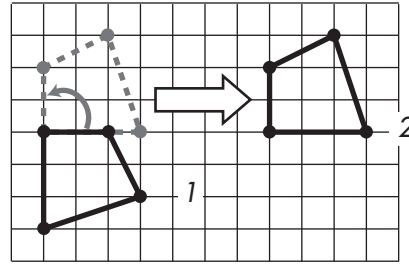
Congruence

The change of position of a figure is called a **transformation**.
Figures that are the same size and shape are called **congruent figures**.

How to recognize a transformation:

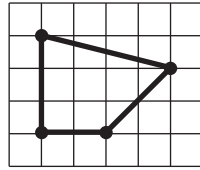
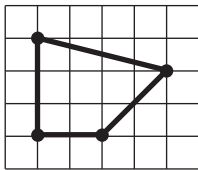


A translation and a rotation of a figure produce a transformation. The size and shape of the figure do not change.

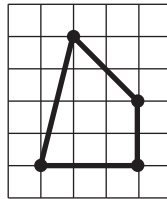


A rotation and a translation of a figure produce a transformation. The size and shape of the figure do not change.

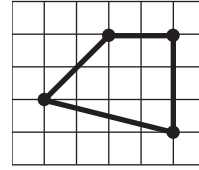
How to recognize congruent figures:



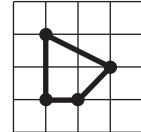
Congruent with Figure 1.



Congruent with Figure 1.



Congruent with Figure 1.

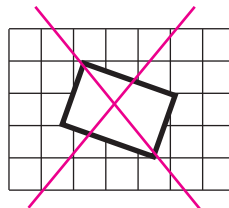
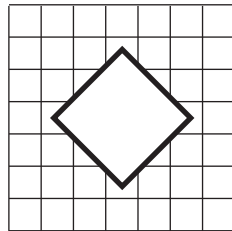
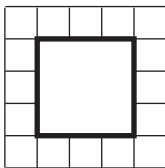


Not Congruent with Figure 1.

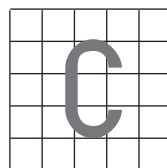
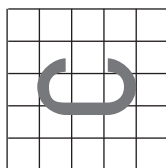
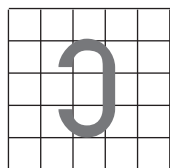
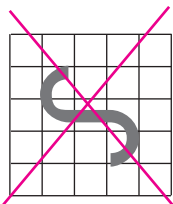
Congruent figures have the same size and shape.

In each group, cross out the figure that is not congruent with the other figures.

1.



2.



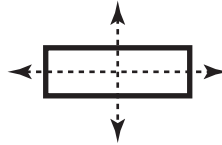
Symmetry

What is line symmetry?

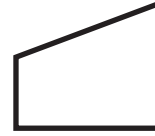
A figure has line symmetry if it can be folded in half so that one side reflects onto the other side. The fold line is called a line of symmetry.



This figure has line symmetry. There is only one way to fold it so that two sides match. It has one line of symmetry.



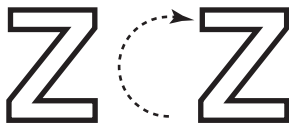
This figure has line symmetry. It can be folded into matching halves two different ways. It has two lines of symmetry.



This figure has no line symmetry. It cannot be folded into matching halves.

What is rotational symmetry?

A figure has rotational symmetry if it turns onto itself in less than a full rotation.



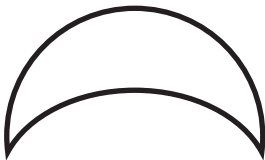
This figure has rotational symmetry. The figure and its rotated shape match after half a rotation.



This figure has no rotational symmetry. The figure and its rotated shape match only after a full rotation.

Circle the number of each figure that has line symmetry.

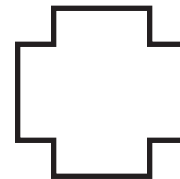
1.



2.



3.

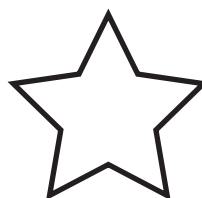


Circle the number of each figure with rotational symmetry.

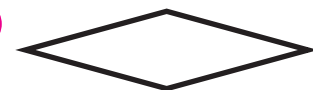
4.



5.



6.

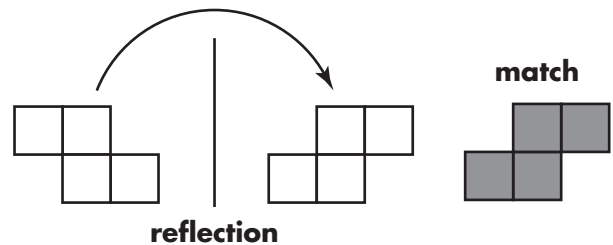
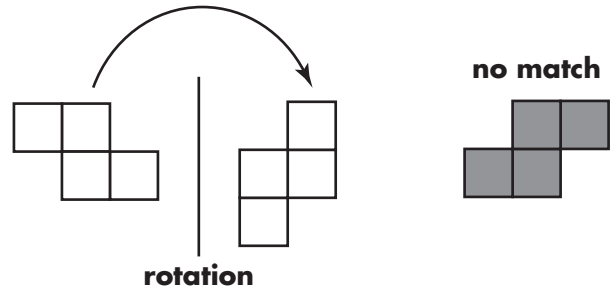


Problem Solving: Use Objects

A tetramino is an arrangement of four identical squares in a plane. The squares are attached to each other edge to edge. A unique tetramino cannot be matched to another by rotating or reflecting. Use square tiles to show if the two figures are unique.



