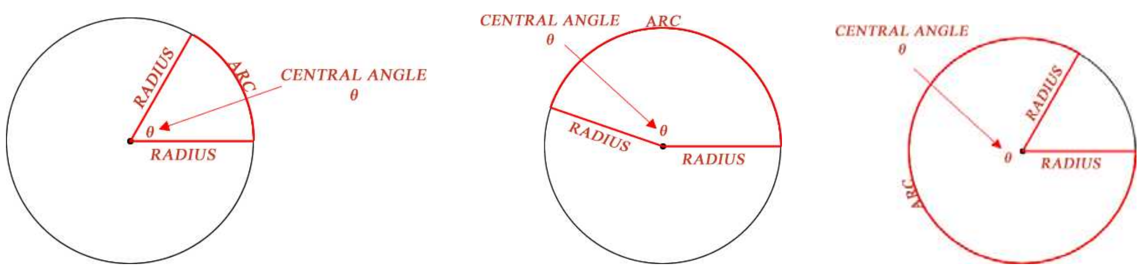
Name:_________________________ **Date**:_____________________________

ACCELERATED PRE-CALCULUS - RADIANS

A CENTRAL ANGLE

If you draw two different radii in a circle and connect those radii with an arc then the angle created between the two radii is called a "**central angle**." I will name this central angle with the variable θ pronounced "tha$\bar{a}$yta." See the images below.



A NEW SYSTEM OF MEASURING ANGLES

The measuring system you have been using to measure angles is the degree system. You will soon be introduced to an alternative system for measuring angles called the "**radian system**."

Just like length can be measured by many different means, e.g. inches, millimeters, feet, meters, etc., angles have two systems of measurement, degrees and radians. **So what is a "radian**?" Let's find out.

WHAT IS A RADIAN?

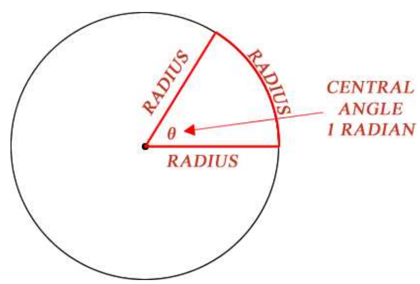
I will now teach you about an alternative means of measuring angles. As you know angles are measured in degrees. But there is another measuring system to measure angles and this system is called the "**radian system**." **The radian system is used by mathematicians**, **the National AP Calculus exam**, **and in ALL university and college Calculus classes**. So get ready to leave degrees behind for a while.

THE FORMAL DEFINITION OF 1 RADIAN

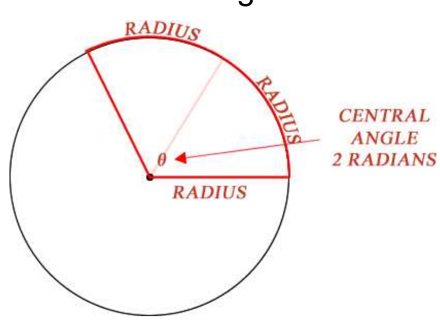
"If you measure the radius of a circle as an arc on the circumference of that circle then the "central angle" created by this arc is called "**1 radian**."

SOME EXAMPLES OF RADIANS

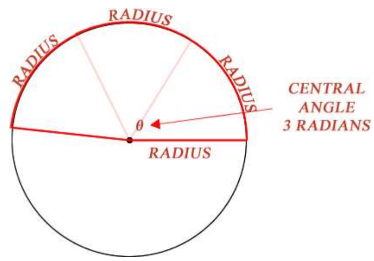
The angle measure you see below is called "**1 radian**."
As you can see this angle is less than 90°.



The angle measure you see below is called **2 radians**.
As you can see this angle is less than 180°.

