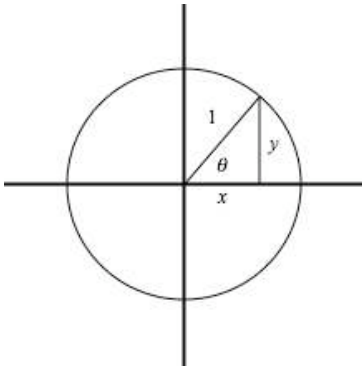


Unit 3 – Right Triangle Trigonometry - Classwork

We have spent time learning the definitions of trig functions and finding the trig functions of both quadrant and special angles. But what about other angles? To understand how to do this, and more importantly, why we do it, we introduce a concept called the unit circle. A unit circle is a circle whose radius is one.



To the left is a unit circle. The angle θ is drawn in the first quadrant but could be drawn anywhere. Suppose $\theta = 40^\circ$. If we were to find $\sin 40^\circ$, we know that it would be defined as $\frac{y}{1} = y$. So when we take $\sin 40^\circ$, we are finding the height of the triangle in a unit circle. The same argument holds when we take $\cos 40^\circ$... we are actually finding the x variable in a unit circle. When we take $\tan 40^\circ$, we are finding the ratio of y to x in a unit circle.

On your calculator, be sure you are in Degree Mode and set your decimal accuracy to FLOAT. Use your calculator to find $\sin 40^\circ$ and $\cos 40^\circ$. Remember what it is you are finding: the y and x variables in the triangle above. And since $x = \cos 40^\circ$ and $y = \sin 40^\circ$, let us show that the Pythagorean theorem holds in this triangle based on the unit circle.

Taking trig functions on the calculator is straightforward: type in the trig function (you will get a left parentheses) and the angle. You do not need to complete the parentheses. Press ENTER and out it comes. Although we can get extreme accuracy, we will find that four decimal places is usually enough. So set your calculator to 4 decimal places. Remember that angles are assumed to be in radians unless in degree format.

Example 1) Find the following:

a) $\sin 29^\circ =$ _____ b) $\cos 131^\circ =$ _____ c) $\tan\left(\frac{7\pi}{8}\right) =$ _____

If angles are input with more accuracy, it is assumed that they are in decimal degrees. Note that parentheses can be used to make the problems clearer in intent.

Example 2) Find the following:

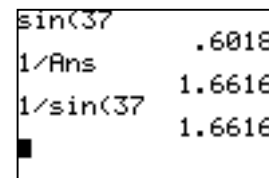
a) $\tan 12.8^\circ =$ _____ b) $\sin(-32.35^\circ) =$ _____ c) $\cos(0.724^\circ) =$ _____

If trig functions of angles that are in degree-minute-second form, use the Angle menu to input them. Remember that seconds are input with ALPHA +. $\cos 38^\circ 40' 29''$ would be input to the calculator thusly:

Example 3) Find the following:

a) $\sin 82^\circ 12' =$ _____ b) $\cos 126^\circ 42' 53'' =$ _____ c) $\tan(-8^\circ 57' 16'') =$ _____

Note that there are no keys for the csc, sec, or cot functions on your calculator. To find them we have to use the fact that sin and csc functions are reciprocals of each other, as are the cos and sec functions, and the tan and cot functions. There are three ways to find, for example $\csc 37^\circ$. Take $\sin 37^\circ$ and then take its reciprocal or simply finding $1/\sin 37^\circ$. The screen on the right shows these two methods. You could also find $\sin 37^\circ$ and then press the reciprocal key x^{-1} .

