## Chapter 6 - Trigonometric Functions

## Cosine and Sine

unit circle: $x^{2}+y^{2}=1$

$$
\begin{aligned}
& x=\cos \theta ; \quad y=\sin \theta \\
& \cos ^{2} \theta+\sin ^{2} \theta=1
\end{aligned}
$$

Circle with center $(0,0)$ radius r :

$$
\begin{aligned}
& x^{2}+y^{2}=r^{2} \\
& x=r \cos \theta ; \quad y=r \sin \theta
\end{aligned}
$$

## Radian Measure

$$
\begin{aligned}
& \pi r a d=180^{\circ} \\
& s=\theta r
\end{aligned}
$$

## Other Trigonometric Functions

$\tan \theta=\frac{\sin \theta}{\cos \theta} ; \quad \cot \theta=\frac{\cos \theta}{\sin \theta}$
$\sec \theta=\frac{1}{\cos \theta} ; \quad \csc \theta=\frac{1}{\sin \theta}$
(Graphs have vertical asymptotes.)

## Inverse Trig Functions

$$
\begin{aligned}
& \cos ^{-1} y=t \leftrightarrow y=\cos t \text { for } 0 \leq t \leq \pi \\
& \sin ^{-1} y=t \leftrightarrow y=\sin t \text { for }-\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \\
& \tan ^{-1} y=t \leftrightarrow y=\tan t \text { for }-\frac{\pi}{2}<t<\frac{\pi}{2}
\end{aligned}
$$

## Sinusoidal Functions

$$
\begin{aligned}
& y=A \sin [B(t-h)]+k \\
& y=A \cos [B(t-h)]+k
\end{aligned}
$$

Amplitude $=|A| ;$ period $=\frac{2 \pi}{|B|}$
midline: $y=k$; horizontal shift $=h$

## Exercises

1. State the period, amplitude, and midline in each of the following.
a. $y=-4 \cos [2(x+1.5)]+6$
b. $y=10 \sin (3 x)-5$
2. The equation of the midline of the function $f(x)=3 \sin [2(x+1)]+4$ is
(1) $y=1$
(2) $y=2$
(3) $y=3$
(4) $y=4$
3. The maximum value of the function $g(t)=40 \cos \left[\frac{2 \pi}{365}(t-91.25)\right]-100$ is
(1) 140
(2) 60
(3) -60
(4) 91.25
$\qquad$
4. As shown in the diagram below, an angle in standard position with degree measure $170^{\circ}$ intersects the unit circle with center $(0,0)$ at point $P$. To the nearest hundredth, determine the $x$ - and $y$-coordinates of point $P$.

5. Convert to degree measure: (a) $\frac{5 \pi}{6} \quad$ (b) $\frac{7 \pi}{12}$
6. Convert to radian measure: (a) $240^{\circ}$
(b) $150^{\circ}$
(Leave in terms of $\pi$.)
7. Find a formula, using the sine function, for your height above the ground after $t$ minutes on the given ferris wheel:

The ferris wheel is 28 meters in diameter and boarded at a height 6 meters above the ground. The wheel completes one full revolution every 10 minutes. At $t=0$, you are in the three o'clock position and ascending.
8. Write an equation, using cosine, for the function whose graph is shown below. Your equation must be of the form $y=A \cos [B(x-h)]+k$.

9. Find, to the nearest hundredth, the coordinates of point $A$ in the diagram below.

10. Solve for the smallest positive value of $\theta$ :
a. $5+2 \cos (3 \theta)=6$
b. $4-2 \tan (2 \theta-5)=12$
11. Solve for all values of $x$, where $0 \leq x<2 \pi$ :
a. $2 \sin ^{2} x+7 \sin x+3=0$
b. $2 \sin x \cos x-\sin x=0$
12. Which of the following vertical lines is an asymptote for the graph of $f(x)=\csc x$ ?
(1) $x=\frac{\pi}{2}$
(2) $x=-\frac{3 \pi}{2}$
(3) $x=\pi$
(4) $x=\frac{\pi}{4}$
13. Express in terms of one trigonometric function only: $\sec \theta \cdot \tan \theta \cdot \cos ^{2} \theta$

