NAME ______________________

Date __________ Period__________

SYLLABUS
ALGEBRA 2 H

Unit 13: Periodic Functions and Trig

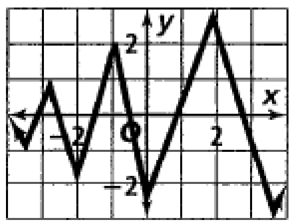
Day	Topic
0	Special Right Triangles and Periodic Function
1	Special Right Triangles Standard Position Coterminal Angles
2	Unit Circle Cosine & Sine (x, y & r)
3	Radians Converting Cosine & Sine of Special Angles
4	Drill days 2-3 Length of Intercepted arc
5	
6	Interpreting the Sine Function
7	Graphing the Sine Function
8	The Cosine Function
9	The Tangent Function
10	Calculators Reciprocal Trig Functions

Unit 12: Standard Position and Coterminal Angles (D1)

Warm Up:

Determine whether each function *is* or *is not* periodic. If it is, find the period.

1.



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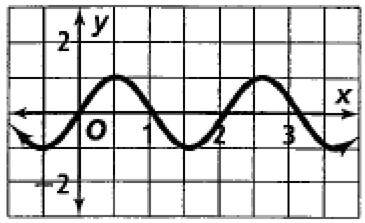
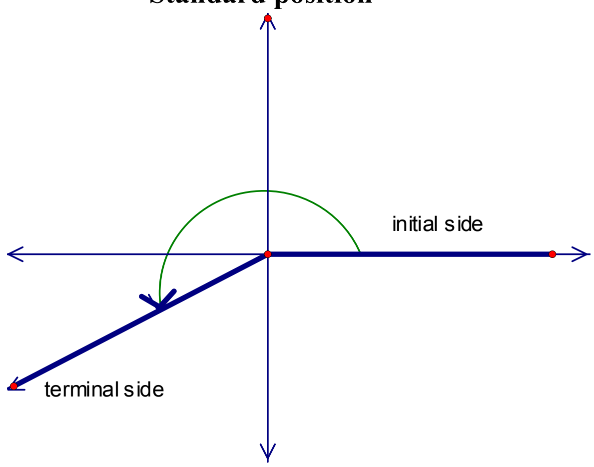


An angle is in standard position when the vertex is at the origin and one ray is on the positive x-axis. The ray on the x-axis is the initial side of the angle; the other ray is the terminal side of the angle.

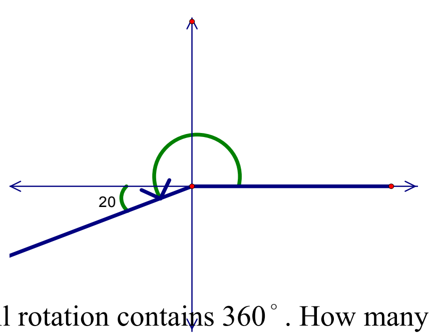
Standard position



To measure an angle in standard position, find the amount of rotation from the initial side to the terminal side.

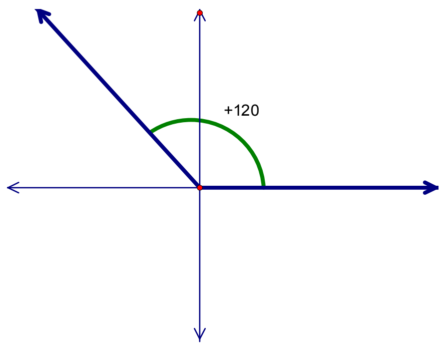
Examples:

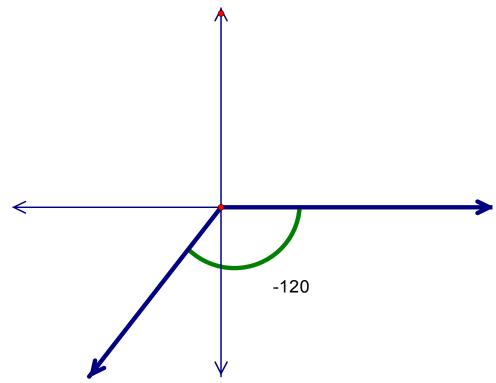
1. Find the measure of the angle.



2. One full rotation contains 360°. How many degrees are in one quarter of a rotation? In one half of a rotation? In three quarters of a rotation?

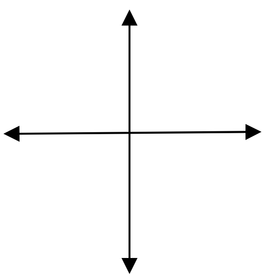
The measure of an angle is positive when the rotation from the initial side to the terminal side is in the counterclockwise direction. The measure is negative when the rotation is clockwise.



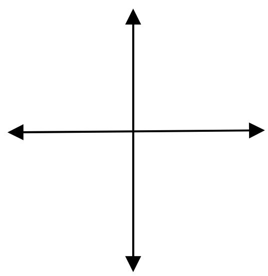


Sketch each angle in standard position.

