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Coordinate analysis of Ptolemy's Geographia Europe Tabula X with respect to geographic graticule and point positioning in a Ptolemaic late 15th century map

Keywords: Ptolemy's *Geographia*; Ptolemy's coordinates; ptolemy maps; history of maps; map comparison; cartometry; optimal fitting

Summary

Claudius Ptolemy, in his *Geographia,* gives a list of geographic coordinates of spherical longitude and latitude of almost ten thousand point locations on the earth surface, as they were known at his times. The list is organized in *Tabulae* which correspond to specific regions of the three known continents at that time, Africa, Asia and Europe. One of these Tabulae is the 10th of Europe (Tabula X) in which almost 500 sets of spherical longitude – latitude coordinates cover the major part of the actual territory of Greece. Having, in one hand, the coordinates of the points and, on the other, relevant map depictions of Tabula X, a computational methodology was developed in order to test the consistency of the coordinates with respect to their point-wise and graticule-wise representation on maps, which are derived on the basis of Ptolemy's *Geographia*. In our study, the list of Tabula X coordinates, coming from the Donnus Nicolaus Germanus 15th century manuscript were cross-checked with respect to the relevant Nobbe's (1843) and Müller's catalogues (1883) and the Tabula X facsimile map from the 1490 de Turre's printed Rome edition of *Geographia* was transformed in digital form. The two digital files (coordinates vs image) were then brought into a best fitting correspondence using, in a first scheme of comparison, the map graticule of meridians and parallels and in a second, the point positioning as represented on the map. The results show an interesting deviation of the two comparison schemes proving that the positioning of the points on the map do not correspond to the geographic graticule and vice-versa. This result raises the question on the graphic correspondence between the graticule and the point positioning from the technical drafting point of view, in preparing the Ptolemaic maps.

Introduction

The geometric part of Claudius Ptolemy's *Geographia* refers mainly to the list of spherical coordinates, in sets of longitude and latitude, which is given in the text concerning almost ten thousand of point positions on the globe as they were known in the Ptolemy's days. The coordinates, rounded-off in five minutes of arc, in both orthogonal primer directions (parallels and meridians), are grouped according to the continental and regional classification followed by Ptolemy in his *Geographia.* Based on these coordinates, the so-called Ptolemaic maps were designed later, the oldest preserved since the late 13th century, almost ten centuries after *Geographia*. In the process of designing a Ptolemaic map, using the listed coordinates as given in the text, the drawing of a geographic graticule of

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parallels and meridians is apparently preceding, as the *guide*, for the placement, in a second step, of the point coordinates. The process is shown in Fig. 1 and it is actually the regular used, in general, for the graphical illustration of coordinates within a relevant grid or graticule of coordinate-lines (e.g. parallels and meridians).

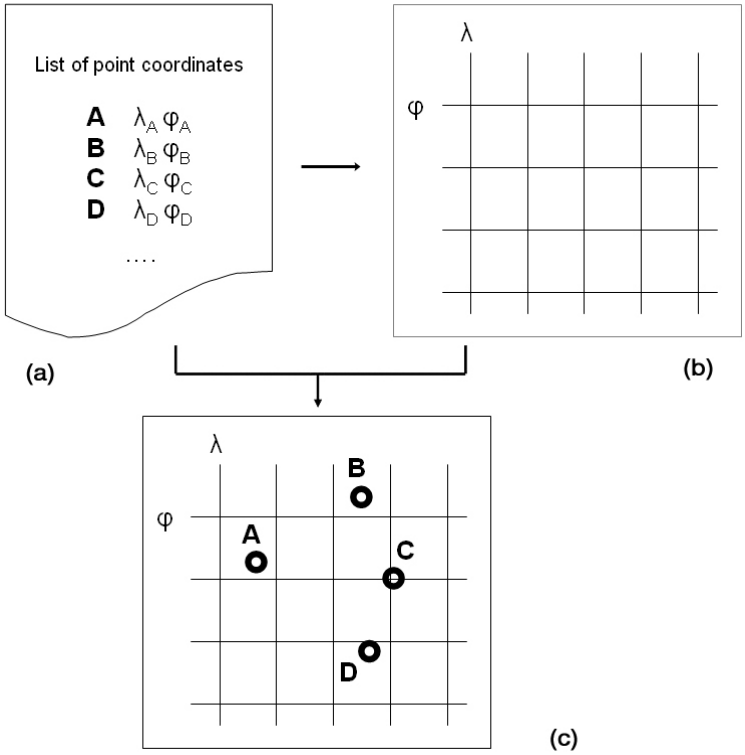

Figure 1. (a) The list of locations A, B, C, … with given longitude and latitude coordinates, (b) the graticule of parallels and meridians drawn with respect to the listed longitudes and latitudes, (c) the graphic placement in the associated graticule of the listed locations A, B, C, …, with given longitude and latitude coordinates.

An interesting question is then raised on the properties of the relation between the given numerical coordinates of positions, as listed in Ptolemy's *Geographia* and the same positions as are graphically depicted in the later derived Ptolemaic maps. A first attempt in answering a similar question is documented in a previous work by one of the co-authors of this paper (Livieratos, 2006: 51-59). His study was focused on a local comparison (island of Crete) of two Ptolemaic maps representing the 'tenth and last' region of Europe in *Geographia,* which covers the major part of the actual territory of Greece. In that work the local part of the maps, representing Crete, were first georeferenced with respect to coordinates given in *Geographia* and with respect to the parallels-meridians graticule (the intersection of the parallels and meridians were taken) and the comparison then followed, first as a point-wise process and second as a graticule-wise process. The results showed an inconformity between the point-wise and the graticule-wise comparison which raise an incertitude on how the coordinates listed in *Geographia* were graphically transferred on the Ptolemaic maps.

