CCGPS Frameworks Student Edition

Mathematics

Second Grade Unit Six
Developing Multiplication

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

Unit 6: Developing Multiplication

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OVERVIEW

In this unit students will:
- Understand and model multiplication as repeated addition and as a rectangular array
- Determine if a number is odd or even (within twenty)
- Create and interpret picture graphs and bar graphs

This standard calls for students to apply their work with doubles addition facts to the concept of odd or even numbers. Van de Walle states, "All too often students are simply told that the even numbers are those that end in 0, 2, 4, 6 or 8 and odd numbers are those that end in 1, 3, 5, 7 or 9. While of course this is true, it is only an attribute of even and odd numbers rather than a definition that explains what even or not even really means" (Teaching Student Centered Mathematics, page 291).

Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, place value cubes, etc.) before moving towards pictorial representations such as circles or arrays.

This standard calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.

This standard calls for students to work with categorical data by organizing, representing and interpreting data. Students should have experiences posing a question with 4 possible responses and then work with the data that they collect.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as counting, time, money, positional words, patterns and tallying should be addressed on an ongoing basis through the use of calendar centers (tubs), and games.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. The tasks in these units illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS FOR MATHEMATICAL CONTENT

Work with equal groups of objects to gain foundations for multiplication.

M2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

M2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Represent and interpret data.

MCC2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. Students are expected to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

ENDURING UNDERSTANDINGS

- There are similarities between skip counting and repeated addition
- Repeatedly adding the same quantity, using a grouping picture or forming a rectangular array is strategies for representing repeated addition equations.
- Arrays are a way of representing both repeated addition and skip counting.
- Arrays should be identified in rows and then columns.
- Explore and be able to explain even and odd numbers while using manipulatives.
- An even number can be decomposed into two equal addends.
- Double addition facts assist in recognizing even numbers.
- Tables and charts can help make solving problems easier.
- Questions can be solved by collecting and interpreting data.

ESSENTIAL QUESTIONS

- How are odd and even number lines identified on the number line?
- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?
- How are arrays and repeated addition related?
- How can rectangular arrays help us with repeated addition?
- How can we model repeated addition on the number line?
- How can we use model repeated addition equation with an array?
- How does skip counting help us solve repeated addition problems?
- What is an array?
- What is repeated addition?

CONCEPTS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

In Grade 1, instructional time focused on four critical areas:
- Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;
- Developing understanding of whole number relationships and place value, including grouping in tens and ones;
- Developing understanding of linear measurement and measuring lengths as iterating length units; and
- Reasoning about attributes of, and composing and decomposing geometric shapes.

Routine topics such as counting, time, money, positional words, patterns, and tallying should be addressed on an ongoing basis through the use of calendars, centers, and games. Organizing and graphing data as stated in MCC.MD.10 should be incorporated in activities throughout the year. Students should be able to draw a picture graph and a bar graph to represent a data set with up to four categories as well as solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Specifically, it is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.
- Fluency with single digit addition/subtraction facts to 20
- Tally marks
- Picture graphs, bar graphs
- Counting to 100
- One to one correspondence
- Comparing sets of objects (equal to, more than, or less than)

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Definitions of the terms are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

- **addends**
- **addition**
- **array**
- **bar graph**
- **columns**
- **data**
- **equal sharing/forming equal sized groups**
- **equation**
- **even**
- **odd**
- **pairing**
- **picture graph**
- **product**
- **rectangular**
- **rows**
- **scale**
- **sum**
- **total**

STRATEGIES FOR TEACHING AND LEARNING
(Information adapted from Mathematics Common Core State Standards and Model Curriculum, Ohio Department of Education Teaching)

Work with equal groups of objects to gain foundations for multiplication.

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Instructional Strategies

Students need to understand that a collection of objects can be one thing (a group) and that a group contains a given number of objects. Investigate separating no more than 20 objects into two equal groups. Find the numbers (the total number of objects in collections up to 20 members) that will have some objects and no objects remaining after separating the collections into two equal groups. Odd numbers will have some objects remaining while even numbers will not. For an even number of objects in a collection, show the total as the sum of equal addends (repeated addition).

A rectangular array is an arrangement of objects in horizontal rows and vertical columns. Arrays can be made out of any number of objects that can be put into rows and columns. All rows contain the same number of items and all columns contain an equal number of items. Have students use objects to build all the arrays possible with no more than 25 objects. Their arrays should have up to 5 rows and up to 5 columns. Ask students to draw the arrays on grid paper and write two different equations under the arrays: one showing the total as a sum by rows and the other showing the total as a sum by columns. Both equations will show the total as a sum of equal addends.

The equation by rows: 20 = 5 + 5 + 5 +5
The equation by columns: 20 = 4 + 4 + 4 + 4 + 4

Build on knowledge of composing and decomposing numbers to investigate arrays with up to 5 rows and up to 5 columns in different orientations. For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends two different ways: by rows, 12 = 4 + 4 + 4; by columns, 12 = 3 + 3 + 3 + 3. Rotate the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, 12 = 3 + 3 + 3 + 3; by columns, 12 = 4 + 4 + 4. Have students discuss this statement and explain their reasoning: The two arrays are different and yet the same.

Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row. Students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2. Then students put two full ten-frames together end-to-end so they form 10 rows of 2 circles or 10 columns of 2 circles. They use this larger array to count by 2s up to 20 and write an equation that shows 20 equal to the sum of ten 2s.

Represent and Interpret Data.

Represent and Interpret Data
MCC.2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Instructional Strategies
At first students should create real object and picture graphs so each row or bar consists of countable parts. These graphs show items in a category and do not have a numerical scale. For example, a real object graph could show the students' shoes (one shoe per student) lined end to
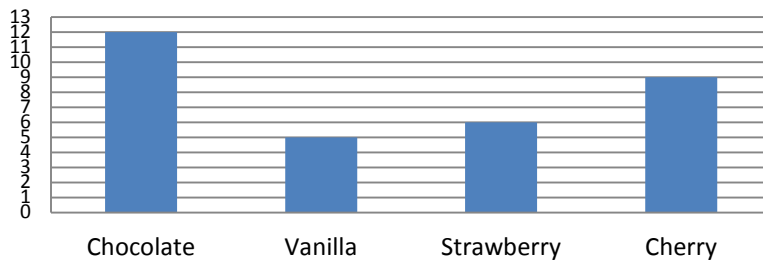
end in horizontal or vertical rows by their color. Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to 2 to 4 rows or bars. Students would then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale.
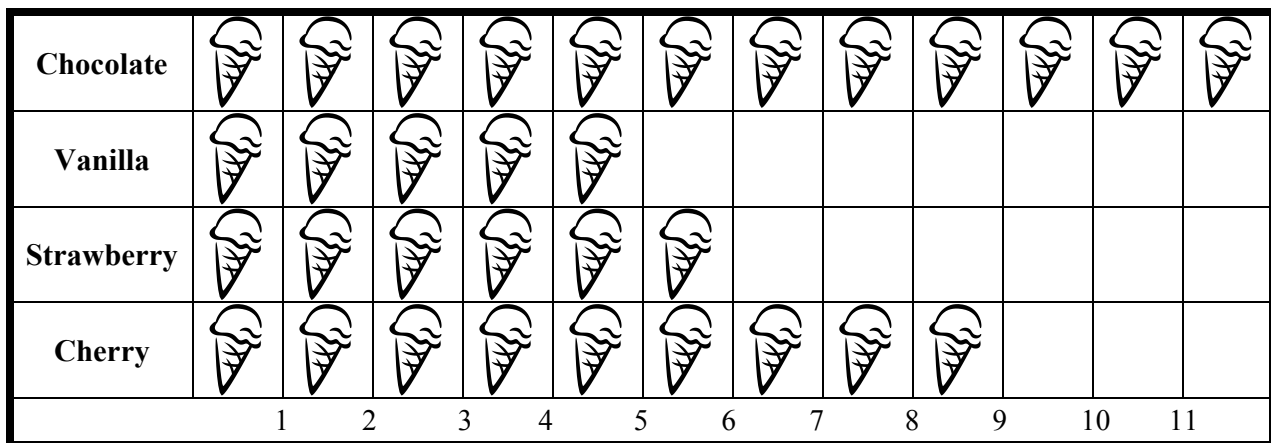
Flavor	Number of People
Chocolate	12
Vanilla	5
Strawberry	6
Cherry	9

Students display their data using a picture graph or bar graph using a single unit scale.