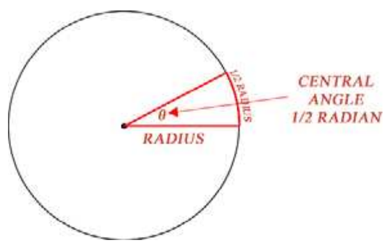


The angle measure you see below is called **3 radians**.
As you can see this angle is more than 180°.

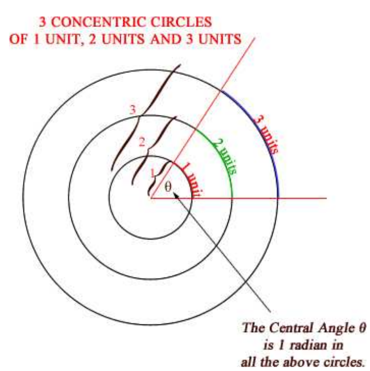


So for each radius on the outside of a circle an angle of one radian is created. It appears that the angle of 3 radians is less than 180° which leads me to the hypothesis there must be at least an angle measure of 6 radians in a 360° circle.

TEACHER COMMENT: Students will ask me if there can be $\frac{1}{2}$ radian or $\frac{1}{3}$ radian or $\frac{2}{5}$ radians or $\frac{12}{7}$ radians. My response is, "Sure." For example, $\frac{1}{2}$ radian is the **central angle** produced from $\frac{1}{2}$ of a radius laid out as an arc on the circumference of a circle.



Comment: Given a specific radian measure, e.g. 1 radian, this radian measure will produce the SAME ANGLE on circles with different radii. You will soon verify this geometric fact.



HOW MANY RADII IN A CIRCLE?

From my experience as a teacher, most students do not "feel" comfortable with the notion of radians. But this feeling will pass as you start connecting the concept of radians with the concept of degrees. So my goal will be to connect radians and degrees. I will need your attention and patience as I reason my way through a number of steps that will produce this connection between radians and degrees. So let’s begin.

As you know the **circumference of a circle**,i.e. the perimeter of the circle, can be determined by the following formula:

$$Circumference = 2\pi(radius\ of\ the\ circle)$$

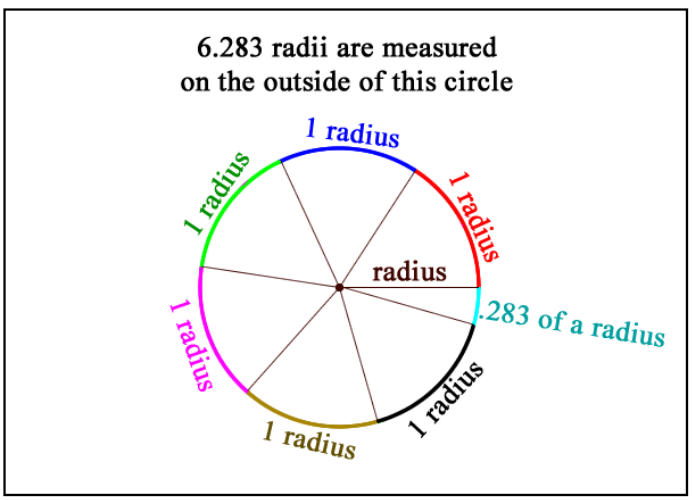
or shortened to

$$C = 2\pi r \approx 6.283r$$

$\approx$ is the approximately equal symbol.
Why do you think I am using $\approx$ above?

So what does this mean? It means that there are 6.283 radii around a circle, i.e. 6 radii plus $.283$ of a radii are on the outside of this circle. See **figure 3** below.

figure 3



COMMENT: Please understand that the circumference formula can perform two tasks for you:

Task 1) Tell you how many radii there are in a circle
Task 2) Tell you the actual length around a circle

...My concern is what task 1 is informing you of. I have not said anything about the actual length around any particular circle, only how many radii there are in a circle.

