Chapter 6 – Trigonometric Functions

Cosine and Sine

unit circle:
$$x^2 + y^2 = 1$$

 $x = \cos \theta$; $y = \sin \theta$
 $\cos^2 \theta + \sin^2 \theta = 1$

Circle with center (0,0) radius r:

$$x^{2} + y^{2} = r^{2}$$
$$x = r \cos \theta; \quad y = r \sin \theta$$

Other Trigonometric Functions

$$\tan \theta = \frac{\sin \theta}{\cos \theta}; \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}; \quad \csc \theta = \frac{1}{\sin \theta}$$

(Graphs have vertical asymptotes.)

Radian Measure

$$\pi \, rad = 180^{\circ}$$

$$s = \theta r$$

Inverse Trig Functions

$$\cos^{-1} y = t \leftrightarrow y = \cos t \text{ for } 0 \le t \le \pi$$

 $\sin^{-1} y = t \leftrightarrow y = \sin t \text{ for } -\frac{\pi}{2} \le t \le \frac{\pi}{2}$
 $\tan^{-1} y = t \leftrightarrow y = \tan t \text{ for } -\frac{\pi}{2} < t < \frac{\pi}{2}$

Sinusoidal Functions

$$y = A \sin[B(t-h)] + k$$

$$y = A \cos[B(t-h)] + k$$
Amplitude = $|A|$; period = $\frac{2\pi}{|B|}$
midline: $y = k$; horizontal shift = h

Exercises

1. State the period, amplitude, and midline in each of the following.

a.
$$y = -4\cos[2(x+1.5)] + 6$$

b.
$$y = 10\sin(3x) - 5$$

2. The equation of the midline of the function $f(x) = 3\sin[2(x+1)] + 4$ is

(1)
$$y = 1$$

(2)
$$y = 2$$

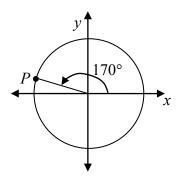
(3)
$$v = 3$$

(4)
$$y = 4$$

3. The *maximum* value of the function $g(t) = 40 \cos \left[\frac{2\pi}{365} (t - 91.25) \right] - 100$ is

$$(3) -60$$

4. As shown in the diagram below, an angle in standard position with degree measure 170° intersects the unit circle with center (0, 0) at point P. To the *nearest hundredth*, determine the x- and y-coordinates of point P.



5. Convert to degree measure: (a) $\frac{5\pi}{6}$

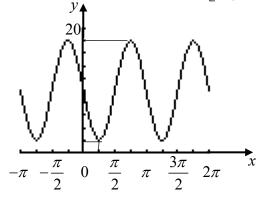
(b) $\frac{7\pi}{12}$

6. Convert to radian measure: (a) 240° (Leave in terms of π .)

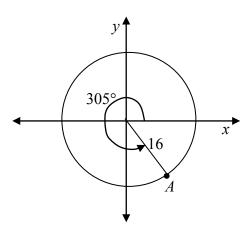
- (b) 150°
- 7. Find a formula, using the sine function, for your height above the ground after *t* minutes on the given ferris wheel:

The ferris wheel is 28 meters in diameter and boarded at a height 6 meters above the ground. The wheel completes one full revolution every 10 minutes. At t = 0, you are in the three o'clock position and ascending.

8. Write an equation, using cosine, for the function whose graph is shown below. Your equation must be of the form $y = A\cos[B(x-h)] + k$.



9. Find, to the *nearest hundredth*, the coordinates of point A in the diagram below.



10. Solve for the *smallest* positive value of θ :

a.
$$5 + 2\cos(3\theta) = 6$$

b.
$$4-2\tan(2\theta-5)=12$$

11. Solve for all values of x, where $0 \le x < 2\pi$:

a.
$$2\sin^2 x + 7\sin x + 3 = 0$$

b.
$$2\sin x \cos x - \sin x = 0$$

12. Which of the following vertical lines is an asymptote for the graph of $f(x) = \csc x$?

(1)
$$x = \frac{\pi}{2}$$

$$(2) \quad x = -\frac{3\pi}{2}$$

$$(3) \quad x = \pi$$

(1)
$$x = \frac{\pi}{2}$$
 (2) $x = -\frac{3\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{\pi}{4}$

13. Express in terms of **one** trigonometric function only: $\sec \theta \cdot \tan \theta \cdot \cos^2 \theta$