Favorite Ice Cream Flavor

Favorite Ice Cream Flavor

	1	2	3	4	5	6	7	8	9	10	11
Chocolate	🍦	🍦	🍦	🍦	🍦	🍦	🍦	🍦	🍦	🍦	🍦
Vanilla	🍦	🍦	🍦	🍦	🍦						
Strawberry	🍦	🍦	🍦	🍦	🍦	🍦					
Cherry	🍦	🍦	🍦	🍦	🍦	🍦	🍦	🍦	🍦		

As students continue to develop their use of reading and interpreting data it is highly suggested to incorporate these standards into daily routines. It is not merely the making or filling out of the graph but the connections made from the date represented that builds and strengthens mathematical reasoning.

Use the information in the graphs to pose and solve simple put together, take-apart, and compare problems illustrated in Table 1.

Table 1: Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together **Take Apart**	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Understand the similarities between skip counting, repeated addition, and multiplication.
- Construct arrays for a given repeated addition sentence.

- Write a repeated addition equation for a given array.
- Determine how the addition sentence for a given array changes when the array is rotated ¼ turn.
- Understand that multiplication is repeated addition.
- Write an equation to express an even number.
- Identify if a number is even or odd by modeling the number in pairs.
- Draw and interpret a picture and a bar graph to represent a data set with up to four categories.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all second grade students. These tasks or a task of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks be reviewed prior to instruction. The tasks in this unit illustrate the types of learning activities that should be conducted to meet the CCGPS. A variety of additional resources should be utilized to supplement these tasks.

Below is a description of the types of tasks you will see in this unit and their purpose.

Scaffolding Task	Constructing Task	Practice Task	Performance Task
Tasks that build up to the task constructing task.	Constructing understanding through deep/rich contextualized problem solving tasks.	Games/activities	Summative assessment for the unit.

Task Name	Task Type/ *Grouping Strategy*	Content Addressed
Bumpy or Not Bumpy?	Scaffolding Task *Large Group, Small Groups*	Odd and Even Numbers
What's in the Bag?	Scaffolding Task *Large Group, Small Groups*	Odd and Even Numbers
Cookie Monster	Scaffolding Task *Large Group, Small Groups*	Odd and Even Numbers
Ten!	Scaffolding Task *Large Group, Small Groups*	Odd and Even Numbers
Add it Up!	Scaffolding Task *Large Group, Small Groups*	Odd and Even Numbers
Seating the Class	Constructing Task *Small Groups*	Repeated addition, Skip counting
Cereal Arrays/ I have__, Who Has __?	Practice Task *Small Groups*	Arrays
Pattern Block Drop	Practice Task *Small Groups*	Arrays

The Queen's Dilemma	Constructing Task *Partners*	Arrays
Mathemagicians	Practice Task *Partners*	Arrays
No, You Can't	Constructing Task *Partners*	Arrays
The Candy Box	Performance Task *Individual*	Arrays

As this unit has no Culminating Task, you may pair/modify tasks to include all unit standards in combination.
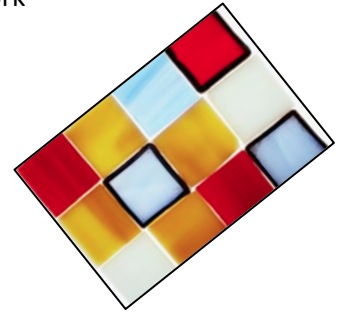
Scaffolding Task: Bumpy or Not Bumpy

Approximately 2 days (Activity originally found in Van de Walle and
Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 292)

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of
members, e.g., by pairing objects or counting them by 2s; write an equation to express an even
number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
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7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics:
Grades K-3, page 291)

"The categorization of numbers as odd or even is an important regularity in our number system.
All too often students are simply told that the even numbers are those that end in 0, 2, 4, 6, or 8
and odd numbers are those that end in 1, 3, 5, 7, or 9. While of course this is true, it is only an
attribute of even and odd numbers rather than a definition that explains what *even* or *not even*
(i.e., *odd*) really means."

After concluding the Bumpy or Not Bumpy Task, "students should be able to classify numbers
into the categories that we call odd and even. **After they have conceptualized these classes of
numbers**, the appropriate labels of *odd* and *even* can be applied."

ESSENTIAL QUESTIONS
- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?

MATERIALS
- Bumpy or Not Bumpy Blackline Master

- Paper, crayons, pencils
- Envelop

GROUPING

Small Group, pairs

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Special Note: This task can be repeated several times in small groups or in a center.

Part I
Duplicate the Bumpy or Not Bumpy Blackline Master for each student. Have students cut them and keep them in an envelope. Explain how each piece (except the single square) is made of two columns of squares. Have students work in pairs or small groups to see how many things they can find to tell about the pieces. (For example: There is a piece for each number 1 to 10. Some are like rectangles. Some have a square sticking out.) For those who might need a start, suggest that they put the pieces in order from one square to ten. Have students share with the whole group what they have discovered.

Part II
Next, have students sort their pieces into two sets. It is very likely that some group will sort their pieces into "bumpy" (odd) and "not bumpy" (even). Refer to the two groups of pieces as "bumpy" numbers and "not bumpy" numbers (or whatever labels your students prefer to use). Students will then share their groupings with their classmates and have the class guess what rule they used to categorize the pieces.

Part III
Next, assign groups of students three or four numbers between 11 and 40 or 50 and have them decide whether two-column cards for these numbers would be bumpy or not bumpy. They can use words and pictures to explain their conclusions.

FORMATIVE ASSESSMENT QUESTIONS
- What did you notice about your pieces?
- How do the pieces differ from each other? What do the pieces have in common?
- What characteristics did you use to sort your pieces?

DIFFERENTIATION
Extension
- Have students create their own pieces to represent various numbers. Then trade their pieces with a classmate and they will describe the pieces a "bumpy" or "not bumpy".

Intervention
- Have students use unifix or connecting cubes to build each piece. The can then manipulate the pieces to pair up each square. This will help then to better understand the meaning of "bumpy" and "not bumpy"

Bumpy or Not Bumpy Blackline Master

Scaffolding Task: What's in the Bag?

Approximately 2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE
(Information adapted from North Carolina DPI Instructional Support Tools)

Students should have had prior experiences and/or instruction with addition. They should begin to relate multiplication as repeated addition. Please see Units 2 and 4 for addition support.
If you have not already done tasks where students have split a group of 20 (or fewer) items into two equal groups then this needs to be done before attempting this task. Provide several experiences where students are able to investigate all the numbers 0-20 to see which ones can be split into two equal groups. This is a good opportunity to review the concepts of "not bumpy" (even) and "bumpy" (odd) numbers and now build on the understanding of how this connects to repeated addition. Having students write addition equations for the even numbers they are able to split into two equal groups is a good way to introduce the concept of repeated addition. Students should recognize that all even numbers can be expressed using two of the same addends (ex. 2+2=4, 3+3=6, again focusing on equal addends sets the stage for repeated addition, leading into multiplication.)

This task will focus on the use of strategies; however, it is important to note the focus is on conversations as students engage in experiences with repeated addition. Initially, students apply base-ten concepts and use direct modeling with physical objects or drawings to find different ways to solve problems. They move to inventing strategies that do not involve physical materials or counting by ones to solve problems. Student-invented strategies likely will be based on place-value concepts, the commutative and associative properties, and the relationship between addition and subtraction. These strategies should be done mentally or with a written record for support. It is vital that student-invented strategies be shared, explored, recorded and tried by

others. Recording the expressions and equations in the strategies horizontally encourages students to think about the numbers and the quantities they represent instead of the digits. Not every student will invent strategies, but all students can and will try strategies they have seen that make sense to them. Different students will prefer different strategies.