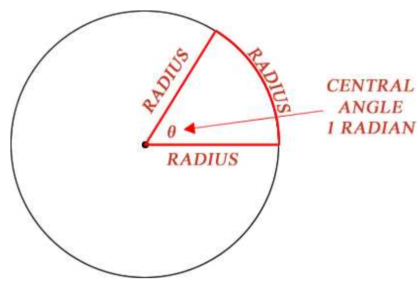
MATHEMATICAL FACT

ANY CIRCLE OF ANY LENGTH RADIUS
HAS 2π RADII OR $6.283\ldots$ RADII ON ITS CIRCUMFERENCE

SO HOW MANY RADIANS IN A CIRCLE?

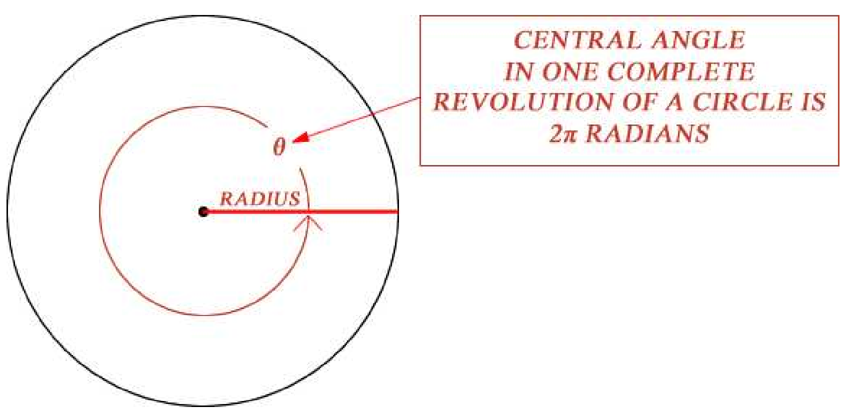
You learned that for each radius placed on the outside of a circle you get a central angle with a measure of 1 radian. See figure 4 below.

figure 4



You now know there are 6.283 or 2π radii on a circle. Also you know for each radius on the circumference of a circle a central angle of 1 radian is produced . Therefore the angle measure in radians of the entire circle must be $6.283\ldots$ or 2π radians. See figure 5 below.

figure 5



THE NEXT TWO IDEAS:
A REVIEW OF A RATIO AND A PROPORTION

I will now introduce you to two ideas, ratios and proportions, which will help you understand the relationship between degrees and radians. My goal is to create a formula that will translate radians to degrees and degrees to radians. So put on your thinking caps.

A REVIEW: **WHAT IS A RATIO**?

A ratio is a fraction that compare one number concept to another number concept. Often these number concepts will name the same "thing." Here are some examples:

ex. 1) $\frac{12 \text{ inches}}{1 \text{ foot}}$ These are two number concepts are used to name the same "length."

ex. 2) $\frac{1 \text{ kilogram}}{2.2 \text{ pounds}}$ These are two number concepts are used to name the same "weight."

If you have a ratio that is true, like the ones above, then you can create a proportion that allow you to translate one number concept into another number concept. This is a very powerful tool. **WATCH AND LEARN**!

A REVIEW: WHAT IS A PROPORTION?

A proportion is an equation that sets one fraction equal to another fraction. But more importantly it allows you to set a RATIO OF NUMBERS **equal to** RATIO OF VARIABLES that are **related to the ratio of numbers**. Let me show you what I mean.

$$\frac{12\ Inches}{1\ Foot} \text{ is a ratio } \rightarrow\rightarrow_{transform\ to} \frac{12}{1} = \frac{Inches}{Foot} \rightarrow\rightarrow \frac{12}{1} = \frac{I}{F}$$

...I now have a formula, $\frac{12}{1} = \frac{I}{F}$, that will allow me to translate inches to feet or feet to inches. I represents inches and F represents feet.

...**ex**. **1**) Change 800 inches to feet

$\frac{12}{1} = \frac{I}{F}$ I will substitute 800 into I and solve for F.