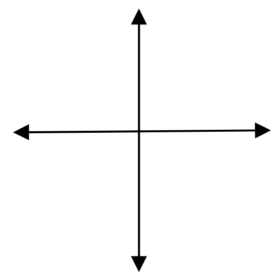
3. 36°



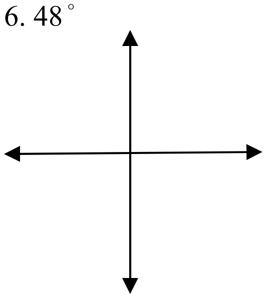
4. 315°



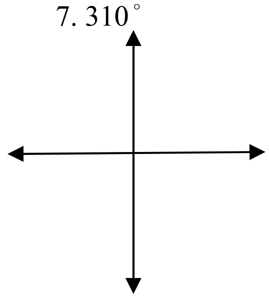
5. -150°



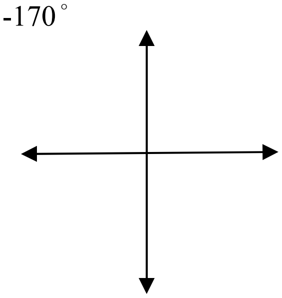
6. 48°



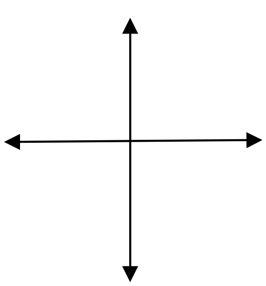
7. 310°



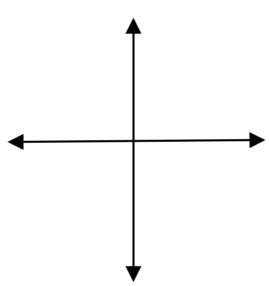
8. -170°



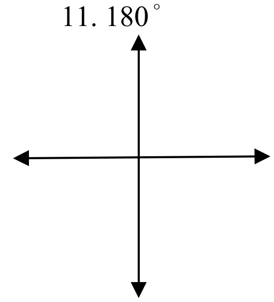
9. 1000°



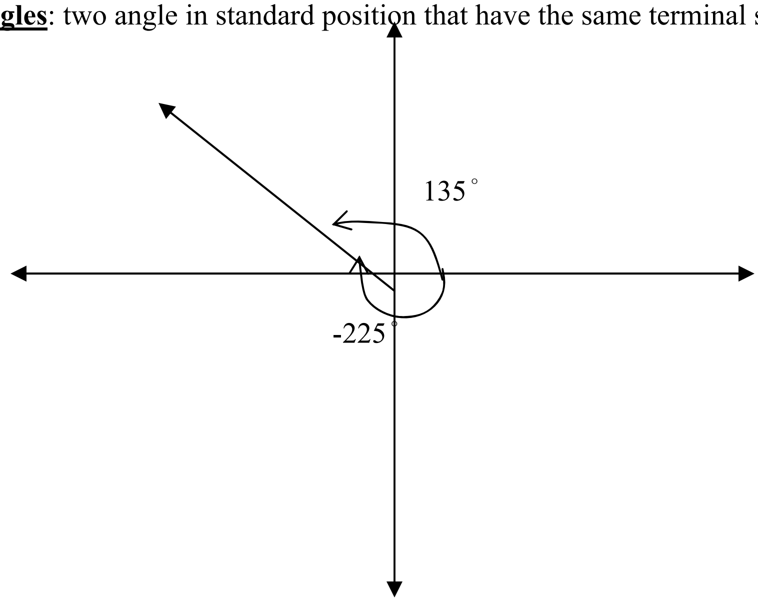
10. -1000°



11. 180°



Coterminal angles: two angle in standard position that have the same terminal side



Find the measure of an angle between 0° and 360° coterminal with each given angle.

12. -100°

13. 372°

14. -20°

15. -60°

16. -15°

17. 421°

18. -225°

19. 482°

20. 409°

21. -145°

22. 484°

23. -38°

Closure:

Sketch each angle in standard position.

1. $-150°$

2. 100°

Find the measure of an angle between 0° and 360° coterminal with each given angle.

3. 409°

4. $-225°$

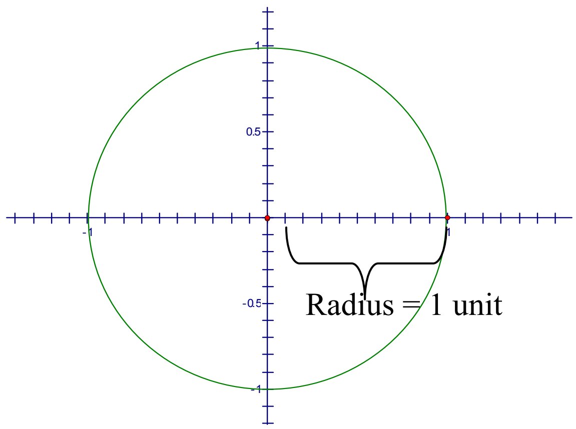
Unit 12: The Unit Circle (D2)

Warm Up:

Find the measure of an angle between 0° and 360° coterminal with each given angle.

1. $-170°$ 2. $387°$ 3. $-127°$

A **unit circle** has a radius of 1 unit and its center at the origin of the coordinate plane. Points on the unit circle are related to periodic functions (you'll read more about periodic functions at a later date)



Sine and Cosine of an Angle

Suppose an angle is standard position has measure θ. The **cosine of** θ ($\cos\theta$) is the **x-coordinate** of the point at which the terminal side of the angle intersects the unit circle. The **sine of** θ ($\sin\ \theta$) is the **y-coordinate**.