ESSENTIAL QUESTIONS
- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?

MATERIALS
- Various manipulatives (counters, base-ten blocks, unifix cubes, beans in bags labeled A – J, 1 set per partner)
- Paper, crayons, pencils

GROUPING

Whole Group, Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Special Note: This task can be repeated several times in small groups or in a center.

Part I
Gather students together in class meeting area. Display the questions, "What is even? What is odd?" The teacher will need to guide discussion into mathematical talk and not story sharing. Be prepared to guide students thinking into conversations about something such as sharing carrot snacks between two friends.

Part II
Have two students come up and practice sharing the cubes the teacher has placed in front of them. For example, the teacher would place 6 cubes in front of 2 students and ask them if they can share the total evenly (fairly). As students are sharing, record each shared quantity on a chart labeled "We can share equally between 2 groups/ We cannot share equally between 2 groups." After several student pairs share the cubes (different quantities each time), lead class in discussion about information on the chart. The conversation should be directed to build the understanding that groups shared evenly are called even numbers and ones which do not share evenly are called odd numbers. The chart can be relabeled as EVEN and ODD.

Part III
Students work in partners with of 10 different bags of items. These should be made in advance and could be shared between various partners. Each bag should be labeled A –J. Once students have determined which groups are odd and which are even, they will work together and create bar graph the number of odd and even draws they had with their partner. Students should be prepared to share their graph with others.

Part IV
Students individually will create their own number line from 0-20. The teacher calls out numbers and students first label the numbers as they teacher calls them out and then students labels as odd or even using red and blue crayons. Students will share with a table partner to check their labeling.

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you using to determine how many _____ are in your group?
- Can you show that answer in a different way?
- How can you demonstrate this with a picture?
- How could you write this in a number sentence?
- Do you have the same number of any of your objects? Why do you think this is the case?
- What information did you use to decide if a number of odd or even?

DIFFERENTIATION

Extension
- If students complete the task, allow them to determine whether or not they can come up with a rule for any number that would tell whether or not the number is odd or even. Have students record their rule on an anchor chart and present their even/odd rule to the class.

Intervention
- Some students will need to use manipulatives to help to determine or represent the number of objects in each group.
- Give the student a 0-20 chart to help them skip count to determine the number of objects in each group.

CONSTRUCTING TASK: Cookie Monster

Approximately 1-2 Days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND kNOWLEDGE

(Information adapted from North Carolina DPI Instructional Support Tools)
The standard addressed in this task calls for students to apply their work with doubles addition facts to the concept of odd or even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, place value cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Example: Is 8 an even number? Prove your answer.

Student 1	Student 2
I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number.	I grabbed 8 counters. I put them into 2 equal groups. There were 4 counters in each group, so 8 is an even number.

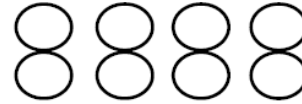| **Student 3**	**Student 4**
I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know 8 is even.	I drew 8 circles. I matched one on the left with one on the right. Since they all match up, I know that 8 is an even number.
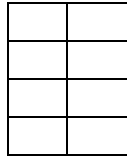
Student 5   I know that 4 plus 4 equals 8. So 8 is an even number.

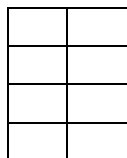Example: Is 7 an even number? Prove your answer.

Student 1	**Student 2**
I grabbed 7 counters. I paired counters up into groups of 2. Since I had 1 counter left over, I know that 7 is NOT an even number. It is an odd number	I grabbed 7 counters. I tried to put them into 2 equal groups. There were 4 counters in one group and 3 in the other group. I know that 7 is NOT an even number. It is an odd number.

Student 3	**Student 4**
I tried to draw a rectangle with 7 boxes. Every box on the left did not match a box on the right, so I know 7 is NOT an even number. It is an odd number.	I drew 7 circles. I matched one on the left with one on the right. Since they all do NOT match up, I know that 7 is NOT an even number. It is an odd number
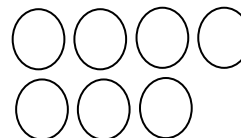
Student 5   I know that 4 plus 3 equals 7. It is not a double fact, so 7 is NOT an even number. It is an odd number.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?

- How can we model repeated addition equation with an array?
- How are arrays used in our daily lives?

MATERIALS

- Counters, square tiles, other manipulatives
- Cookie Monster Student Recording Form
- *Stacks of Trouble* by Martha F. Brenner

GROUPING

Small Group

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Review the concepts of arrays by reading *Stacks of Trouble* by Martha F. Brenner aloud to the class. With each situation have students discuss how the dishes are stacked to create an array.

Part II
In small groups or partners, present the student with this task:

The cookie monster loved cookies. Every 3rd day he would take out one more cookie than he did the previous time. Each time before eating the cookies, he would line them up on the table and make an array with only two rows.

On the first day, the cookie monster had 2 cookies and he was able to build his array with two equal rows. On day 4, he had 3 cookies but he couldn't make an array with two equal rows. If the pattern continued 9 more times, on what days would the cookie monster be able to make an array with his cookies using only two rows? Show and explain your mathematical thinking.

Row 1

Row 2

Day 1   Day 4   Day 7
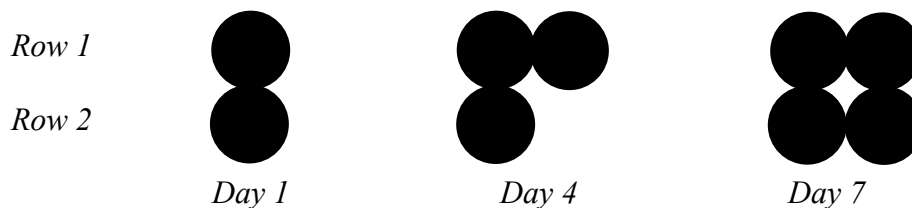
Be sure to describe and explain any patterns you recognize. Can you make a rule?

Part III
When students have completed the task, have them explain their rule on chart part and share with the class. Students will vary in their explanations, which should give the teacher a better understanding of their misconceptions.

FORMATIVE ASSESSMENT QUESTIONS

- What arrays do you see?

- How can we create an equation for this?
- What are you noticing about the pattern?
- How are the days similar? How are they different?

DIFFERENTIATION

Extension
- Have students create their own cookie monster pattern that alternates between even and odd, but this time the Cookie Monster takes more than one cookie. How many cookies will he take?

Intervention
- Allow students to use circular counters to manipulate to better understand that variety of arrays.
- Allow students to use grid paper to ensure that their representation of the pattern is correctly aligned.

Name: _____

Me Love Cookies

The cookie monster loved cookies. Every 3rd day he would take out
one more cookie than he did the previous time. Each time before
eating the cookies, he would line them up on the table and make an array with only
two rows.

On the first day, the cookie monster had 2 cookies and he was able to build his
array with two equal rows. On day 4, he had 3 cookies but he couldn't make an
array with two equal rows. If the pattern continued 9 more times, on what days
would the cookie monster be able to make an array with his cookies using only two
rows? Show and explain your mathematical thinking.

Row 1

Row 2

Day 1   Day 4   Day 7
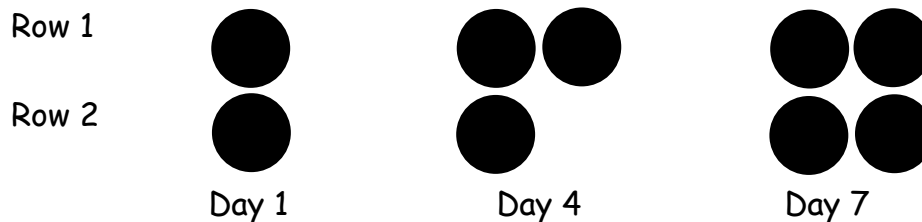
Be sure to describe and explain any patterns you recognize. Can you make a rule?