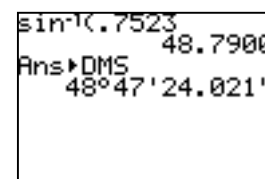


Example 4) Find the following:

- a) $\csc 81^\circ =$ _____ b) $\sec 122^\circ =$ _____ c) $\cot 34.2^\circ =$ _____
d) $\sec 338.292^\circ =$ _____ e) $\cot 14^\circ 29' 36'' =$ _____ f) $\csc 149^\circ 50' =$ _____

Many times, we want to reverse the process. We know the sine of an angle and we wish to find the angle itself. To accomplish this, we use **inverse trig functions** or **arc trig functions**. These are found on your calculator above the sin, cos, and tangent keys. We use the blue (2^{nd}) key to input them.

For instance, let us find the first quadrant angle whose sine is .7523. Note the screen on the right. Our answer would be 48.79° (expressed in decimal degrees). If we wanted our answer in degree – minute – second format, note how we would accomplish that by using the Angle menu.



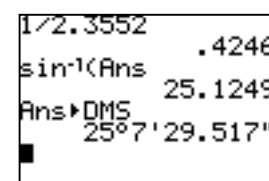
Example 5) Find the following (decimal degrees):

- a) $\sin^{-1}.9099 =$ _____ b) $\arccos 0.4231 =$ _____ c) $\tan^{-1} 1.8089 =$ _____

Example 6) Find the following (Degrees – minutes – seconds)

- a) $\arctan 4.002 =$ _____ b) $\sin^{-1}.0809 =$ _____ c) $\cos^{-1}.4998 =$ _____

Finally, if we wish to find an arccsc , arcsec , or arctan function, again, there is no one keystroke that will give it to you. To find $\csc^{-1} 2.3552$, for instance, we must first take the reciprocal of 2.3552, and then take the arcsin of that. On the right is the way you would accomplish this (with the optional changing into degrees-minutes-seconds):



Example 7) Find the following (decimal degrees):

- a) $\sec^{-1} 1.76 =$ _____ b) $\text{arccot } 3.4221 =$ _____ c) $\csc^{-1} 1.1102 =$ _____

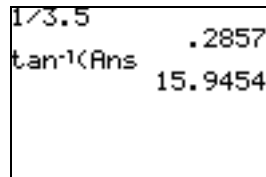
Example 8) Find the following (Degrees – minutes – seconds)

- a) $\text{arccsc } 3.8621 =$ _____ b) $\cot^{-1} 0.7501 =$ _____ c) $\text{arcsec } 5.8621 =$ _____

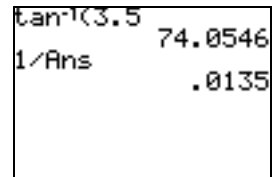
About errors:

Your calculator can take trig functions and arc trig functions to extreme accuracy. However, if you input the problem into the calculator incorrectly, one of two things will happen. One – an error. Assuming you typed it in the correct syntax, the calculator is telling you that it cannot perform the operation. This is actually good for

you. For instance, if you take $\cos^{-1} 1.4231$, the calculator gives you a domain error because we know that the cosine of any angle cannot be greater than one. Hopefully, you would be smart enough to realize what is happening. But if you input the problem incorrectly into the calculator and the calculator can perform the operation, you will get an answer and *you will believe it*. For instance, suppose you wanted to find $\cot^{-1} 3.5$. The correct way to accomplish this is on the screen below on the left. But many students know that a reciprocal is needed and do the steps on the right. They write down their answer of 0.0135° and they never know they are wrong. That is why it is vital that you learn these steps now and learn them well.

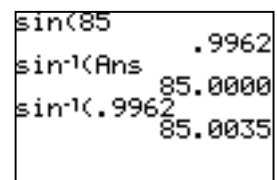


right way of calculating $\cot^{-1} 3.5$



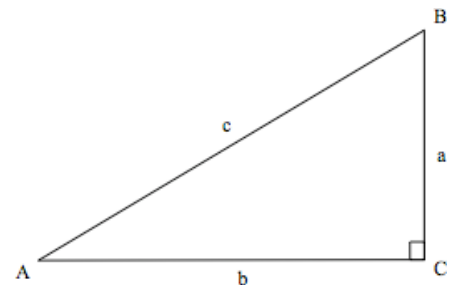
wrong way of calculating $\cot^{-1} 3.5$

Prove to yourself that $\sin$ and $\sin^{-1}$ are indeed inverses by taking the sine of an angle and immediately taking the inverse sin of that answer. Or reverse the process. In both cases, you should end up with the value you started. Note that you must use the **Ans** key on your calculator to achieve this effect. Type in the value and you will not get the number you started with because of round-off error. This is a good way to determine whether you truly understand how to take trig functions and inverse trig functions using the calculator.