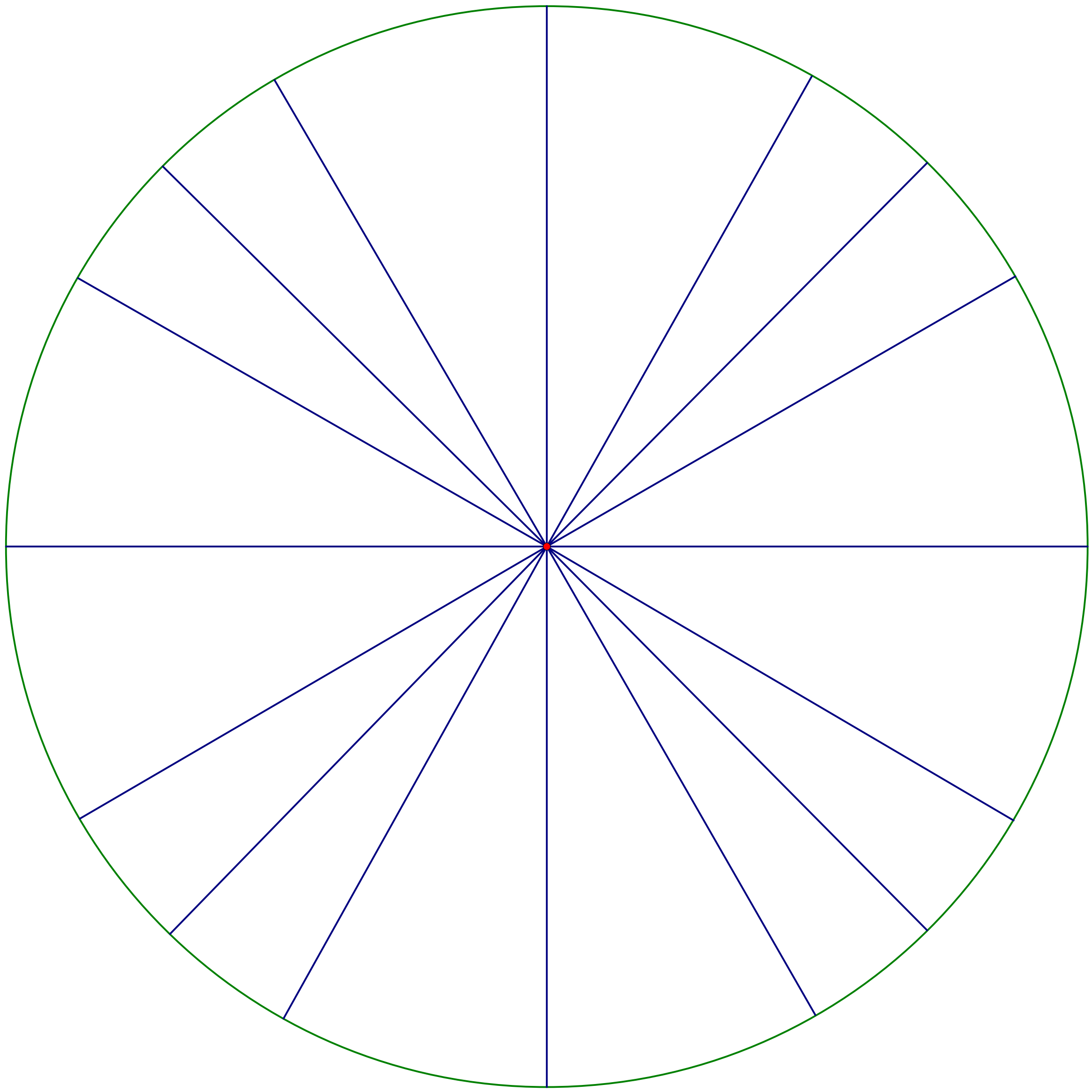
Example:

1. Find the cosine and sine of each angle. (see unit circle handout)

In summary:

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$



Unit 12: Radian Measures (D3)

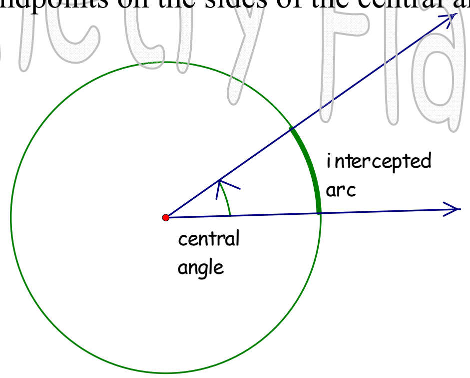
Warm Up

Find the exact coordinates of the point where the terminal side of the given angle intersects the unit circle. Then find the decimal equivalents. Round your answers to the nearest hundredth.

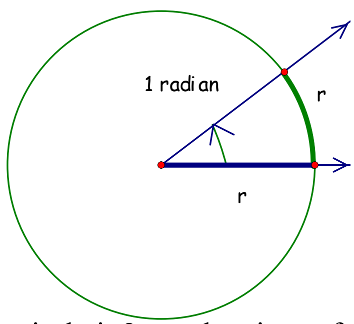
1. $225°$ 2. $-45°$ 3. $120°$

Geometry Flashback

A **central angle** of a circle is an angle with a vertex at the center of a circle. An **intercepted arc** is the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle



When a central angle intercepts an arc that has the same length as the radius of the circle, the measure of the angle is defined to be one **radian.**
Radians, like degrees, measure the amount of rotation from the initial side to the terminal side.



Because the circumference of a circle is $2\pi r$, the circumference of the unit circle is 2π meaning there are 2π radians in the unit circle. . Since $2\pi = 360^\circ$, and therefore π radians $= 180^\circ$, you can use the proportion

$$\frac{d^\circ}{180^\circ} = \frac{r \text{ radians}}{\pi \text{ radians}}$$ to convert between degrees and radians.