Scaffolding Task: Ten!

Approximately 2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE

Students explore odd and even numbers in a variety of ways including the following: students may investigate if a number is odd or even by determining if the number of objects can be divided into two equal sets, arranged into pairs, or counted by twos. After these experiences, students may derive that they only need to look at the digit in the ones place to determine if a number is odd or even, since any number of tens will always split into two even groups.
(AZ Explanations and Examples)

ESSENTIAL QUESTIONS

- How are odd and even number lines identified on the number line?
- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?

MATERIALS

- *How Many Feet in the Bed?* By Diane Hamm or similar book about counting by 2s
- Various manipulatives (counters, base-ten blocks, unifix cubes)
- Group Recording Sheet

GROUPING
Whole Group, Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Read a story like, *How Many Feet in the Bed*? By Diane Hamm. Ask the students to suggest other things that come in 2's. Brainstorm ideas on a class chart.

Part II
Place children in small groups of three to four students and have them create a strategy to find the total number of eyes, ears, wings on birds, wheels on bicycles etc. for a given number of objects not to extend beyond 10 (as students only work with skip counting to 20 in grade 2, establish understanding first before moving on.) Give students a half sheet of chart paper to use to explain the number of objects from the scenario given. For example, a teacher could give a group of students bicycles and the number 7 and ask them to determine how many wheels in all, using pictures, words and numbers. As students are working, look for the use of strategies to determine the number of each item in the group. Look to see if students are:
- counting by ones and pointing (1,2,3,4,5,6,7,8 .. 8 eyes)
- skip counting by 2s (2, 4, 6, 8), 4s (4,8,12,16), or 5s (5,10,15,20)
- using repeated addition (2+ 2 + 2 + 2 = 8)
- drawing pictures of equal groups. ⊕ ⊕ ⊕

While students are working, ask formative assessment questions from list below.

Part III
Students will share their scenarios with the class. Teacher should ask questions as students present information.

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you using to determine how many _____ are in your group?
- Can you show that answer in a different way?
- How can you demonstrate this with a picture?
- How could you write this in a number sentence?
- Do you have the same number of any of your objects? Why do you think this is the case?
- What makes a number even? What makes a number odd?

DIFFERENTIATION

Extension
- Students work independently to show quantities larger than 20 that come in 2s.

Intervention
- Students use manipulatives to show their work.

Constructing Task: Add it Up!

Approximately 2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle, Karp, and Bay-Williams, Elementary and Middle School Mathematics: Teaching Developmentally, page 266-267)

"An interesting category of number structures is that of odd and even numbers. Students will often observe that the sum of two even numbers is even, that the sum of two odd numbers is even, or that the sum of an even and odd number is always odd. Similar statements can be made about multiplication.

Students will provide a variety of interesting proofs of odd/even conjectures. As with other conjectures, they typically begin by trying lots of numbers. But here it is a bit easier to imagine that there just might be two numbers 'out there' that don't work. Then students turn to the definition or a model that illustrates the definition. For example, if a number is odd and you split in two, there will be a leftover. If you do this with the second odd number, it will have a leftover also. So if you put these two together, the two leftovers will go together so there won't be a leftover in the sum. Students frequently use models such as bars of snap cubes to strengthen their arguments."

ESSENTIAL QUESTIONS

- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?
- What is repeated addition?

- How do I use what I know about odd and even to help me with repeated addition?

MATERIALS

- Various manipulatives (counters, base-ten blocks, unifix cubes, beans) in bags. Need multiple bags with even number of items and multiple bags with an odd number of items
- Group Recording Sheet

GROUPING

Whole Group, Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Special Note: This task will take a few days as students repeat the work several times. Each opportunity will provide students will more time to describe their thinking and deepen their conceptual understanding of how these two concepts connect.

Part I
Teacher will gather students together for large group work. In advance, the teacher should create multiple bags of even materials and multiple bags of odd bags. Guide students in making connections of what happens when we combine two bags with even amounts, what happens when we combine bags with odd amounts, what about when we combine bags with one of each. What is the result? Students will need to record their findings as they combine bags such as: Bag A had 7 and Bag B had 4, therefore adding an odd and an even number which resulted in an odd number; however when I combined Bag A with 7 and Bag C with 3, I had a total of 10 which I know is an even number. This is all with manipulatives, students are not writing the number sentences yet. Scaffolding the conversation to allow students to begin to see how odd sets do not all have partners, while even number sets do have partners. The teacher should allow time for students to predict whether the total will be odd or even and why they think the number will be odd or even.

Part II
After students have had extensive work describing all these combinations, go back to the bag combinations with writing addition sentences talking about addition sentences. This time students will connect the combinations with writing the number sentences, connecting repeating addition to even and odd. Use rich math language as you question students and repeat the questions about combinations. What are our addends? What do we know about those numbers? How will knowing if its even or odd help us determine the answer?

This task is one that can be moved to a center once students have had amble time in class discussion. This task is designed to be repeated several times.

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you using to determine how many _____ are in your group?

- Can you show that answer in a different way?
- How can you demonstrate this with a picture?
- How could you write this in a number sentence?
- Do you have the same number of any of your objects? Why do you think this is the case?
- What makes a number even? What makes a number odd?
- How can knowing if a number is even or odd help you with addition sentences?
- What is an addend?

DIFFERENTIATION

Extension
- Students work independently to grab bags of items and justify their answers.

Intervention
- Students use manipulatives to show their work.

Constructing Task: Seating the Class

Approximately 3-5 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MCC2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 83)

"In the beginning, children will be able to use the same models- sets and number lines- for all four operations. A model not generally used for addition but extremely important and widely used for multiplication and division is the array. An *array* is any arrangement of things in rows and column, such as a rectangle of square tiles or blocks.

To make the clear connection to addition, early multiplication activities should also include writing an addition sentence for the same model."

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we model repeated addition equation with an array?
- How can we determine if a number is odd or even?

MATERIALS

- Counters, square tiles, other manipulatives
- Half-Sheet of Chart Paper

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I

Review the concepts of repeated addition or equal groups by demonstrating examples with students on the whiteboard using the following task:

Mrs. Evans wants to rearrange the desks in her classroom. She wants to organize the students to sit in an array, with each row having the same number of desks. If her class has twenty students, how many different arrays could Mrs. Evans make her desks? Explain your thinking using pictures, equations and words.

Put the students into small groups to allow them to decide on ways that Mrs. Evans could organize her class into equal rows. You may want to incorporate the word "group" so students can begin to focus on the grouping as it associates with rows and repeated addition. **You can expose them to the word divide (separate), but it should not be a focus as you are talking to the students about what they are doing.** Encourage students to come up with as many ways possible. Have students use the chart paper to demonstrate the various ways they divided up the 20 students, and help them explain the strategy they used. Make sure to have students explain the arrays they created using pictures, words, and repeated addition. Encourage conversations on how to use their understanding of odd and even to assist in this task.

Teacher note: Understanding the connection between repeated addition and equal groups is the goal; this will develop a foundation for multiplication. As students are working, look for students who have found various ways to organize the desks.

Part II

After students have completed the task, choose students who used a variety of strategies to share with the class. Act out some of their strategies. Keep a class chart to document the different ways the class could arrange the 20 students into rows. Ask students what they notice about the number 20. Is there only one way to separate the 20 students? What addend combinations created a total of 20? In other words, how many different repeated addition equations were we able to write?

FORMATIVE ASSESSMENT QUESTIONS

- How many total students does Mrs. Evans have?
- What is your plan to arrange up the students?
- Will your rows be equal? Should they be? Why?