In this paper we generalize the issue using all the 530 pairs of longitude and latitude coordinates listed in *Geographia,* under *Decima et Ultima Europe Tabula* (Tabula X)*,* which covers the major part of the actual territory of Greece. The coordinates are coming from three distinct sources. The first is the Donnus Nicolaus Germanus mid-15th century

manuscript of Ptolemy's *Geographia* as given in *Codex Ebnerianus* (Stevenson 1991: 92) which, according to Fischer (1991: 10), served as the basis for the later printed Rome editions, 1478, 1490 (is used in this study), 1507, 1508, Ulm editions, 1482, 1486 and Strasbourg editions, 1513, 1520, 1522, 1525. The second and third sources are two 19th century editions by Nobbe (1843, 1966) and Müller (1883).

The coordinates from the three sources, properly auto- and cross-checked and evaluated, are then projected onto a map with a relevant graticule of parallels and meridians. In this way, three Ptolemaic *reference* maps of Tabula X are designed, which are directly derived from the coordinates, and are then used as the base maps in order to study their deviations from the printed Ptolemaic de Turre's Tabula X map (1490), as far as the consistency of the point placements is concerned, with respect also to the relevant geographic graticule as depicted on de Turre's map.

It is needless to say that, the process developed in this work can be easily generalized and applied to any other manuscript and/or printed Ptolemaic map, transformed in digital raster form.

On the Tabula X coordinates

In this study, longitude and latitude coordinates are taken into consideration, referred to Ptolemy's *Geographia* both to inhabited locations (i.e. towns and cities) and to point-physical features (i.e. debouchments, mountain rises, cape-ends). In Tabula X, this concerns 530 points or 530 pairs of longitude and latitude. As input coordinates are taken those given in the Germanus, Nobbe and Müller lists. All coordinates, in digital form, were independently and mutually checked (Fig. 2) in order to detect discrepancies in the point placement and especially the gross errors.

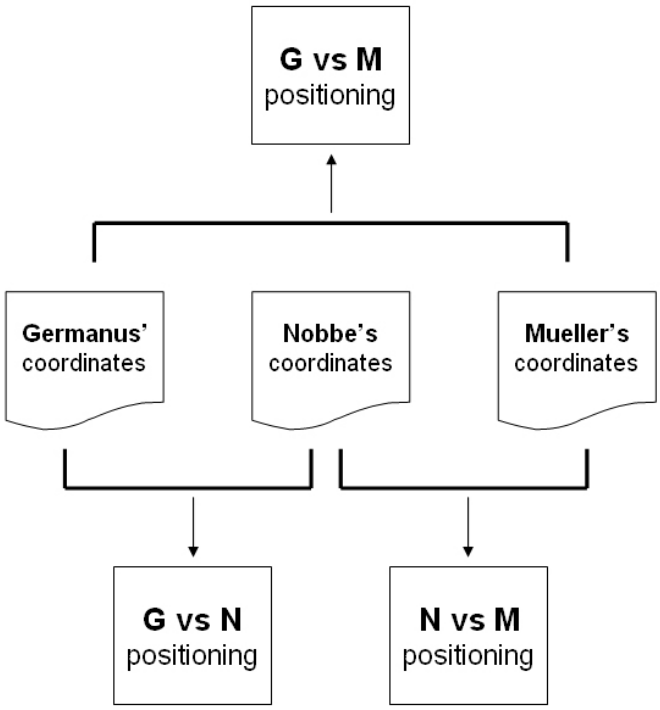

Figure 2. Mutual coordinate check for consistency using Germanus, Nobbe and Müller lists. The outputs are three georeferenced maps of positioning discrepancies, namely 'G vs N', 'N vs M' and 'G vs M'.

From the independent check of Germanus coordinate list, 6 gross errors were detected. These errors and the corrections are given in Tab. 1.

Point site	Germanus		**Corrected**	
	longitude	latitude	longitude	latitude
Central part of Oeta mountains	50° 30′	28° 35′	**50° 30′**	**38° 35′**
Thessaloniki	49° 50′	49° 20′	**49° 50′**	**40° 20′**
Saso island	41° 10′	39° 30′	**44° 10′**	**39° 30′**
Zakynthos island	47° 30′	30° 30′	**47° 30′**	**36° 30′**
Strophades, two islands	47° 20′	30° 30′	**47° 20′**	**36° 00′**

Table 1. The gross errors found from the independent coordinate consistency-check in Germanus list.

In the Nobbe list, two values are given for some single points. These values were checked in order to choose the most appropriate value for each point as it shown in Tab. 2.

Point site	Nobbe		**Corrected**	
	longitude	latitude	longitude	latitude
Mouth of the Lydias river	49° 30′	40° 00′ (10′)	**49° 30′**	**40° 00′**
Oreus	53° 00′ (40′)	38° 20′	**53° 00′**	**38° 20′**
Methone	48° 10′	35° (25′) 5′	**48° 10′**	**35° 25′**
Zarex mountains	51° 00′	35°(50′) 20′	**51° 00′**	**35° 20′**
Herea	49° (40′) 20′	36° 00′	**49° 20′**	**36° 00′**
Cnosos	54° 45′	35° (10′)	**54° 45′**	**35° 10′**
Criumetopon promontory	52° 30′ (45′)	34° 10′	**52° 30′**	**34° 10′**

Table 2. The gross errors and the double values, in minutes of arc, detected from the independent coordinate consistency-check in Nobbe list.