Examples:
1. Find the radian measure of each angle

a. 60° b. 85° c. 45°

2. Find the degree measure

a. $\frac{5\pi}{2}$ b. $\frac{-3\pi}{6}$ c. $\frac{13\pi}{6}$

Basically, when solving the proportion, you are cross-multiplying. So we can shorten the proportion to:

To convert degrees to radians $\frac{\pi \text{ radians}}{180^\circ}$ To convert radians to degrees $\frac{180^\circ}{\pi \text{ radians}}$

Convert:

1. 95° 2. $\frac{2\pi}{3}$

Closure:

Write each measure in radians. Express your answer in terms of π.

1. 45° **2.** 90° **3.** 30° **4.** 150° **5.** 180°

Write each measure in degrees. Round your answer to the nearest degree, if necessary.

16. π **17.** 2π **18.** $\frac{5\pi}{6}$ **19.** $\frac{3\pi}{4}$ **20.** $\frac{3\pi}{2}$

Unit 12: Review and Arc Length (D4)

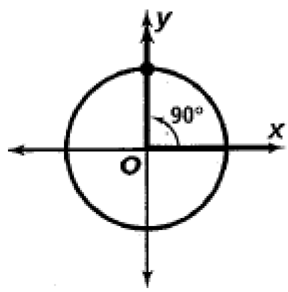
Find the exact coordinates of the point where the terminal side of the given angle intersects the unit circle. Then find the decimal equivalents. Round your answers to the nearest hundredth.

51. 45° **52.** 225° **53.** −225° **54.** −45° **55.** 330°

56. −330° **57.** 150° **58.** −150° **59.** 300° **60.** −300°

61. 240° **62.** 120° **63.** −90° **64.** 360° **65.** 720°

66.



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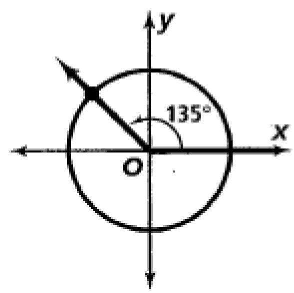

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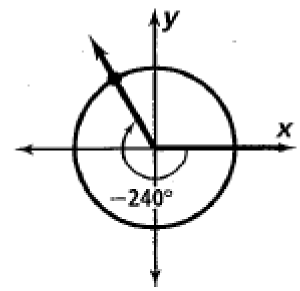
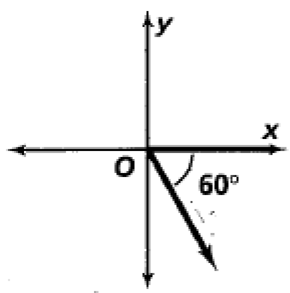


Find the measure of each angle in standard position.

69.



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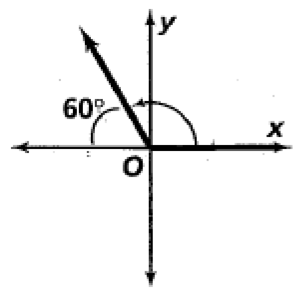

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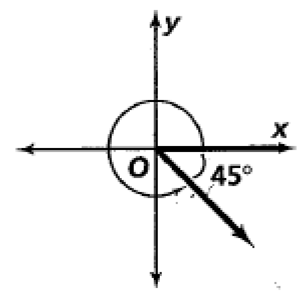
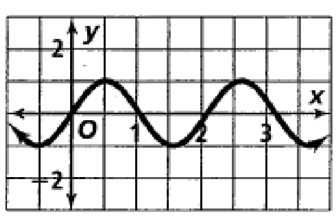


Find the period and amplitude of each periodic function.

7.



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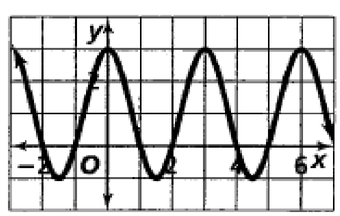

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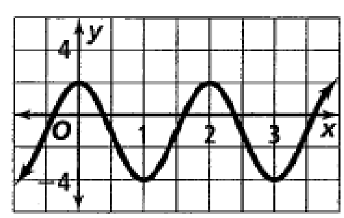

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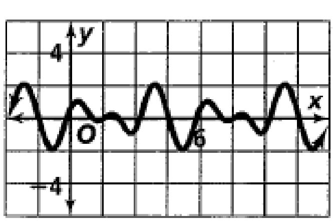

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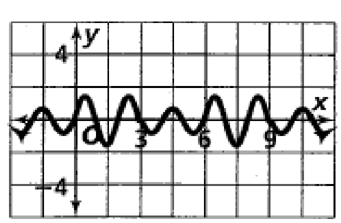

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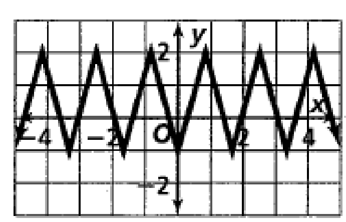


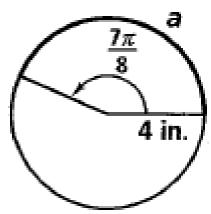
Arc Length: You can find the length of an intercepted arc by using the proportion

________________ = ________________ OR

For a circle of radius r and a central angle of measure θ **(in radians),** the length s of the intercepted arc is $s = \theta r$

Use each circle to find the length of the indicated arc. Round your answer to the nearest tenth.

39.