Solving Right Triangles:

If we know two pieces of information of a right triangle, we can solve that triangle – that is find all the missing information from that triangle. For this section, we will make some generalizations: that the triangle will be labeled $\triangle ABC$ where the side opposite angle A is labeled a, the side opposite B is labeled b and the right angle is angle C and the hypotenuse is labeled c.



We can solve triangles if we are given a) an angle and the hypotenuse, b) the angle and a leg, c) a leg and the hypotenuse, and d) two legs. Our tools will be our trig functions and the Pythagorean Theorem: $a^2 + b^2 = c^2$.

You should always start by drawing a picture of the given information. While it doesn't have to be to scale, it should be somewhat close. Work should be shown and answers placed in appropriate places. We will assume that accuracy in sides is two decimal places and angles are to be in decimal degrees unless otherwise stated (if an angle is given in degree-minute-second format, assume that the other angle should also be in that format).

To best understand the process, expect problems to be given in this format: We will draw the picture and do the work on the right side. While the calculator does the number crunching, you **must** show how you are calculating the answers.

A = _____	a = _____
B = _____	b = _____
C = 90°	c = _____

Example 9) Angle and hypotenuse

$$\begin{array}{ll} A = 21^\circ & a = \underline{\hspace{2cm}} \\ B = \underline{\hspace{2cm}} & b = \underline{\hspace{2cm}} \\ C = 90^\circ & c = 14 \end{array}$$

Example 10) Angle and leg

$$\begin{array}{ll} A = 77.2^\circ & a = 29.5 \\ B = \underline{\hspace{2cm}} & b = \underline{\hspace{2cm}} \\ C = 90^\circ & c = \underline{\hspace{2cm}} \end{array}$$

Example 11) Angle and leg

$$\begin{array}{ll} A = 38^\circ 12' 44'' & a = 102.35 \\ B = \underline{\hspace{2cm}} & b = \underline{\hspace{2cm}} \\ C = 90^\circ & c = \underline{\hspace{2cm}} \end{array}$$

Note that we can find B:

90-38°12'44"
Ans►DMS 51.7878
51°47'16"

Example 12) Leg and hypotenuse

$$\begin{array}{ll} A = \underline{\hspace{2cm}} & a = 8 \\ B = \underline{\hspace{2cm}} & b = \underline{\hspace{2cm}} \\ C = 90^\circ & c = 13 \end{array}$$

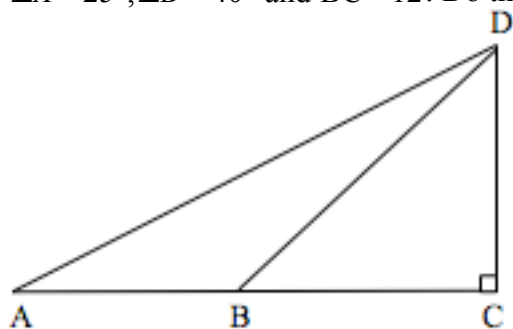
Example 13) Leg and leg

$$\begin{array}{ll} A = \underline{\hspace{2cm}} & a = 2 \text{ feet} \\ B = \underline{\hspace{2cm}} & b = 15 \text{ inches} \\ C = 90^\circ & c = \underline{\hspace{2cm}} \end{array}$$