Finally, in the Müller list, the only obvious misprint in point-coordinates is that of 'Thessaloniki' as shown in Tab.3.

Point site	Müller		**Corrected**	
	longitude	latitude	londitude	latitude
Thessaloniki	49° 50′	49° 20′	**49° 50′**	**40° 20′**

Table 3. The gross errors found from the independent coordinate consistency-check in Müller list.

The gross errors and other positioning differences between the three lists of coordinates are depicted in the map of Fig. 3.

The coordinates, from the three lists, were plotted in the same projection, e.g the geographic projection ($y = R\phi, x = R\lambda$), assuming a unit radius reference sphere ($R = 1$) for the earth's model. From the first screening of the point placements, some obvious misprints in point-coordinates were more than evident, like e.g. Thessaloniki, which in both Germanus and Müller lists is given with latitude 49° 20′ instead of 40° 20′.

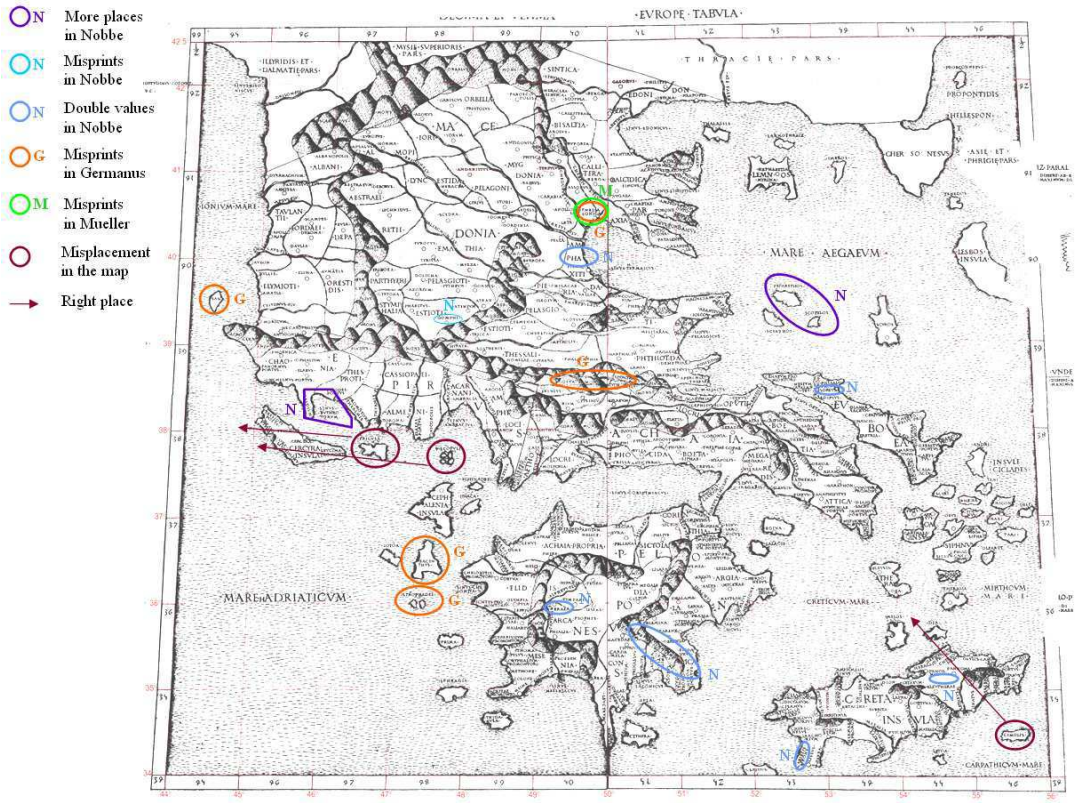

Figure 3. The map of positioning differences in Germanus, Nobbe and Müller.

From the cross comparison of the three coordinate lists, three relevant maps are plotted, depicting the differences in point positioning, as they come out from the comparison, namely 'G vs N', 'N vs M' and 'G vs M', respectively shown in Fig. 4, Fig. 5 and Fig. 6.

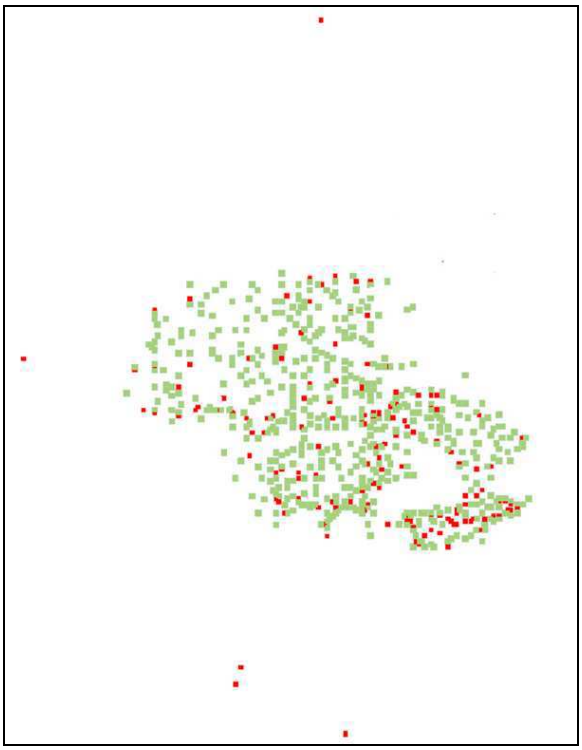

Figure 4. 'G vs N': In red are the different point positioning in Germanus list with respect to Nobbe's positioning (in green)