1. Construct the two figures using tiles.
2. Construct a rotation of the first figure. Try other rotations.
3. Check for a match. *Think: If figures match, they are not unique. If figures do not match, figures may be unique.*
4. Construct a reflection of the first figure.
5. Repeat Step 3.
6. Write your answer in a complete sentence.



The two figures are not unique, because they match by reflection.

1. Show if these two figures are unique tetraminoes. Explain your answer.

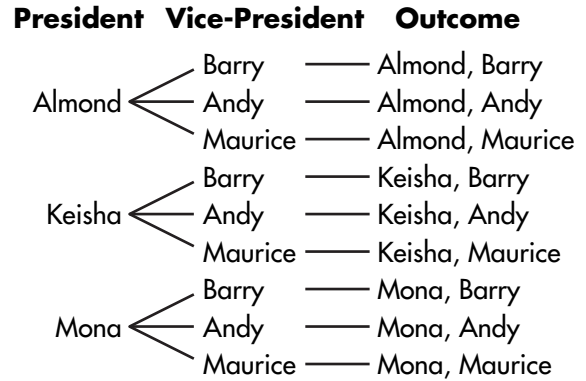


The two figures do not match either by reflection or rotation. They are unique tetraminoes.

Outcomes

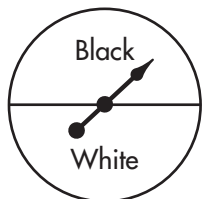
Almond, Keisha, and Mona are running for student council president. Barry, Andy, and Maurice are running for vice-president. Each student has an equal chance of being elected.

You can use a tree diagram to find all the possible outcomes. The set of all possible outcomes is called the sample space.

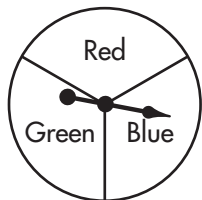


There are 9 possible outcomes in the sample space.

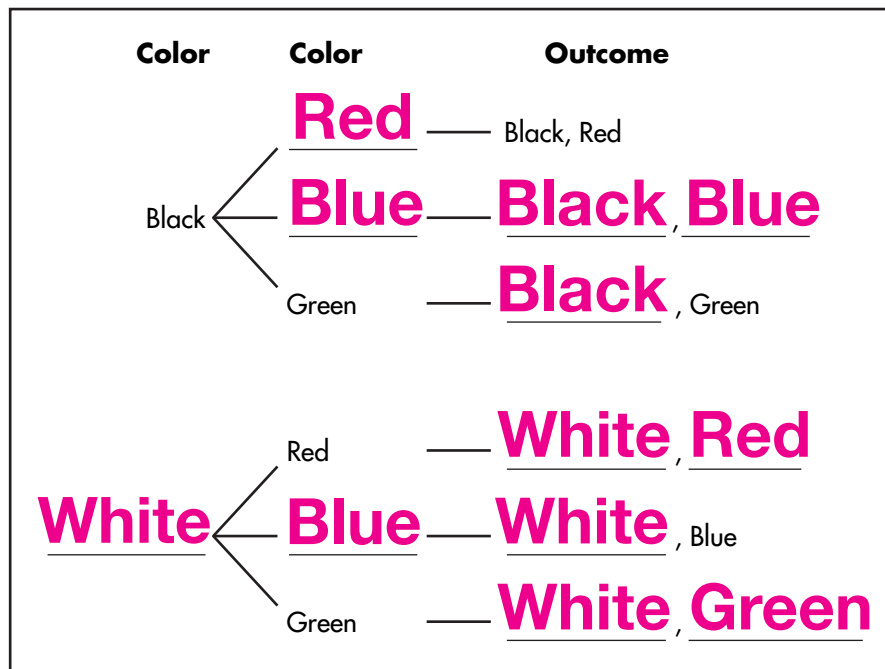
- Complete the tree diagram to show the possible outcomes when Spinner A and Spinner B are spun.



Spinner A



Spinner B



- How many times does the outcome black/green occur in the tree diagram?