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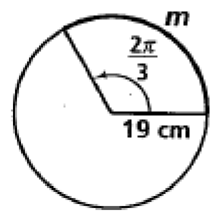

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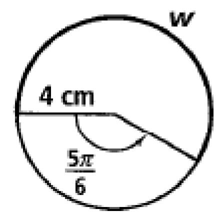

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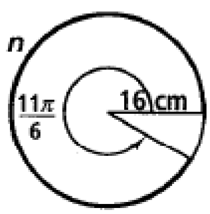

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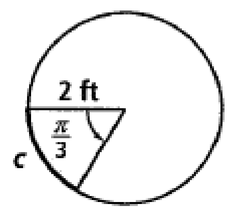

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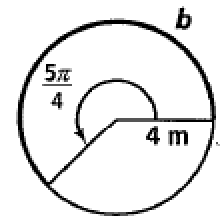


Unit 12: Interpreting the Sine Curve (D6)

Terms:

a. Periodic functions repeat a ___________________ at regular intervals.

b. The ________________ determines the horizontal length of one cycle.

c. The ___________________ is half the distance from the minimum and maximum values of the function in the y direction.

Investigating with the Graphing Calculator:

1. Set your mode to "Radian".
2. Graph the "parent" function $y = \sin(x)$
3. Use the Zoom Trig (#7) window

Using your calculator's picture and the following key points, sketch the graph to the right. Draw the same graph that is on your screen and include all of the key points in your sketch.

$\sin(-2\pi) =$

$\sin\left(-\frac{3\pi}{2}\right) =$

$\sin(-\pi) =$

$\sin\left(-\frac{\pi}{2}\right)$

$\sin 0 =$

$\sin\frac{\pi}{2} =$

$\sin\pi =$

$\sin\frac{3\pi}{2} =$

$\sin 2\pi =$

A. How many cycles are drawn on the graph? __________

B. What is the length (in radians) of one period? _________

C. What is the amplitude of the graph? __________

D. What do you think would change on the graph if you change the equation to **y = 2sin(x)?**
Test your thought with the calculator.

Changing the Period of the Sine Function:

Now change the function to **y = sin(2x)**

Draw that graph here, and include
key points on the axis.

Describe in words how the graph changed.

What is the new period of the function?

If y = sin(bx), where b is a real number, explain how to find the period of the sine functions in terms of b.

Sketching on your own:

Without your calculator, draw two full cycles of each of the following graphs.

1. y = 3sin(x)

Amplitude = ________

Period = _________

2. $y = \sin\left(\dfrac{x}{2}\right)$

Amplitude = ________

Period = _________

Unit 12: Graphing the Sine Curve (D7)

Definitions to become familiar with given $y = a\sin b\theta$, *with* $a \neq 0, b > 0$, *and* θ *in radians*

1. **cycle:**

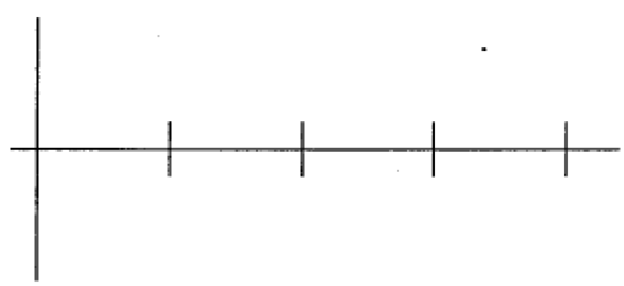
2. **period:**

3. **amplitude**

Example:

1. Sketch one cycle of $y = \frac{1}{2}\sin 2\theta$

2. Sketch one cycle of $y = 1.5\sin 2\theta$



Unit 12: Graphing the Sine Curve (D7: homework assignment)

Use the axis below to draw each graph. Include the 5 key points in radian measure.

Sketch one cycle of each sine curve. Assume $a > 0$. Write an equation for each graph.

16. amplitude 2, period $\frac{2\pi}{3}$

17. amplitude $\frac{1}{3}$, period π

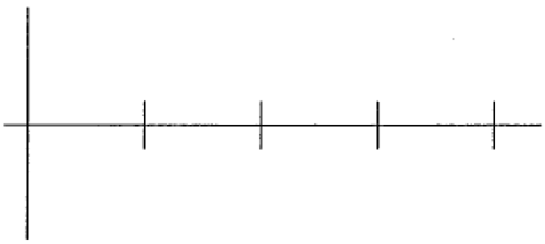
18. amplitude 4, period 4π

19. amplitude 3, period 2π

20. amplitude 1, period 2

21. amplitude 1.5, period 3

16.



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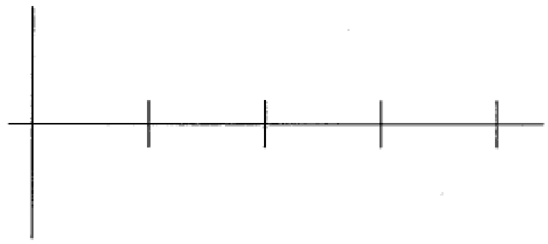

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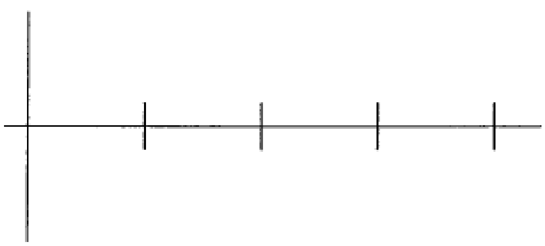

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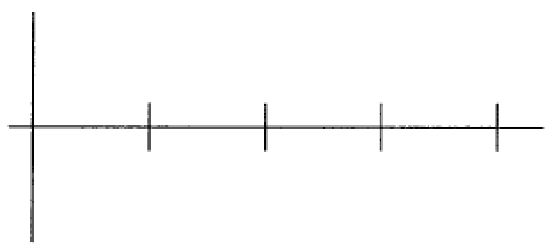