- How many students will be in each row? How did you figure that out?
- How can you show this through repeated addition?
- Is there another way you could group/divide up the students?
- What strategies did you use to group the 20 students?
- Is there only one way to separate the 20 students?
- What addend combinations created a total of 20?
- How many different repeated addition equations were we able to write?

DIFFERENTIATION

Extension
- If students complete the task, tell the students the class just received a new student. How will they rearrange the desks to fit 21 students?
- How many students could Mrs. Evans have if she is unable to arrange the desks in an array?

Intervention
- Allow students to use square tiles to manipulate to better understand that variety of arrays.

Name: _____

Mrs. Evans wants to rearrange the desks in her classroom. She wants to organize the students to sit in an array, with each row having the same number of desks. If her class has twenty students, how many different arrays could Mrs. Evans make her desks? Explain your thinking using pictures, equations and words.
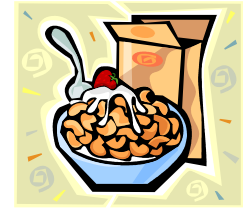
Practice Task: Cereal Arrays

Approximately 3-4 days

STANDARDS ADDRESSED FOR MATHEMATICAL CONTENT

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL CONTENT

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 80)

"When students solve simple multiplication story problems before learning about multiplication symbolism, they will most likely write repeated-addition equations to represent what they did as an equation. This is your opportunity to introduce the multiplication sign and explain what the two factors mean." Although this is true, the focus in second grade is not learn the multiplication symbolism or fact, it is to gain understanding through repeated addition.

"The usual convention is that 4x8 refers to four sets of eight, not eight sets of four. There is absolutely no reason to be rigid about this convention. The important thing is that the students can tell you what each factor in their equations represents."

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- Cereal
- Glue

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Each part of this task is designed to take one class period. Once the students have completed the task a version of it can be placed in a center for repeated practice throughout the year.

Part I
Explain to students that sometimes people use arrays as a model for repeated addition and that today we are going to make sure they understand what columns and rows look like in arrays. Tell students that arrays are made up of rows and columns. Ask students to think of some places that they have gone to where they might have had to sit in rows. Allow them to share and record their experiences. Show students pictures of rows in a movie theatre and at a football stadium (some examples are provided). Explain to students that rows are horizontal or they go in direction from left to right or right to left. Next ask the students if anyone knows what a column is or if they can point to an example of a column in the room or the school. If they need a hint tell them that columns travel in the opposite direction as rows. Once students have figured out that columns run "up and down." tell them that we use the word vertical to describe this direction (up and down). Ask students to think of places where they may have seen columns, and again record their ideas. Show students pictures of columns on various buildings such as a local home or the White House. (Some examples have been provided)

Allow students time to demonstrate their understanding of these new words (rows/horizontal; columns/vertical) by playing a short game of *Simon Says*. Have students get into a horizontal position by lying on the floor when you say "Simon says show me a row" or have them point to or locate things in the room that run in a horizontal direction (i.e. tray of the chalk board, bottom of the doorframe, edge of a rug, etc). For a vertical position they can just stand up when you say, "Simon says show me a column", or point to places in the room that have lines running in a vertical direction (i.e. the flag pole, a music stand, leg of a table, etc.).

Once it is clear that students understand what columns and rows are, and the difference between them you can introduce what an array is to them using a picture that is a combination of rows and columns. Demonstrate for students an example of objects organized in an array. You may have tubs on a shelf or desks organized in an array or using a block of tiles on the hallway floor. Ask students how these things organized as an array helps you. One possible reason would be to help you find things faster. Create a chart where you can list additional arrays they may notice in the classroom or mention having seen elsewhere. Draw several arrays on chart paper or white board large enough for students to see and be able to discuss in small groups.

Give the students some items (buttons, cubes, counters, etc.) and have them organize them into a rectangle array. You can partner students for this or have them work independently. Once they have their items in an array ask the students figure out what shape they have created (rectangle). Ask them to describe their array to the class by telling how many columns and how many rows they have made. As they are describing what they have made, have a student draw it on the chart paper or white board. Be sure to ask, "Does the picture I have drawn match your array?"

Part II
Assign each student a certain number of cereal pieces and have them decide the number of rows and columns that the array would have and then create the repeated addition. They could also create more than one based on their number. Once students come together and share their work, discuss that the number of rows is the addend that will be repeated and the number of columns is how many times you will repeat the addend. Have a student share their addition sentence and explain their reasoning for writing it that way. For example, 5+5+5 creates an array that is 5 rows and three columns. Once they show emergent understanding, allow students to explain or describe the array with an addition sentence. You can challenge students to describe the array with different kind of clues. For example, "I have an array that is 3+3+3+3." Or, "Who has an array that is 2+2+2?" You will need to have several examples of different arrays on the board for them to use as examples.

Part III
Tell students that they will now have the opportunity to create their own arrays using cereal. Have each student draw a repeated addition sentence. Encourage each student to create an array using the cereal to illustrate the card. As students create their cereal arrays, move about the room and ask questions from the formative assessment list.

Have students glue down their cereal arrays so they can hold them up to share with the class. After students have completed the task, allow them to share their arrays. Invite discussion about how two students might view or describe the same array differently. For example:

Student 1
I see 3 pieces of cereal in each column and there are 4 columns. So I added: 3 + 3 + 3+ 3. That equals 12.
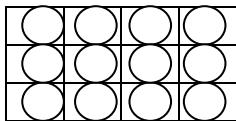
Student 2
I see 4 pieces of cereal in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.
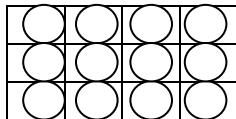
Encourage students to ask questions of their peers and make comments about the work and strategy used to figure out how to make their arrays.

Part IV
Start off with the question, "What happens if you rotate/turn the cereal array that you made? Does it change the number sentence you write? How and why?"
Invite further discussion about what happens when you rotate or turn someone's array a ¼ turn. Allow students to come up and act it out by actually rotating their paper. Have the class generate the new repeated addition equation that now goes with the array. Record these new repeated addition sentences on the board. Have students return to their seats, give them enough cereal to create the "new" array (the rotated one) and have them record the repeated addition equation that goes with this recreated array. This supports understanding of the commutative property, and allows students to remain flexible in their thinking.

Part V
Students will work together to play, *I have, Who has?* on the following pages.
Tell students that they are going to have to do some detective work. Detectives are really good at figuring things out, therefore they are going to have to put on their thinking caps as they pretend to be "Detective Dan" and "hunt/search" to find out, "Who Has It?"

FORMATIVE ASSESSMENT QUESTIONS

- Describe how you know how to rotate/turn your array? How does the sentence change when it turns? Does the answer change?
- What is your repeated addition number sentence?
- How many rows should you include?
- How many columns should you include?
- Why are arrays important?

DIFFERENTIATION

Extension
- Supply cards with larger addends. These students may also be encouraged to create as many arrays as possible with the same number of items.
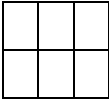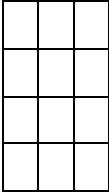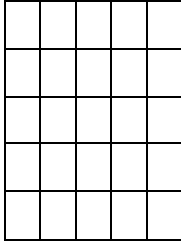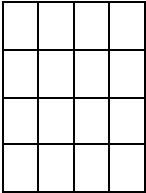
Intervention
- Use 1 inch graph paper to line up the cereal correctly in arrays.