1 time

Writing Probability as a Fraction

The probability of an event is a number that describes the chance that an event will occur. Probability can be expressed as a fraction.

$$\text{Probability of an event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

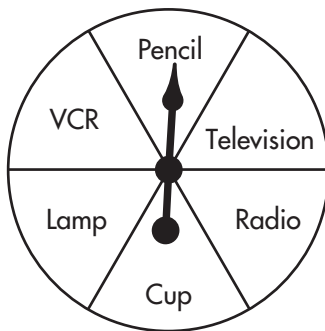
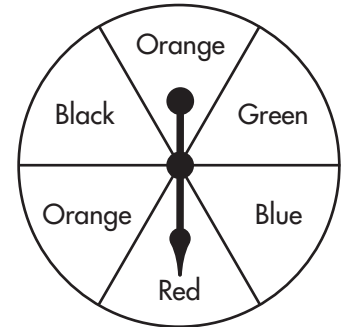
If Felice spins the spinner, what is the probability of landing on Orange?

There are 6 possible outcomes (spaces) and 2 favorable outcomes (Orange spaces).

$$\text{Probability (landing on Orange)} = \frac{\text{number of Orange spaces}}{\text{number of spaces}} = \frac{2}{6}$$

The probability of landing on Orange is $\frac{2}{6}$ or $\frac{1}{3}$.

The probability of landing on Orange can be written as $P(\text{Orange}) = \frac{1}{3}$.



1. Find $P(\text{object that does not use electricity})$

$\frac{2}{6}$ _____

2. Find $P(\text{object that uses electricity})$

$\frac{4}{6}$ _____

3. Find $P(\text{object used for writing})$

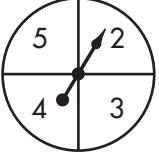
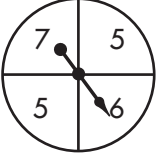
$\frac{1}{6}$ _____

4. **Explain It** What does $P(\text{radio})$ mean?

The probability of the pointer stopping on the segment marked *radio* in one spin.

Experiments and Predictions

You can use an experiment to predict outcomes.

 <p>Spinner A</p> <p>Compare the chances of spinning an even number and spinning an odd number.</p>	<p>Event: Spinning an even number Favorable outcomes: 2, 4 2 out of 4 possible outcomes are favorable, so in 2 out of 4 spins, you can expect an even number.</p>	<p>Event: Spinning an odd number Favorable outcomes: 3, 5 2 out of 4 possible outcomes are favorable, so in 2 out of 4 spins, you can expect an odd number.</p>	<p>Spinning an even number and spinning an odd number are equally likely events.</p>
 <p>Spinner B</p> <p>Compare the chances of spinning an even number and spinning an odd number.</p>	<p>Event: Spinning an even number Favorable outcomes: 6 1 out of 4 possible outcomes are favorable, so in 1 out of 4 spins, you can expect an even number.</p>	<p>Event: Spinning an odd number Favorable outcomes: 5, 5, 7 3 out of 4 possible outcomes are favorable, so in 3 out of 4 spins, you can expect an odd number.</p>	<p>Spinning an even number is less likely than spinning an odd number. Spinning an odd number is more likely than spinning an even number.</p>

Think about tossing a standard number cube.



1. What are the possible outcome numbers? Are the numbers equally likely? Explain.

The possible outcome numbers are 1, 2, 3, 4, 5, and 6. They are equally likely because there is one of each on the six faces of the number cube.

2. **Number Sense** In 6 tosses, how many times would you expect to toss the number 6?

1 time

Problem Solving: Solve a Simpler Problem

A farmer has 4 garden plots in a row. He wants to build paths connecting each pair of plots. He also doesn't want the paths to cross. How many paths must he build?

Read and Understand

What do you know?

There are four plots in a row.

Each pair of plots must be connected by a path.

The paths cannot cross.

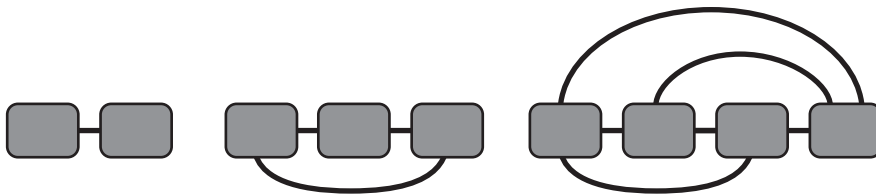
What are you trying to find?

The total number of paths

Plan and Solve

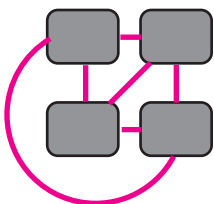
Solve a simpler problem.

1. Construct a picture of two garden plots as shown below.
2. Then draw another picture of three plots. Look for a pattern for the design and number of paths.
3. Construct a diagram of the four plots.



A farmer has four garden plots arranged in a square. Look at the following diagrams. Use them to help answer this question. Can the four garden plots be connected by six paths that do not cross?

yes



Credits

Pages I-III Taken from *Scott Foresman-Addison Wesley Mathematics, Grade 5: Review From Last Year Masters* by Scott Foresman-Addison Wesley

Pages 21-300 Taken from *enVisionMATH™ Teacher Resource Masters, Topics 1-20, Grade 5* by Scott Foresman-Addison Wesley