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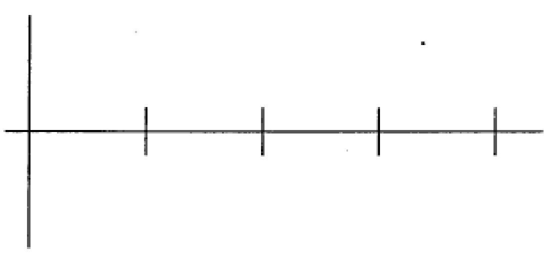

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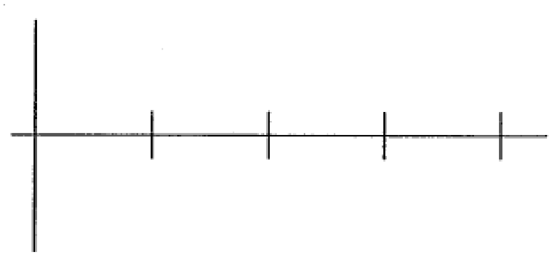


Sketch one cycle of the graph of each sine function.

22. $y = 2 \sin \theta$

23. $y = \sin 3\theta$

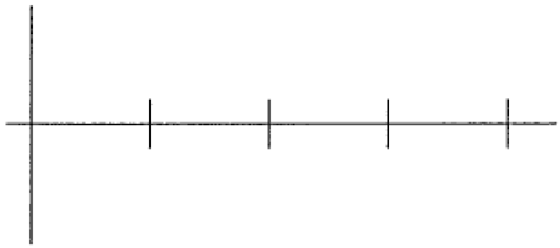
24. $y = -\sin \frac{\pi}{2}\theta$

25. $y = 2 \sin \pi\theta$

26. $y = 4 \sin \frac{1}{2}\theta$

27. $y = -4 \sin \frac{1}{2}\theta$

22.



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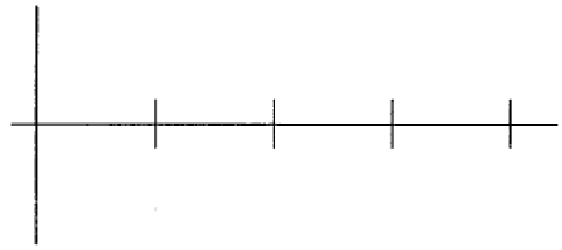

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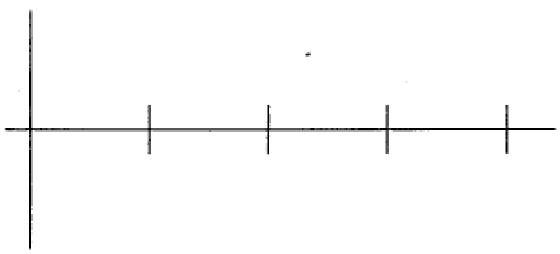

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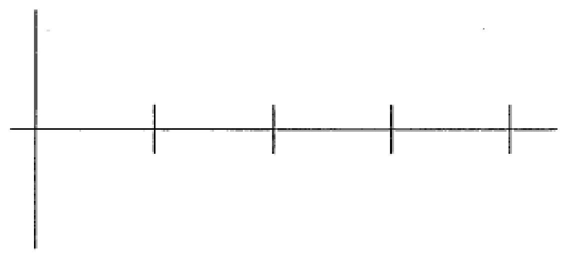

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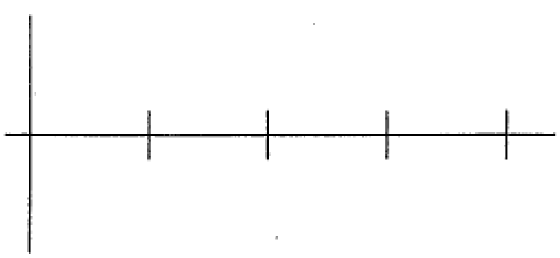

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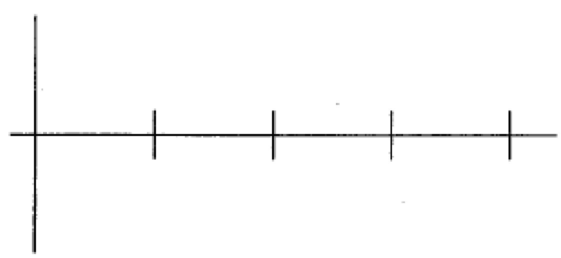
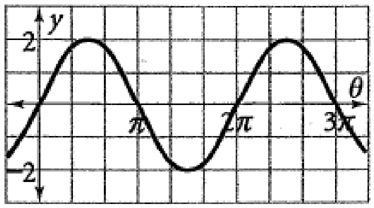


Find the period of each sine curve. Then write an equation for each sine function.

28.


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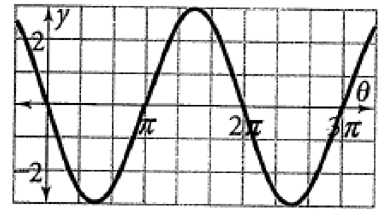

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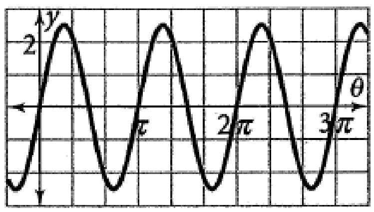

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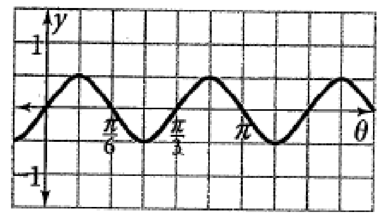