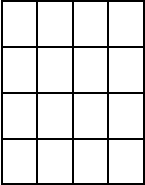**Sample Pictures for Repeated Addition**
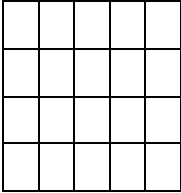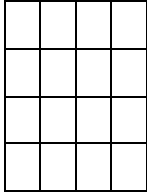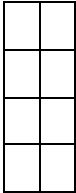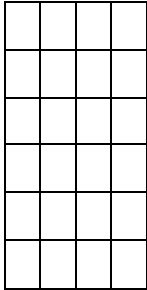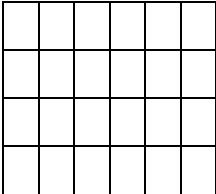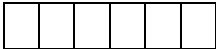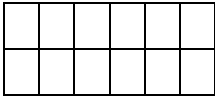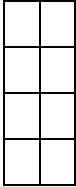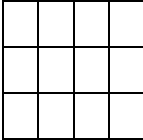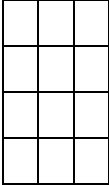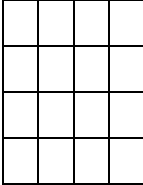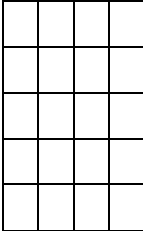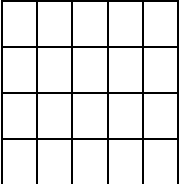
Clue Cards

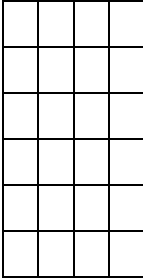|	
Teacher Clue Card: My clue card has 2 rows: Who has a card with 3 rows?	My clue card has Who has a card with 3 columns?
My clue card has Who has a card with 5 rows?	My clue card has Who has a card with 4 columns?
My clue card has Who has a card with 1 row?	My clue card has Who has a card with 5 columns?

My clue card has Who has a card with 4 rows?	My clue card has Who has a card with 2 columns?
My clue card has Who has a card with 6 rows?	My clue card has Who has a card with 6 columns?
My clue card has Who has a card with 1 row?	My clue card has Who has a card with 1 column?
My clue card has Who has a card with 1 row?	My clue card has Who has a card with 6 columns?
My clue card has Who has a card with 2 rows?	My clue card has Who has a card with 2 columns?

My clue card has Who has a card with 3 rows?	My clue card has Who has a card with 3 columns?
My clue card has Who has a card with 4 rows?	My clue card has Who has a card with 4 columns?
My clue card has Who has a card with 5 rows?	My clue card has Who has a card with 5 columns?
My clue card has Who has a card with 6 rows?	My clue card has Who has a card with 6 columns?

Practice Task: Pattern Block Drop

Approximately 2 days
Adapted from Even/Odd Pattern Block Grab from
K5MathTeachingResources.com

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE
(Information adapted from Mathematics Common Core State Standards and Model Curriculum, Ohio Department of Education Teaching)

Build on knowledge of composing and decomposing numbers to investigate arrays with up to 5 rows and up to 5 columns in different orientations. For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends two different ways: by rows, $12 = 4 + 4 + 4$; by columns, $12 = 3 + 3 + 3 + 3$. Rotate the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, $12 = 3 + 3 + 3 + 3$; by columns, $12 = 4 + 4 + 4$. Have students discuss this statement and explain their reasoning: The two arrays are different and yet the same.

Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row. Students count by rows to 10 and write the equation $10 = 2 + 2 + 2 + 2 + 2$. Then students put two full ten-frames together end-to-end so they form 10 rows of 2 circles or 10 columns of 2 circles. They use this larger array to count by 2s up to 20 and write an equation that shows 20 equal to the sum of ten 2s.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

## MATERIALS

- 1 inch Graph paper
- Glue

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I

Review with the students what arrays are and why we use arrays to organize things. Ask students to describe what they know about rows and columns. Use the cereal arrays from the last task to review how to write repeated addition equations to represent arrays. Review how to use arrays to represent objects. You may wish to model creating an array to represent the number of sides on 4 triangles. Say something like, "I have 4 triangles. I want to create an array that represents how many sides I have on these triangles. How many columns will I have in my array? How many rows in each column? Draw the 4 by 3 array and write repeated addition sentences for the array.
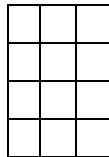
Part II

Each student will reach into a bag of pattern blocks and grab a handful. They will drop the handful of blocks onto their workspace and sort them by shape. They will then take each group of shapes (make sure to have all the same shapes in each group) and create an array to represent the number of sides included in the group. For example, I might grab 3 triangles, 7 squares and 5 hexagons. After sorting them, I will choose the first group to represent with an array. I choose the 5 hexagons. I have 5 sets of 6 sides in the hexagon group. I will create an array of 5 columns with 6 in each row. I will then describe the array with a repeated addition equation. As students create their arrays, circulate the room and ask questions from the formative assessment list.

Part III
After students have completed part II of the task, allow them time to share their arrays. Invite discussion about the different strategies they might use to count the number of sides represented by the array. Encourage students to ask questions of their peers and make comments about the work and strategy used to figure out how to make their arrays. Lead a class discussion about how if you rotated these arrays ¼ of a turn that they would not represent the same group of pattern blocks. In this activity, allow students to describe their thinking and then lead them to an understanding of the columns represent the number of blocks and the rows represent the number of sides. If you rotate the array, it changes what is represented.

FORMATIVE ASSESSMENT QUESTIONS

- Is the same information represented when you rotate the array? Why or why not?
- What is your repeated addition number sentence?
- How many rows should you include?
- How many columns should you include?
- Why are arrays important?
- How can arrays help us to multiply?

DIFFERENTIATION

Extension
- Have students create a graph to represent the shapes they drew from the bag.

Intervention
- Some students may need to use other strategies like counting by 1s, skip counting or repeated addition to determine the number of sides in the group instead of creating arrays.

Constructing Task: The Queen's Dilemma
Approximately 3 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addend

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson
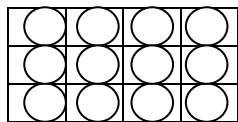
BACKGROUND KNOWLEDGE

This standard calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.
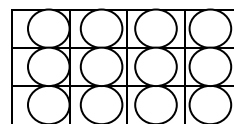Example below:

Student 1

I see 3 counters in each column and there are 4 columns. So I added: 3 + 3 + 3+ 3. That equals 12.

Student 2

I see 4 counters in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?

- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- *A Remainder of One* by Elinor J. Pinczes (Houghton Mifflin Co., 1995) or similar book
- Array recording sheet (per group)
- Half sheet of chart paper (per group)
- Various manipulatives (pop cubes, counters, tiles, etc.)

GROUPING

Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I

Begin the lesson with a review of arrays. Share pictures of arrays that can be seen in the real world and discuss the difference between a row and a column. Read aloud the book, *A Remainder of One*. Throughout the book, discuss the arrays with the students, focusing on the number of rows and columns and the product of each array. After reading the book, explain and demonstrate how the ants were not able to make equal rows with 2, 3 or 4 in a row but they could make equal rows with 5. Model the language 5 by 5. Indicate that the repeated addition sentence for this is 5+5+5+5+5. Remind the children this is called an array because you can make a rectangle with the number. Tell the students that now they will get to act as the Queen and will be given a certain number of ants to divide into an equal number of rows and an equal number of columns with nothing left over. For example, if she had only 6 soldiers in the group they could march in 2 rows with 3 in each row, a 2 by 3 array (2x3) Orthey could march in 3 rows with 2 in each row, a 3 by 2 array (3x2). Keep in mind writing a multiplication equation is not something second grade students are required to master at this point. The focus is on the concept of multiplication and how we can model it with both repeated addition sentences and arrays.

(They can also march in 1 row of 6 or 6 rows of 1, in other words, single file as when going to lunch.)
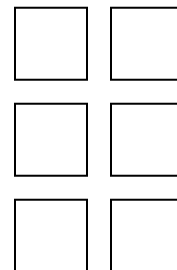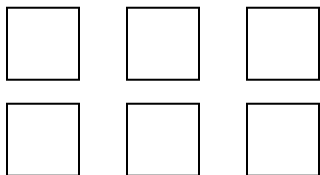
So a group of six makes a rectangle (or an array).