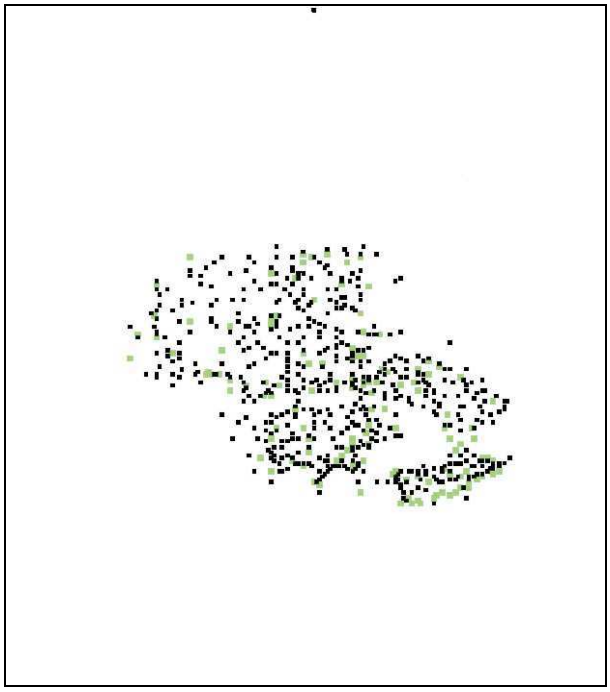

Figure 5. 'N vs M': In green are the different point positioning in Nobbe list with respect to Müller's positioning (in black).

Figure 6. 'G vs M': In red are the different point positioning in Germanus list with respect to Müller's positioning (in black).

Observing the positional discrepancies shown in the above maps (in same scale) we detect a less than 3% gross misplacement in the 'G vs M' case (only 14 points out of 530) while in 'G vs N' and in 'N vs M', the percentage of gross misplacements is almost 10 times higher (about 140 points out of 530). This shows the high degree of deviation in point-positioning inherent in Nobbe list of coordinates as referred in Tabula X.

In Fig. 7, Fig. 8 and Fig. 9 the point-positioning differences are shown, as displacements, for 'G vs M', 'G vs N' and 'N vs M' schemes, analyzed in eight classes of displacement in minutes of arc, namely 5′, 10′, 15′, 20′, 25′, 30′- 40′, 45′- 60′ and > 60′.

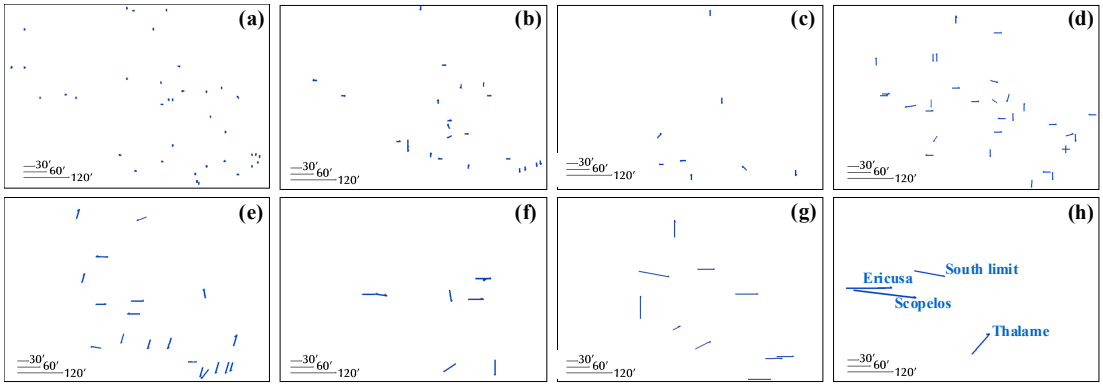

Figure 7. 'G vs N' point-positioning differences in eight classes of displacements in minutes of arc: (a) 5′ (b) 10′ (c) 15′ (d) 20′ (e) 25′ (f) 30′- 40′ (g) 45′- 60′ (h) > 60′

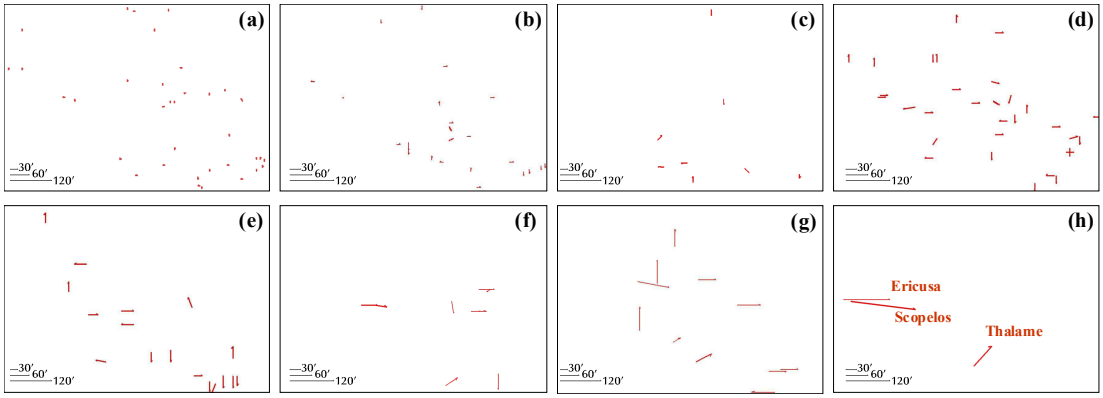

Figure 8. 'N vs M' point-positioning differences in eight classes of displacements in minutes of arc: (a) 5′ (b) 10′ (c) 15′ (d) 20′ (e) 25′ (f) 30′- 40′ (g) 45′- 60′ (h) > 60′