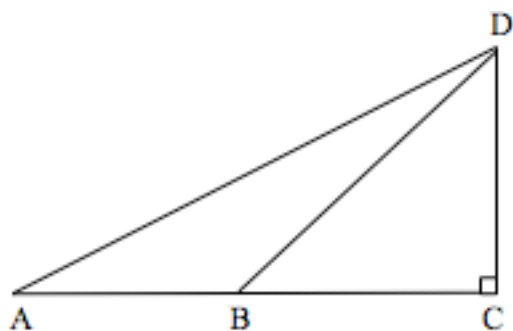
Angles in degree-minutes-seconds

Multi-Step Problems:

Example 14) Consider the picture below. I want to find the length of segment $\overline{AB}$. Suppose $\angle A = 25^\circ$, $\angle B = 40^\circ$ and $\overline{BC} = 12$. Do the necessary work on the right to find $\overline{AB}$.



Example 15) Let's tweak the problem slightly. I want to find the length of segment $\overline{CD}$. $\angle A = 25^\circ$, $\angle B = 40^\circ$ and $\overline{AB} = 12$. Note that we do not have any information about sides of either right triangle. And yet, it is possible to solve this problem. How?



Example 16) Using the same picture, $\angle A = 35^\circ$, $\angle B = 68^\circ$ and $\overline{AB} = 76.5$, find the length of segment $\overline{CD}$.

Real-Life Applications

Guidelines for solving a triangle problem:

- Draw a sketch of the problem situation. Don't be afraid to make it large.
- Look for right triangles and sketch them in.
- Mark the known sides and angles and unknown sides and angles using variables.
- Express the desired sides or angles in terms of trig functions with known quantities using the variables in the sketch.
- Solve the trig equation you generated and express the answer using correct units.

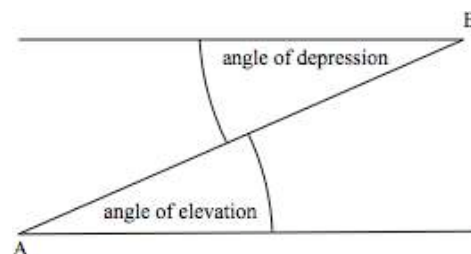
Surveying Problems: Problems involving finding quantities that would be too difficult to measure using rulers and other instruments.

- 17) I am standing at point A on one side of a river and wish to measure the distance across a river to a house H on the other side. I walk 50 feet along the riverbank to point B and measure the angle ABH to be $18^\circ 23'$. Find the distance across the river (assume a right triangle).

- 18) A guy-wire must be attached to a 50 foot pole. The angle that the guy wire must make with the ground has to be 75.5° . Find the length of the wire required to do the job.

Angle of Elevation and Depression. As a person at point A looks up at point B , an angle of elevation with the ground is created. When B looks down at point A , an angle of depression is created (with the horizontal). These angles are congruent. Why?