Share the task with the students:

The queen of the ___(insert school name)____ Ant Colony has 16 ants in her army. The queen is attempting to organize her ants into arrays. She wants to know how many arrays she can create using her 16 ants. How many arrays can she make with 16 ants? Using manipulatives, drawings and words, explain your work.

Note: This task could be interpreted that the queen must use all 16 ants. This task can also be used to allow students to leave some ants out. This would give students multiple different arrays. Consider this for your differentiation.

Allow the students to work with a partner to experiment with different arrays that the queen could create when arranging the ants. Have the students use manipulatives to create their arrays. Then, encourage the students to draw their arrays on the chart paper and record the repeated addition equation on the recording sheet. While students are working, circulate the room and ask questions from the formative assessment list.

Part II
After students have completed Part I of the task, choose several students to share their discoveries and observations with the class.

If students used numbers up to 16, use this to lead discussion: The teacher or students can record the arrays the students have found on a larger version of the "The Queen's Dilema" sheet.Ask questions which require them to look for patterns. Some things that they may notice are:

- 5, 7, 11 and 13 can only march in single file

- 4, 9 and 16 can be made into squares with equal sides

- 2,4,6,8,10,12,14 and 16 can all be divided into 2 equal rows

As the students "discover" this information, use this data to create a graph that shows how many of the numbers could only create single file lines, made squares, 2 equal rows or others. Graphing this data will reveal patterns about numbers, allowing for discussion of odd and even, among other things.

FORMATIVE ASSESSMENT QUESTIONS

- What manipulatives are you using to help solve this problem?
- Why are you arranging the tiles in that way?
- What are some ways the queen could arrange 6 ants? 10? 16? etc.
- What are some ways that she cannot arrange them?
- How many rows does this array have?
- How many columns does this array have?
- How can you tell the difference between rows and columns?
- What strategies are you using to help figure out ways the queen could arrange the ants?
- Are you noticing anything about the numbers that she is or is not able to use?

- How could you use repeated addition to help you solve this problem?
- Could this number be arranged in a different way?
- How would the equation be different if this array were rotated a ¼ turn?
- How are you communicating the results you have found?
- What patterns are you noticing from your chart?
- Why are we able to make different arrays for some numbers but not others?
- Do any of the arrays you have made have the same product?

DIFFERENTIATION

Extension
- If students complete the assigned task, ask students what additional arrays the ants could march in if the queen allowed 14 more ants to join the army.
- Have students create all the arrays up to 16 ants. Use the included recording sheet.

Intervention
- Students who are having difficulty may need additional questioning with the use of manipulatives.

Group Members: _____ Date: _____

The Queen's Dilemma Recording Sheet

Number	Arrays	Equation
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		

Practice Task: "Mathemagicians"
Approximately 3 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addend
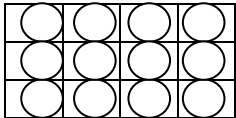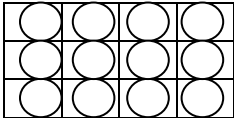
STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE

This standard calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.
Example below:

Student 1

I see 3 counters in each column and there are 4 columns. So I added: 3 + 3 + 3+ 3. That equals 12.

Student 2

I see 4 counters in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?

- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- 25 index cards (per group)
- Various manipulatives (snapcubes, counters, tiles, coins etc.)

GROUPING

Partners

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Begin the lesson with a review of arrays. Share the pictures of arrays created in the task "The Queen's Dilemma" and discuss the difference between a row and a column. Make sure you model the language 3 by 3. Indicate that the equation for this is 3+3+3. Ask the children this is called an array because you can make a rectangle with the number. Tell the students that now they will be the Mathemagician and will magically build all of the arrays possible for each number 1-25. Remind them to divide the numbers into an equal number of rows and an equal number of columns with nothing left over. For example,with the number 10 one can create 1 row with 10 in it, 2 rows with 5 in each row, 5 rows with 2 in each row, and 10 rows with 1 in each row. Encourage the children to look for the real magic in the numbers and discover ALL of the possibilities for their given numbers and to describe the array in all of the ways possible (repeated addition, a 5 by 2 array, etc.) **Keep in mind writing a multiplication equation is not something second grade students are required to master at this point**. The focus is on finding the total number of objects and how we can model it with both repeated addition sentences and arrays.

Part II
Allow the students to work with a partner to experiment with different arrays that the Mathemagician could create when describing numbers 1-25. Have the students use manipulatives to create their arrays. Then, encourage the students to draw their arrays on the construction paper and record the repeated addition equation next to the array. While students are working, circulate the room and ask questions from the formative assessment list. Each group of students will have one index card for each number 1-25 showing the arrays possible for that given number. You may want to consider having them create books with these index cards so that they may continue to share them after the lesson is complete.

Part III
After students have completed Part II of the task, choose several students to share their discoveries and observations with the class.

Lead a discussion about what the discovered about each number. Compare the information/data to the discoveries from the task "The Queen's Dilemma", is there any new data/information to

add to the graph? Ask questions which require them to look for patterns. Some things that they may notice are:

- 1, 2, 3, 5, 7, 11, 13, 17, 19 and 23 can only have in single file rows/columns

- 4, 9, 16 and 25 can be made into squares with equal sides

- 2,4,6,8,10,12,14, 16, 18, 20, 22, and 24 can all be divided into 2 equal rows

- 15, and 21 are odd numbers but they have more arrays than just single file rows/columns

FORMATIVE ASSESSMENT QUESTIONS

- What manipulatives are you using to help solve this problem?
- Why are you arranging the tiles in that way?
- What are some ways the Mathemagician could arrange the number 6? 10? 16? etc.
- What are some ways that she/he cannot arrange them?
- How many rows does this array have?
- How many columns does this array have?
- How can you tell the difference between rows and columns?
- What strategies are you using to help figure out ways the Mathemagician could arrange the numbers?
- Are you noticing anything about the numbers that she is or is not able to use?
- How could you use repeated addition to help you solve this problem?
- Could this number be arranged in a different way?
- How would the equation be different if this array were rotated a ¼ turn?
- How are you communicating the results you have found?
- What patterns are you noticing from your chart?
- Why are we able to make different arrays for some numbers but not others?
- Do any of the arrays you have made have the same product?

DIFFERENTIATION

Extension
- Have the students identify the arrays that are similar or that have been rotated ¼ of a turn. They are different, yet the same. Ask them to tell why.

Intervention
- Students who are having difficulty may need additional questioning with the use of manipulatives.

CONSTRUCTING TASK: No, You Can't
Approximately 1-2 Days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1 Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle, Karp, and Bay-Williams, Elementary and Middle School Mathematics: Teaching Developmentally, page 266-267)

This task requires the students to build conjectures of their own and then participate in meaningful discussion. This is a direct reflection of the Standards of Mathematical Practices which should be incorporated within every task. While the

"It is important that all students initiate conjectures. It is important that all students actively consider the validity of all conjectures made by classmates. When deciding if s conjecture is always true, have students write their ideas before sharing with the class. If you begin with a class discussion, only a few students are likely to participate, with others content to listen whether or not they are following the arguments. You can then use both what the students write as well as their input in discussions to assess what level of reasoning they are at: authority, use of examples, or an appeal to logic."

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?

- How can we use model repeated addition equation with an array?

MATERIALS
- Recording Sheet with table

GROUPING

Small Group, Partners

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Within a small group, provide students with the problem:

Lou, Stu, and Moe are triplets who love to argue. They always argue until one would prove the other two wrong. This time they're stuck and need your help.
- *Lou says every number (0 to 25) can be represented in an array that has two or more rows.*
- *Stu says you can make an array with 2 or more rows for less than 16 of the numbers (0 to 25).*
- *Moe says you can make an array with 2 or more rows for more than 16 of the numbers (0 to 25).*

Who is right? If you know a brother is wrong, you must prove it to them using numbers, pictures and words to show your thinking! Otherwise they will keep arguing!!!!!

