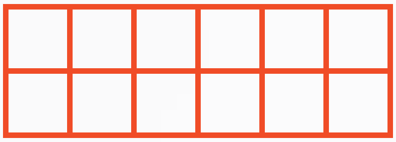
Exploring Prime Factors

By Dr. Alice C. Raganas

Here are 12 squares . . .

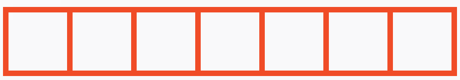


Use the 12 squares to form rectangles of different shapes. How many different forms of rectangle can you make?

Did you come up with these three different rectangles?

A 3 by 4 or 4 by 3, a 2 by 6 or 6 by 2, and a 1 by 12 or 12 by 1.

Now, form as many different rectangles as you can with seven of your squares.



Have you noticed that just one rectangle is formed? A 1 by 7 or 7 by 1 rectangle.

12 is a composite number with factors 1, 2, 3, 4, 6, and 12.

7 is a prime number with factors 1 and 7.

A number with more than two factors is a *composite number*.
A number with exactly two factors is a *prime number*.

Prime Less than 100				
2	13	31	53	76
3	17	37	59	79
5	19	41	61	83
7	23	43	67	89
11	29	47	71	97

To factor a number, you need to test only prime numbers to find prime factors. From these, all the whole numbers, between 1 and 100, are given in the table above.

To find the factors of 42 in this way, we test the primes in succession using either of the methods below:

(Inverted Short Division)

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

(Factor Tree)

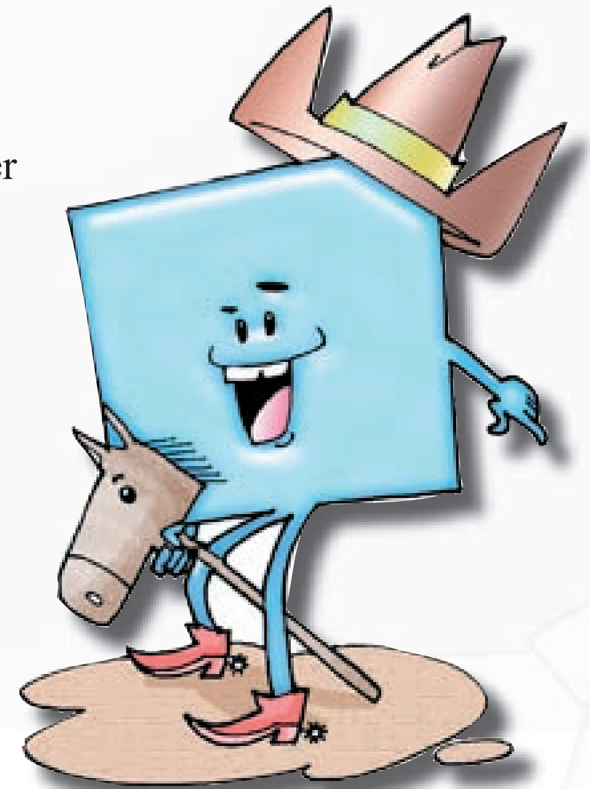
$$\begin{array}{ccc} & 42 & \\ 2 & & 21 \\ & 3 & \quad 7 \end{array}$$

$\rightarrow 42 = 2 \times 3 \times 7$

We find the remaining whole number factors of 42 by taking all possible products of the prime factors:

$$2 \times 3 = 6$$
$$2 \times 7 = 14$$
$$3 \times 7 = 21$$
$$2 \times 3 \times 7 = 42$$

Including 1, the whole-number factors of 42 are 1, 2, 3, 4, 6, 7, 14, 21, and 42. The statement $42 = 2 \times 3 \times 7$ gives a prime factorization of 42.



Try This!

A. State whether each number is prime or composite.

1.	9	________	6.	23	________
2.	57	________	7.	19	________
3.	21	________	8.	17	________
4.	12	________	9.	33	________
5.	10	________	10.	11	________

B. Write the prime factorization of the given number.

1.	14 =	________	6.	51 =	________
2.	26 =	________	7.	22 =	________
3.	72 =	________	8.	34 =	________
4.	18 =	________	9.	144 =	________
5.	39 =	________	10.	135 =	________

C. Express each number as the sum of two primes in as many ways as possible.

Example: $16 = 3 + 13$ or $16 = 5 + 11$

1.	8 =	________	6.	18 =	________
2.	22 =	________	7.	30 =	________
3.	10 =	________	8.	20 =	________
4.	24 =	________	9.	32 =	________
5.	12 =	________	10.	28 =	________

Answer

A.
1. composite
2. composite
3. composite
4. composite
5. composite
6. prime
7. prime
8. prime
9. composite
10. prime

B.
1. 2×7
2. 13×2
3. $2 \times 2 \times 2 \times 3 \times 3$
4. $2 \times 3 \times 3$
5. 3×13
6. 3×17
7. 2×11
8. 2×17
9. $2 \times 2 \times 2 \times 2 \times 3 \times 3$
10. $3 \times 3 \times 3 \times 5$

C.
1. $5 + 3$
2. $11 + 11$
3. $5 + 5$
4. $11 + 13$
5. $7 + 5$
6. $7 + 11$
7. $13 + 17$
8. $13 + 7$
9. $19 + 13$
10. $11 + 17$