_____. Angles of elevation and depression are always created with the horizontal.

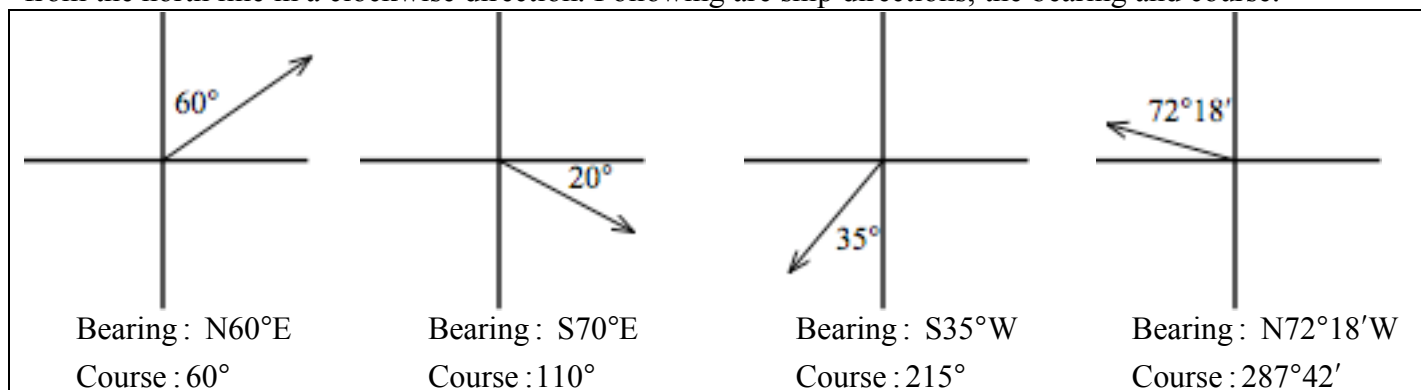


- 19) A device for measuring cloud height at night consists of a vertical beam of light that makes a spot on the clouds. That spot is viewed from a point 140 feet along the ground and the angle of elevation is $64^\circ 40'$. Find the height of the cloud.

- 20) While standing on a cliff 163 feet above the ocean, I see a sailboat in the distance at an angle of depression of 21.2° . What is the horizontal distance to the sailboat?

- 21) A wire holding up a 40 foot telephone pole is 18 feet long. The wire attached to the telephone pole 10.5 feet below the top. What is the angle of elevation of the wire?
- 22) From a window in building A, I observe the top of building B across the 50 foot wide street at an angle of elevation of $74^{\circ}25'$. I observe the base of building B at an angle of depression of $52^{\circ}18'$. Find the height of building B.

Bearing and Course: When ships or planes navigate, they need to have a simple way of explain what direction they are traveling. One way is called bearing. The bearing will be in the form (N or S angle E or W). A bearing is always drawn from the nearest north or south line. A heading (or course) is always drawn from the north line in a clockwise direction. Following are ship directions, the bearing and course.



Tip: in word problems, whenever you see or are asked for a bearing or heading, always look for the word “from” and draw an x - y axis at that point.

Tip: if the bearing from A to B is N65°E, the bearing from B to A will be the same angle but the opposite direction: S65°W.

- 23) A jeep leaves its present location and travels along bearing N62°W for 29 miles. How far north and west of its original position is it?

24) An airplane leaves an airport and travels due east for 255 miles. It then heads due south for 330 miles. From its current position, along what heading should it travel to reach the airport and how far is it?

25) Two small boats leave an island at the same time. Boat A travels due North for 21 miles and Boat B travels due west for 18 miles. How far apart are the boats and what is the bearing of boat A from boat B? How about the bearing of boat B from boat A?

26) A plane leaves an airport and travels 2 hours along heading 120° at 175 mph. It then turns onto heading 30° and travels 2.5 hours at 200 mph. How far from the airport is it and from its last position, what is the heading to the airport?

Unit 3 – Right Triangle Trigonometry – Homework

1. For each problem, calculate the answer to 4 decimal places. In the case of inverse trig, assume decimal degrees unless otherwise stated. If a problem is impossible, state so. (answers are given on the last page of this packet because you need to know whether or not you are getting these correct).

#	Problem	Answer
a.	$\sin 82^\circ$	
b.	$\cos 77^\circ$	
c.	$\tan 13.6^\circ$	
d.	$\cos 25^\circ 12'$	
e.	$\tan 225^\circ 28' 49''$	
f.	$\sin 95^\circ 30''$	
g.	$\csc 49^\circ$	
h.	$\sec(-24^\circ)$	
i.	$\cot 0.543^\circ$	
j.	$\sec 45^\circ 18' 9''$	
k.	$\csc 11^\circ 59''$	
l.	$\cot 2$	