Part II
Have groups create an anchor chart stating which brother they feel is right and what lead them to this understanding. After all the students have created their anchor charts, allow each group to present their discoveries. Once all the groups have shared their thoughts, open the class to a group discussion in which they can carry on a constructive, respectful, debate. This will directly enforce the Standards for Mathematical Practices.

FORMATIVE ASSESSMENT QUESTIONS
- What information is important?
- Which brother do you think will be correct?
- Can you see any mistakes that the brothers made?

DIFFERENTIATION

Extension
- What do all the numbers that have an array with 2 or more rows have in common? Create a rule and test it with larger numbers.

Intervention
- Provide students with a limited list of numbers 1-10. This will allow the students to still see the patterning,

Name: _____

No, You Can't

Lou, Stu, and Moe are triplets who love to argue. They always argue until one would prove the other two wrong. This time they're stuck and need your help.

- Lou says every number (0 to 25) can be represented in an array that has two or more rows.

- Stu says you can make an array with 2 or more rows for less than 16 of the numbers (0 to 25).

- Moe says you can make an array with 2 or more rows for more than 16 of the numbers (0 to 25).

Who is right? If you know a brother is wrong, you must prove it to them using numbers, pictures and words to share your thinking! Otherwise they will keep arguing!!!!!
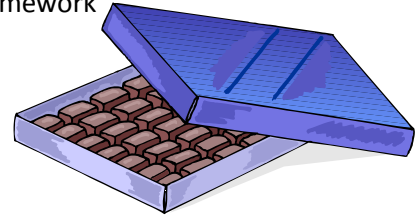
Name: _____

Write the repeated addition sentence for the numbers that can be modeled in an array with 2 or more rows

	Equation			Equation
0			13	
1			14	
2			15	
3			16	
4			17	
5			18	
6			19	
7			20	
8			21	
9			22	
10			23	
11			24	
12			25	

Performance Task: The Candy Box
Approximately 2 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MCC2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL CONTENT

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

BACKGROUND KNOWLEDGE

Students should have had multiple experiences with repeated addition. This task serves as the final opportunity for students to express their understanding of the connection between arrays and repeated addition.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we model repeated addition on the number line?
- How can we use model repeated addition equation with an array?

MATERIALS

- Task Description
- Half a sheet of chart paper per student.
- Various 1inch by 1 inch construction paper squares

GROUPING

Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Bring in various containers from home that utilize arrays, including candy boxes, egg cartons, etc. Show these containers to students. Have students discuss why they think the boxes are organized in this way.

Give students this task:

The candy store wants to package candies in different sized boxes. The owner is thinking about the different rectangular boxes he could use. How do you think the owner should organize the candy? Show the different ways that the candy store owner could package __ candies in a rectangular box and construct a viable argument to persuade the owner to organize the candy the way that you selected.

Part II
Give students materials and allow for students to come up with as many ways possible for their number. Encourage students to use the chart paper to demonstrate the ways and their strategy use. Have students demonstrate the candy boxes they would create and describe them in rows and columns, repeated addition, and skip counting. Ask students questions from the formative assessment list.

Part III
After students have completed the task, choose students who used a variety of strategies to share with the class. Keep a class chart to document the different ways the candy store owner could arrange his box. Ask students what they notice about their number.

FORMATIVE ASSESSMENT QUESTIONS

- How many total candies does the candy store owner want in each box?
- How have you arranged your squares?
- How many rows do you have?
- How many columns do you have?
- How can you show this through repeated addition?
- Is there another way you could arrange the candy box?
- Is there only one way to arrange the candies?

DIFFERENTIATION

Extension
- Allow the students to create a 3-D model of one of their candy box designs. Then, allow students to imagine that they are trying to sell the box model to the owner. Have them write a proposal explaining their model and why it is the best choice for the candy store.

Intervention

- Some students may need to use the example boxes that you brought in from home to gain ideas about the arrangement of the candy box.

One Inch Grid
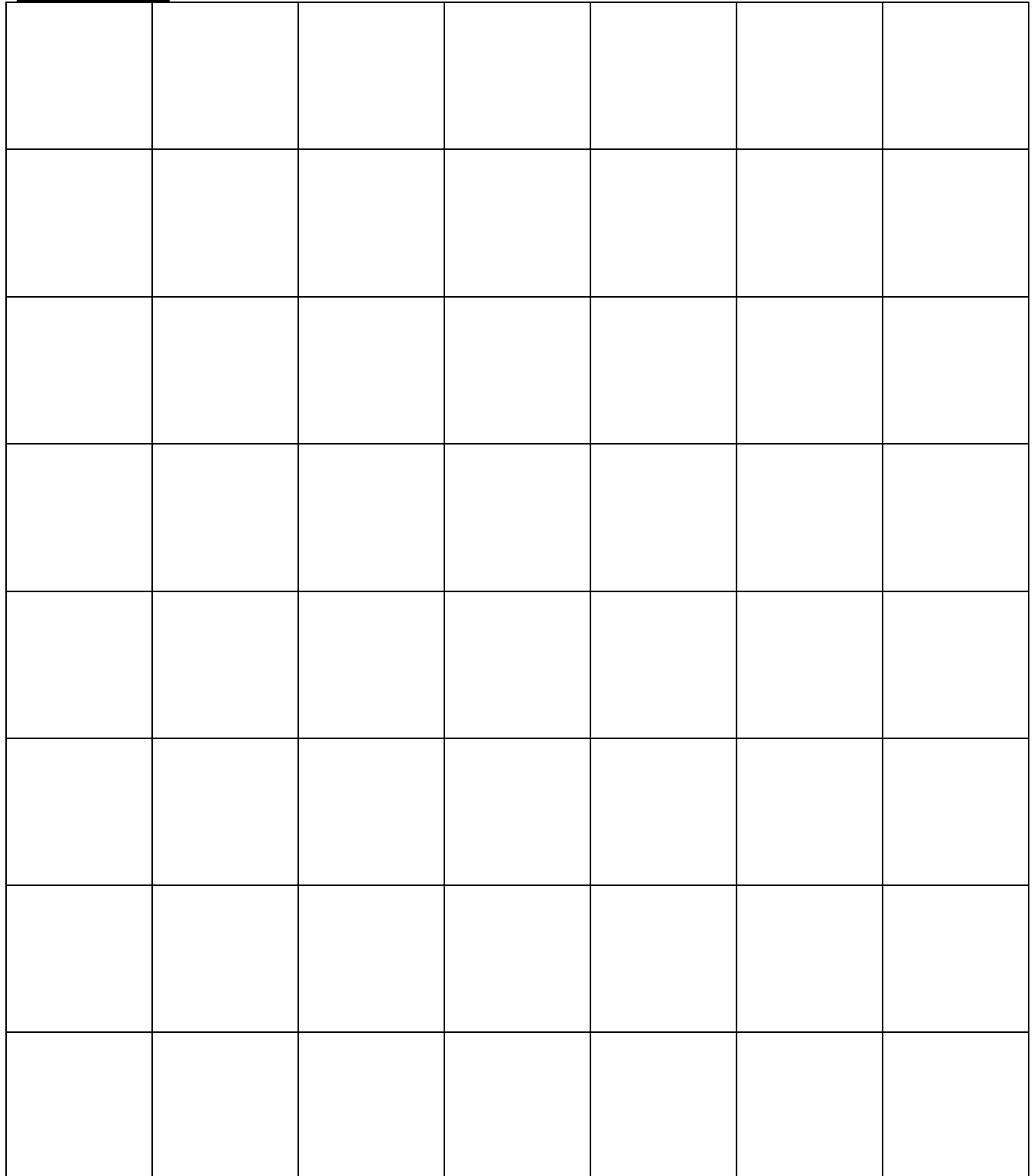
Name: _____

The candy store wants to package 24 candies in every box, with each box holding a perfect array of candies. The owner is thinking about the different rectangular boxes he could use and how the candies would be arranged. How many different boxes can the owner use? Explain using numbers, pictures, and words.