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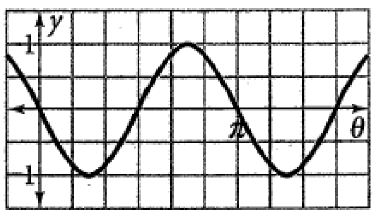

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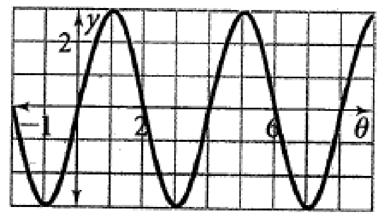


41. a. Graph the functions $y = 3\sin\theta$ and $y = -3\sin\theta$ on the same screen. How are the two graphs related?

b. Graph the functions $y = \sin 3\theta$ and $y = \sin(-3\theta)$ on the same screen. How are the two graphs related?

c. **Critical Thinking** How does the graph of $y = a\sin b\theta$ change when a is replaced with its opposite? How does the graph change when b is replaced with its opposite?

42. Use the formula period $= \frac{2\pi}{b}$ to find the period of each sine function.

a. $y = 1.5\sin 2\theta$

b. $y = 3\sin\frac{\pi}{2}\theta$

Unit 12: The Cosine Function (D8)

Investigating with the Graphing Calculator:

1. Set your mode to "Radian".
2. Graph the "parent" function y = $\cos\theta$
3. Use the Zoom Trig (#7) window

Using your calculator's picture and the following key points, sketch the graph to the right. Draw the same graph that is on your screen and include all of the key points in your sketch.

$\cos(-2\pi) =$

$\cos\left(-\frac{3\pi}{2}\right) =$

$\cos(-\pi) =$

$\cos\left(-\frac{\pi}{2}\right)$

$\cos 0 =$

$\cos\frac{\pi}{2} =$

$\cos\pi =$

$\cos\frac{3\pi}{2} =$

$\cos 2\pi =$

A. How many cycles are drawn on the graph? _________

B. What is the length (in radians) of one period? ________

C. What is the amplitude of the graph? _________

D. What do you think would change on the graph if you change the equation to **y = 2cos(x)?**
Test your thought with the calculator.

Changing the Period of the Sine Function:

Now change the function to **y = cos(3x)**

Draw that graph here, and include key points on the axis.

Describe in words how the graph changed.

What is the new period of the function?

If y = cos(bx), where b is a real number, explain how to find the period of the sine functions in terms of b.

Sketching on your own:

Without your calculator, draw two full cycles of each of the following graphs.

1. y = -4cos(x)

Amplitude = ________

Period = _________

2. $y = \cos\left(\frac{x}{3}\right)$

Amplitude = ________

Period = _________

Solving Trig Equations: Solve each of the following equations from 0 to 2π. Round to the nearest hundredth if needed.

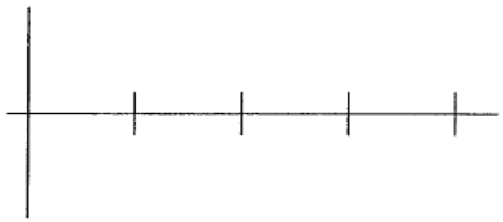
1. sin x = .5
2. cos 2x = -.5
3. sin x = 0
4. $\cos .5x = \frac{\sqrt{2}}{2}$
5. sin3x = .2
6. $\cos \pi x = 1$

Sketch the graph of each function in the interval from 0 to 2π.

5. $y = \cos 2\theta$ **6.** $y = -3\cos\theta$ **7.** $y = -\cos 3t$ **8.** $y = \cos\frac{\pi}{2}\theta$ **9.** $y = -\cos\pi\theta$

Sketch the graphs for 5 – 9 here. Use the 5 key points.

5.



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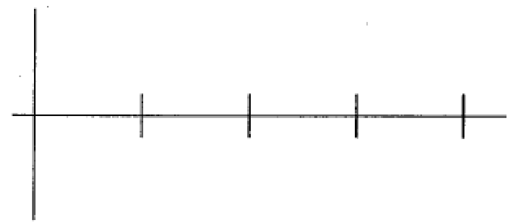

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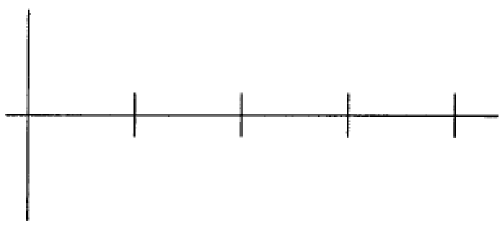

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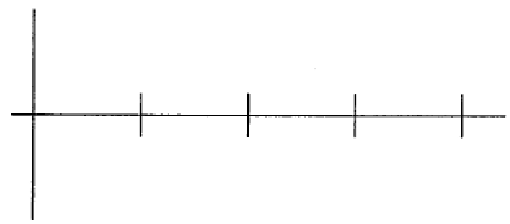

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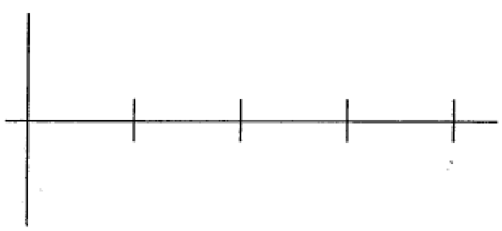
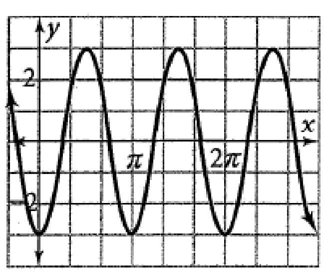


Write a cosine function for each description. Assume that $a > 0$.