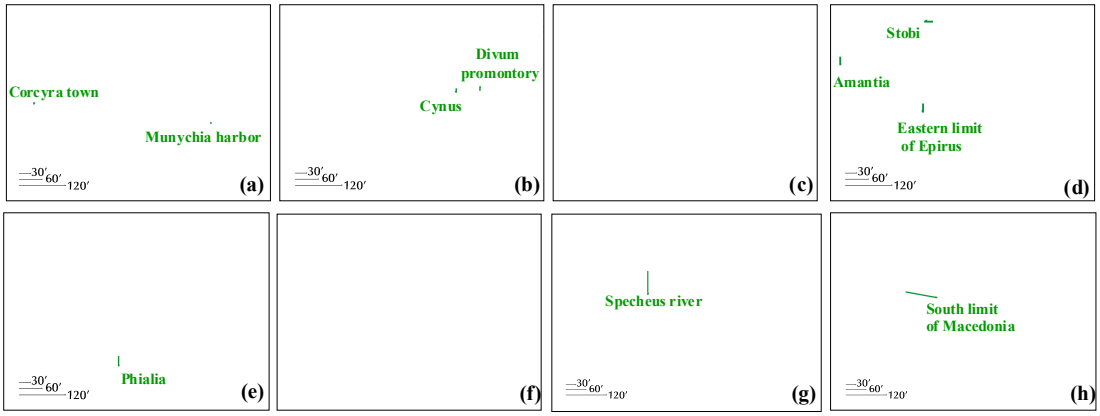

Figure 9. 'G vs M' point-positioning differences in eight classes of displacements in minutes of arc: (a) 5′ (b) 10′ (c) 15′ (d) 20′ (e) 25′ (f) 30′- 40′ (g) 45′- 60′ (h) > 60′

From the above figures we observe the superior degree of agreement between the Germanus and the Müller coordinate lists.

Fitting de Turre's Tabula X map to the coordinates

From the Tabula X coordinate analysis, we conclude in accepting as *operational* a set of 379 points identifying mainly inhabited locations and some cape-ends and debouchments. These points present a high degree of coordinate consistency in the Germanus and Müller coordinate lists and are used:

a. For the creation of a *reference* base map of Tabula X, and
b. For the optimal fitting of de Turre's Tabula X image-map, taken from Nordenskiöld's (1978: 163-4) facsimile atlas, into the *reference* Tabula X.

The *reference* Tabula X map, derived according to the scheme in Fig. 12, is used in order to test the fitting of de Turre's Tabula X representation both point-wise and graticule-wise. For the fitting test, de Turre's image map is georeferenced, to the *operational* set of coordinates, keeping the geographic projection ($y = R\phi, x = R\lambda$) with a unit radius reference sphere in two ways (Fig. 13):

a. With respect to the *operational* points (point-wise, P-w), and
b. With respect to the points of parallel and meridian intersections (graticule-wise, G-w).

The results of this best fitting process, using a 2nd order polynomial transformation model (Boutoura and Livieratos 2006: 60-70) are illustrated in Fig. 12 and Fig. 13.