#	Problem	Answer
m.	$\sin^{-1}.5$	
n.	$\cos^{-1} 0.1976$	
o.	$\arctan \sqrt{6}$	
p.	$\arccos(1.3761)$ DMS	
q.	$\arcsin(0.7901)$ DMS	
r.	$\tan^{-1} \sqrt[3]{10}$ DMS	
s.	$\csc^{-1} 4.25$	
t.	$\sec^{-1} 0.275$	
u.	$\operatorname{arccot} 2$	
v.	$\operatorname{arcsec}(3.895)$ DMS	
w.	$\cot^{-1} \sqrt{5}$ DMS	
x.	$\operatorname{arccsc}(2.95)$ DMS	

2. Solve the following right triangles. Sides to 2 decimal places, angles in decimal degrees unless otherwise stated. Show how you got your answers.

$A = 62^\circ$ $a =$ _____
 a) $B =$ _____ $b =$ _____
 $C = 90^\circ$ $c = 20$

$A =$ _____ $a =$ _____
 b) $B = 31^\circ 32'$ $b =$ _____
 $C = 90^\circ$ $c = 7.24$

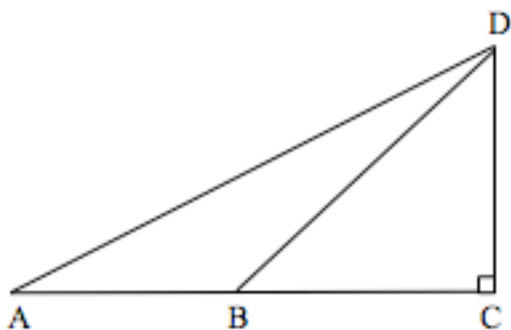
$A = 29.3^\circ$ $a =$ _____
 c) $B =$ _____ $b = 13.5$
 $C = 90^\circ$ $c =$ _____

$$\begin{array}{ll} A = \underline{\hspace{2cm}} & a = \underline{\hspace{2cm}} \\ \text{d) } B = 68.25^\circ & b = 2.45 \text{ inches} \\ C = 90^\circ & c = \underline{\hspace{2cm}} \end{array}$$

$$\begin{array}{ll} A = \underline{\hspace{2cm}} & a = 8 \\ \text{e) } B = \underline{\hspace{2cm}} & b = \underline{\hspace{2cm}} \\ C = 90^\circ & c = 17 \end{array}$$

$$\begin{array}{lll} A = \underline{\hspace{2cm}} & a = 9.25 & \\ \text{f) } B = \underline{\hspace{2cm}} & b = 8.75 & (\text{Angles in Deg-Min-Sec}) \\ C = 90^\circ & c = \underline{\hspace{2cm}} & \end{array}$$

$$\begin{array}{lll} A = \underline{\hspace{2cm}} & a = 2.25 \text{ miles} & \\ \text{g) } B = \underline{\hspace{2cm}} & b = 8,000 \text{ feet} & \\ C = 90^\circ & c = \underline{\hspace{2cm}} & \end{array}$$



3. In the figure above find segment CD if $\angle A = 27^\circ$, $\angle B = 51^\circ$ and $\overline{AB} = 8.2$ miles

For each word problem, draw a picture, fill in given sides and use variables for sides and angles you don't know, then show equations (and/or trig functions) to find out the desired information and circle your answer(s) being sure to use proper unit.

4. A surveyor wants to determine the horizontal distance that the top of a slope is from a tall house at the bottom of the slope. He measures the distance along the slope to be 125 feet. The angle of depression created from the top of the slope to the top and bottom of the house is 17° . Find the horizontal distance from the top of the slope to the house.

5. What is the angle of elevation of the sun when a 6'3" man casts a 10.5-ft shadow?

6. A street in San Francisco has a 20% grade – that is it rises 20 feet for every 100 feet horizontally. Find the angle of elevation of the street?

7. A kite is 120 feet high when 650 feet of string is out. What angle of elevation does the kite make with the ground?

8. From a balloon 2500 ft high, a command post is seen with an angle of depression of $8^\circ 45'$. How far is it from a point on the ground below the balloon to the command post?