10. amplitude 2, period π **11.** amplitude $\frac{\pi}{2}$, period 3 **12.** amplitude π, period 2

Write an equation of a cosine function for each graph.

13.



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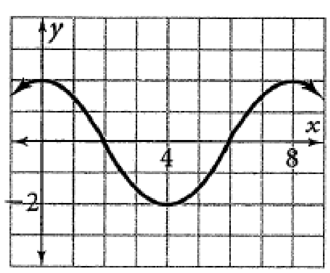


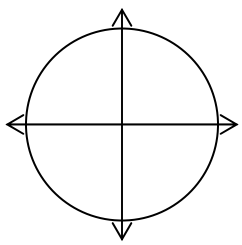
15. Wave Motion Suppose 8-in. waves pass every 3 s. Write an equation that models the height of a water molecule as it moves from crest to crest.

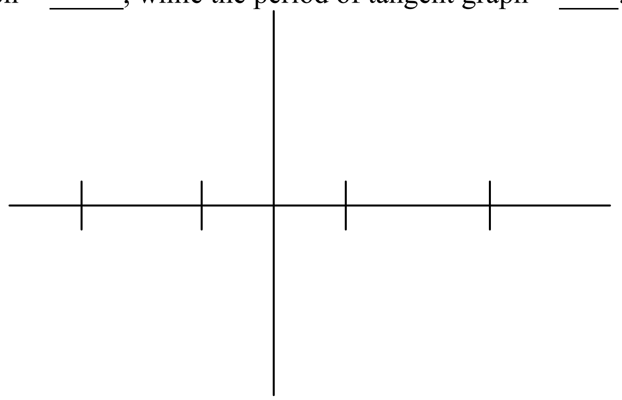
Unit 12: The Tangent Function (D9)

Tangent: the y-coordinate of the point where the line containing the terminal side of the angle intersects the tangent line x=1.

$$\text{Tan } \theta = \frac{\sin\theta}{\cos\theta}$$

The period of a sine and cosine graph = ______, while the period of tangent graph = _____.

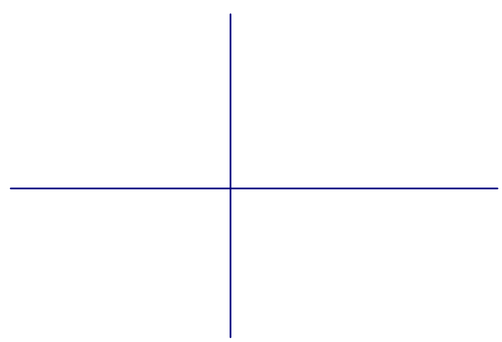




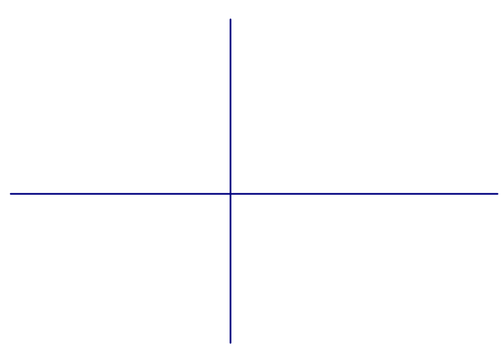
Tangent facts:
$y = a\tan b\theta,\ with\ b > 0\ and\ \theta\ in\ radians$

- $\frac{\pi}{2}$ is the period of the function
- One cycle occurs in the interval from $-\frac{\pi}{2b}\ to\ \frac{\pi}{2b}$
- There are vertical asymptotes at each end of the cycle

Examples:
1. Sketch a cycle for the graph of $y = \tan\pi\theta$



2. Sketch a cycle for the graph of $y = \tan 3\theta$



3. Evaluate the following:

a. $\tan\dfrac{\pi}{2}$

b. $\tan\left(-\dfrac{3\pi}{4}\right)$

c. $\tan\left(-\dfrac{\pi}{4}\right)$

d. $\tan\left(\dfrac{3\pi}{2}\right)$

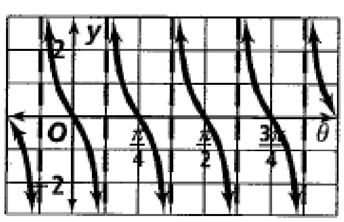
Sketch two cycles of the graph of each function.

11. $y = 2\tan\theta$

12. $y = -\tan\theta$

Identify the period of each tangent function.

28.



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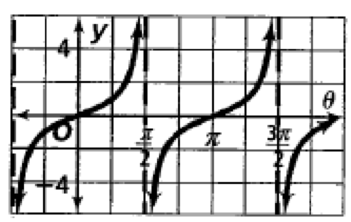

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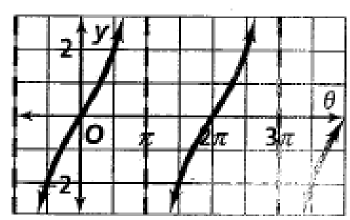

Closure: How is the graph of the tangent function similar to the graph of the sine function and cosine function? How are they different?

Unit 12: Reciprocal Trig Functions (D10)

The following are reciprocal trig functions:

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

Degree θ	Radian θ	Sin(θ)	Cos(θ)	Tan(θ)	Csc(θ)	Sec(θ)	Cot(θ)
0°							
30°							
45°							
60°							
90°							
120°							
135°							
150°							
180°							
210°							
225°							
240°							
270°							
300°							
315°							
330°							
360°							