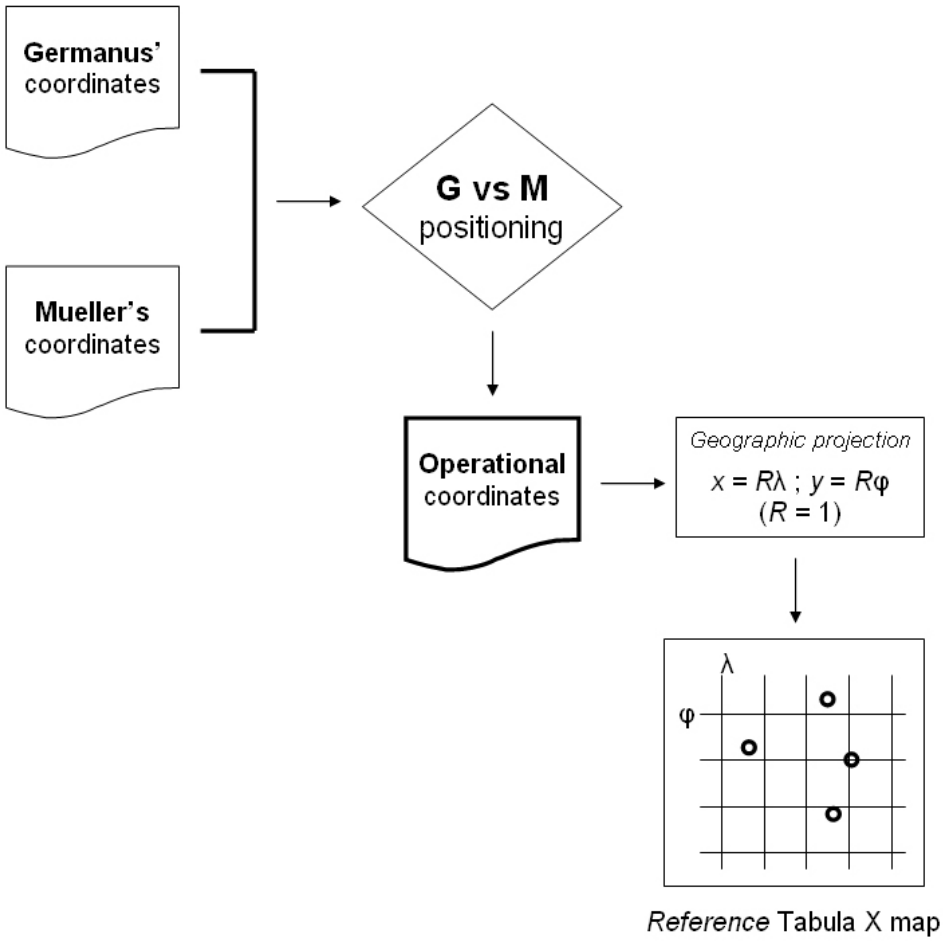

Figure 10. The generation of the *reference* Tabula X map, using the operational coordinates as derived from the 'G vs M' least coordinate discrepancy process.

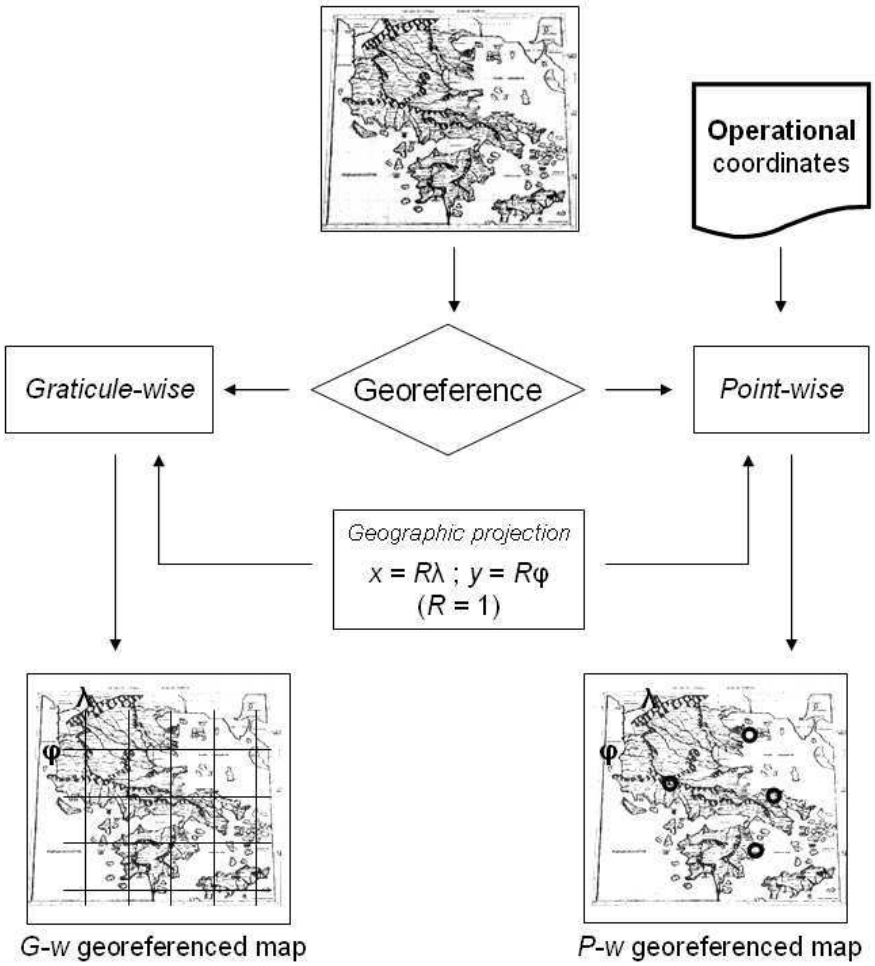

Figure 11. The point-wise (P-w) georeference using the *operational* coordinates as derived from the 'G vs M' least coordinate discrepancy process and the graticule-wise (G-w) georeference of de Turre's Tabula X representation.

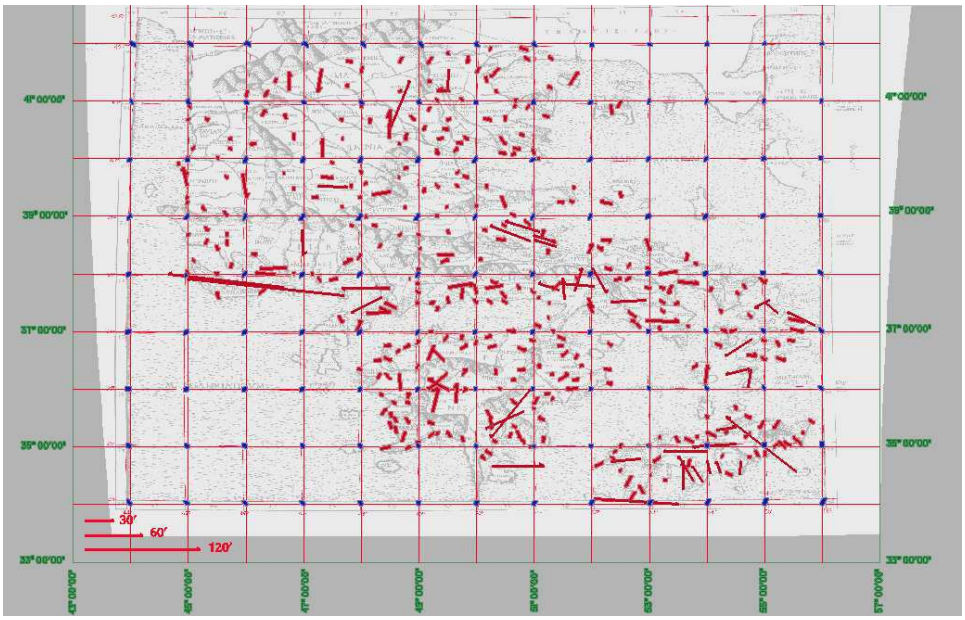

Figure 12. The point-wise (P-w) georeference of de Turre's Tabula X representation using the *operational* coordinates as derived from the 'G vs M' least coordinate discrepancy process and the displacements of the points (in red). The graticule (in red) is not traced on the digital version of the map. These deviations are depicted in blue.