9. An observer on a ladder looks at a building 100 ft away, noting that the angle of elevation of the top of the building is $28^{\circ}40'$ while the angle of depression of the bottom of the building is $7^{\circ}30'$. How tall is the building?
10. From a plane 2 miles high, the angles of depression to two towns in line with each other are $82^{\circ}55'$ and $12^{\circ}8'$. How far apart are the two towns?
11. Jay and Sam who are both on one side of a hill are staring at the top of that 180 foot tall hill. Jay, who is nearer the hill sees the top of the hill at an angle of elevation of $72^{\circ}58'$ while Sam, who is further from the hill sees the top of the hill at an angle of elevation of $41^{\circ}23'$. How far apart are Jay and Sam?
12. Jay and Sam are still staring at the top of the hill. Jay and Sam are 75 feet apart. The angle of elevation of the top of the hill for Jay is 22° while the angle of elevation of the top of the hill for Sam is 57° . Find the elevation of the hill.

13. An airplane travels at 165 mph for 2 hours in a direction of $S62^\circ W$ from Chicago. How far south and west is the plane from Chicago?
14. An airplane travels at 445 mph for 4.5 hours in a direction of 124° from Seattle. How far south and east is the plane from Seattle?
15. A ship leaves a port at 1:00 PM traveling at 13 knots directly north. Another ship leaves the same port at 2:00 PM traveling due east at 15 knots. At 9:00 PM, how far apart are the ships? Along what heading will the northern ship have to travel to reach the eastern ship which is now stopped?
16. A man is orienteering (traveling through forest area with only a compass). He leaves his base and travels due north for 2.5 miles. He then travels due west for .8 miles. He then travels due south for 4 miles. How far is he from his base and along what heading must he travel to reach it?

17. A small Cessna plane leaves Naples, Florida and travels 99 miles east to Miami, Florida. It then heads due south for 123 miles where it crashes into the ocean. If helicopter rescue is to be effected from Naples, how far and along what heading will the helicopter have to travel?
18. John and Ellen have a big fight and swear they will never talk to each other again. John leaves the airport traveling 180 miles along heading 25° . Ellen leaves the airport and travels 580 miles along heading 295° . If Ellen decides she was wrong and decides to catch a flight to John that can travel at 500 mph, how long will it take her to fly into his arms?

Answers to Problem 1

#	Problem	Answer
a.	$\sin 82^\circ$	0.9903
b.	$\cos 77^\circ$	0.2250
c.	$\tan 13.6^\circ$	0.2419
d.	$\cos 25^\circ 12'$	0.9048
e.	$\tan 225^\circ 28' 49''$	1.0169
f.	$\sin 95^\circ 30''$	0.9962
g.	$\csc 49^\circ$	1.3250
h.	$\sec(-24^\circ)$	1.0946
i.	$\cot 0.543^\circ$	105.5139
j.	$\sec 45^\circ 18' 9''$	1.4217
k.	$\csc 11^\circ 59''$	5.2331
l.	$\cot 2$	-0.4577

#	Problem	Answer
m.	$\sin^{-1} .5$	30°
n.	$\cos^{-1} 0.1976$	78.6034°
o.	$\arctan \sqrt{6}$	67.7923°
p.	$\arccos(1.3761)$ DMS	impossible
q.	$\arcsin(0.7901)$ DMS	$52^\circ 11' 41''$
r.	$\tan^{-1} \sqrt[3]{10}$	65.1012°
s.	$\csc^{-1} 4.25$	13.6090°
t.	$\sec^{-1} 0.275$	impossible
u.	$\operatorname{arccot} 2$	26.5651°
v.	$\operatorname{arcsec}(3.895)$ DMS	$75^\circ 7' 24''$
w.	$\cot^{-1} \sqrt{5}$ DMS	$24^\circ 5' 41''$
x.	$\operatorname{arccsc}(2.95)$ DMS	$19^\circ 48' 54''$