$\frac{12}{1} = \frac{800}{F} \rightarrow 12F = 800 \rightarrow F = \frac{800}{12} \rightarrow F = 66.667$

So 800 inches equals 66.677 feet.

...**ex**. **2**) Change 325 feet to inches

$\frac{12}{1} = \frac{I}{F}$ I will substitute 325 into F and solve for I.

$\frac{12}{1} = \frac{I}{325} \rightarrow 325(12) = I \rightarrow I = 325(12) \rightarrow I = 3900$

So 325 feet equals 3900 inches.

I will now create a weight proportion.

$$\frac{1 \text{ Kilogram}}{2.2 \text{ Pounds}} \rightarrow\rightarrow_{transform\ to} \frac{1}{2.2} = \frac{\text{Kilograms}}{\text{Pounds}} \rightarrow\rightarrow \frac{1}{2.2} = \frac{K}{P}$$

...I now have a formula, $\frac{1}{2.2} = \frac{K}{P}$, that will allow me to translate kilograms to pounds and pounds to kilograms. K represents kilograms and P represents pounds.

...**ex**. **3**) Change 647 kilograms to pounds

$\frac{1}{2.2} = \frac{K}{P}$ I will substitute 647 into K and solve for P.

$\frac{1}{2.2} = \frac{647}{P} \rightarrow P = 647(2.2) \rightarrow P = 1423.4$

So 647 kilograms equals 1423.4 pounds.

...**ex**. **4**) Change 987 pounds to kilograms

$\frac{1}{2.2} = \frac{K}{P}$ I will substitute 987 into P and solve for K

$\frac{1}{2.2} = \frac{K}{987} \to 1(987) = 2.2K \to K = \frac{987}{2.2} = 448.636$

So 987 pounds equals 448.636 kilograms.

THE RELATIONSHIP BETWEEN DEGREES AND RADIANS

A circle is a beautiful geometric creature due to its symmetry. This symmetry allows us to reason with it in many different ways. I propose that I can establish a relationship between degrees and radians. Consider the following:

The angle measure around a circle in the degree system is 360°
The angle measure around a circle in the radian system is 2π radians.
In effect, 360° and 2π radians are measuring the SAME ANGLE.

Therefore, we can create a ratio of $\frac{360\ Degrees}{2\pi\ Radians}$ or simplified to $\frac{180\ Degrees}{\pi\ Radians}$.

So $\frac{180\ Degrees}{\pi\ Radians} \to\to \frac{180}{\pi} = \frac{Degrees}{Radians} \to\to \frac{180}{\pi} = \frac{D}{R}$

Problem 1) How many degrees are in 1 radian?

ans: I will replace R in $\frac{180}{\pi} = \frac{D}{R}$ with 1 and solve for D.

$\frac{180}{\pi} = \frac{D}{1} \to\to 180 = D\pi \to\to D = \frac{180}{\pi} \approx 57.296$

So 1 radian is approximately equal to 57.296°.

Problem 2) How many radians are in 45°?

ans: I will replace D in $\frac{180}{\pi} = \frac{D}{R}$ with 45 and solve for R.

$\frac{180}{\pi} = \frac{45}{R} \to\to 180R = 45\pi \to\to R = \frac{45\pi}{180} = \frac{\pi}{4}$ radians

So 45° is the same as $\frac{\pi}{4}$ radians.

IMPORTANT TEACHER COMMENT: *It is the general practice to leave all radian measures in terms of* π and NOT attach the word "radians." From this point on a radian measure will ALWAYS be represented as a REAL NUMBER with NO units attached to the number.

YOUR ASSIGNMENT

You learned that there were angle measures that you were expected to learn on the unit circle as well as their associated symmetric degrees and ordered pairs. You will translate EVERY one of those degree measures on the unit circle to radians. You MUST be able to work in radians as easily as degrees. See the radian measure for 45° below. We found this radian measure above in exercise 2.