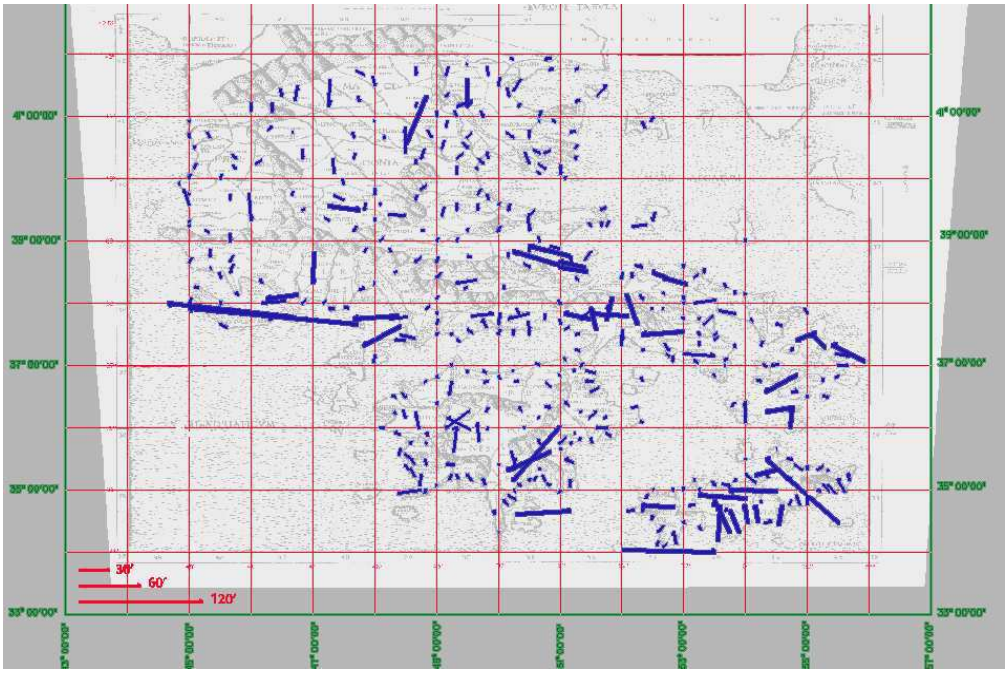

Figure 13. The graticule-wise (G-w) georeference of de Turre's Tabula X representation using the graticule of the digital version of the map and the displacements of the points (in blue). The graticule (in red) is traced exactly on the digital version of the map.

As we can see in Fig. 12, the graticule of meridians and parallels from de Turre's Tabula X representation does not fit exactly to the grid of the digital version of the map, whereas in Fig. 13, the graticule of the georeferenced representation is traced exactly on the digital map. The next step is now the comparison of the two de Turre's georeferenced image maps with respect to the *reference* Tabula X map (Fig. 14) following first, a best (optimal) fitting process, a 2nd order polynomial transformation, as described in Boutoura and Livieratos (2006: 60-70). The results of this comparison process are illustrated in Fig. 15.

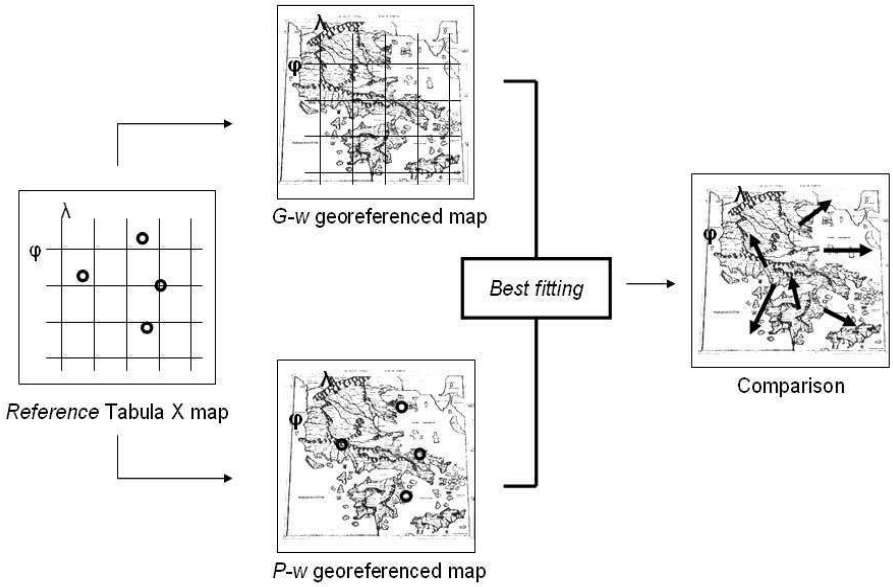

Figure 14. The comparison process of P-w and G-w georeferenced de Turre's Tabula X image maps.

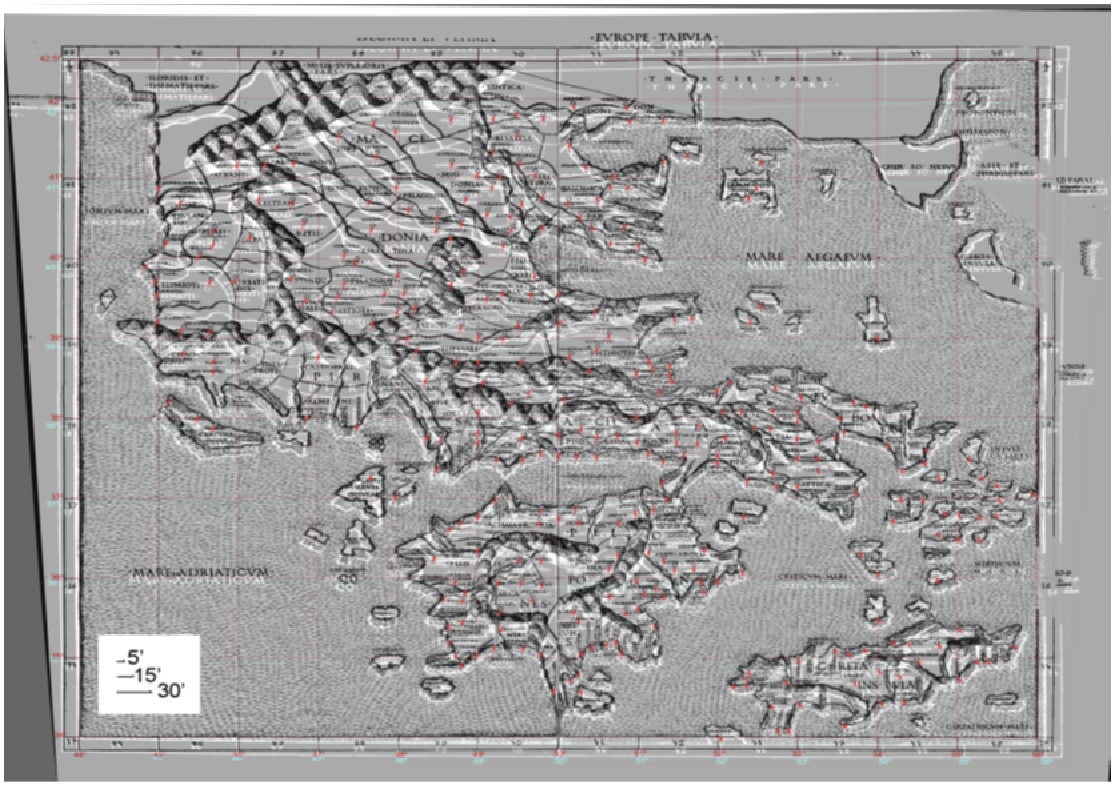

Figure 15. The comparison of the point-wise (P-w) georeference using the *operational* coordinates as derived from the 'G vs M' least coordinate discrepancy process and the graticule-wise (G-w) georeference of de Turre's Tabula X representation. The deviations between the inhabited locations are depicted in red.

Comparing the point-wise (P-w) georeference using the *operational* coordinates as derived from the 'G vs M' least coordinate discrepancy process and the graticule-wise (G-w) georeference of de Turre's Tabula X representation (Fig.14), it is obvious that the deviations between the two georeferenced representations are of the same order, ca 5′, but their directions differ from point to point according to the geographical place they are depicted in the map. These deviations and their different directions become more apprehensible in Fig. 16.

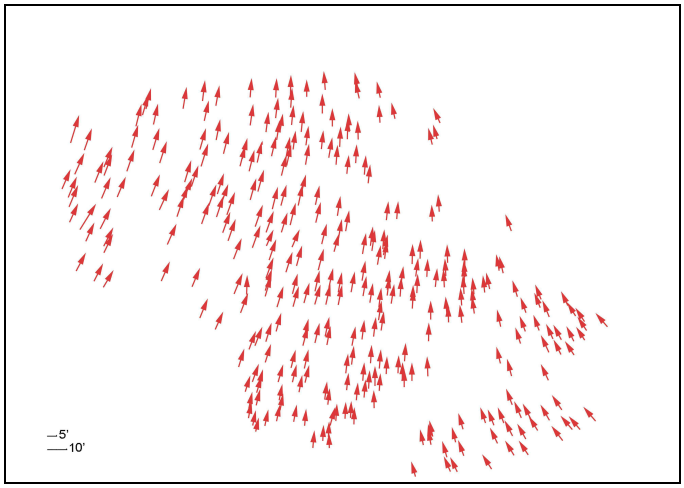

Figure 16. The deviation and their directions between the point-wise (P-w) georeference and the graticule-wise (G-w) georeference of de Turre's Tabula X representation concerning the inhabited locations.

Concluding remarks

Proper analytical transformations implemented under certain acceptable geometric conditions can contribute in showing the relation between the geographical coordinates of longitude and latitude listed in various manuscripts and printed editions of Ptolemy's *Geographia* and the relevant regional cartographic representations drawn with the help of these coordinates. This can be done by bringing the coordinates and their pictorial representations in one-to-one correspondence allowing thus, useful tests of comparison.
Generalizing a previous local test (Livieratos 2006: 51-59) it looks that the hypothesis addressed there, that the map designer followed distinct procedures for the drawing of the geographic graticule and for the plotting of the map content, has now well founded, concerning de Turre's representation of Tabula X. This work should be obviously extended by testing and comparing digitally other relevant representations of Ptolemy's *Geographia* in order to develop a global perspective on the issue.

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