M308 – Advanced Algebra 2
Second Semester Review
Packet – Short Answer

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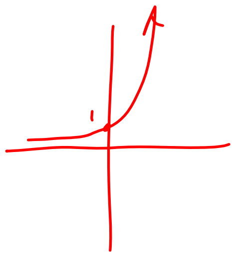
Teacher ____________________

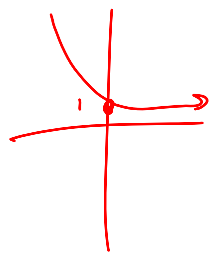
Chapter 4: Inverse Functions, Exponential Functions, and Logarithmic Functions

1) (calculator) Sketch a graph of the following:

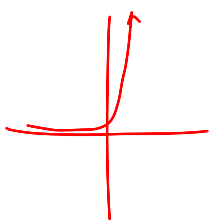
a) $f(x) = 4^x$



b) $f(x) = e^{-x}$

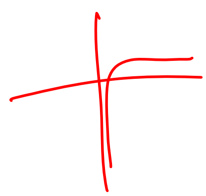


c) $f(x) = e^{3x}$ make sure to enter as $y = e \wedge (3x)$



d) $f(x) = \log_3 x$

enter as log(x) / log(3) by using the change of base formula



2) Solve $4^x = 2$

take ln of both sides

$\ln 4^x = \ln 2$

bring exponent to front, then divide

$\underline{x \ln 4} = \underline{\ln 2}$

$x = \dfrac{\ln 2}{\ln 4} = \dfrac{1}{2}$

OR rewrite with same base

$2^{2x} = 2^1$

Set exponents = + solve

$2x = 1$

$x = \frac{1}{2}$

3) Write the following in logarithmic form.

a) $3^4 = 81$

$\log_3 81 = 4$

b) $5^{-3} = \dfrac{1}{125}$

$\log_5 \frac{1}{125} = -3$

4) Write the following in exponential form.

a) $\log_3 \sqrt[3]{9} = \frac{2}{3}$

$3^{2/3} = \sqrt[3]{9}$

b) $\log_c t = a$

$c^a = t$

c) $\log_4 \left(\frac{1}{64}\right) = -3$

$4^{-3} = \frac{1}{64}$

5) Find the value of the following.

a) $\log_{25} 5 = x$

$25^x = 5$

$5^{2x} = 5^1$

$2x = 1$

$\boxed{x = \frac{1}{2}}$

b) $\log_8 8 = x$

$\boxed{1}$

c) $\ln e^2$

$\boxed{2}$

d) $\log_3 81$

$3^x = 81$

$3^x = 3^4$

$\boxed{x = 4}$

e) $\log_7 1$

$\boxed{0}$

log 1 always = 0

f) $\ln e^1$

$\boxed{1}$

6) Write the following expressions as a sum, difference, or product of logarithms.

a) $\log_5\left(\frac{5\sqrt{7}}{3}\right)$

$\log_5 5 + \log_5 \sqrt{7} - \log_5 3$

$\boxed{1 + \frac{1}{2}\log_5 7 - \log_5 3}$

b) $\log_z \frac{x^5 y^3}{3a^2}$

$\log_z x^5 + \log_z y^3 - \log_z 3 - \log_z a^2$

$\boxed{5\log_z x + 3\log_z y - \log_z 3 - 2\log_z a}$

7) Write the following as a single logarithm.

a) $3\log_a x - 2\log_a y - \log_a m$

$\log_a \frac{x^3}{y^2 m}$

b) $\log_b (x-2) + 3\log_b x - \log_b 8$

$\log_b \frac{(x-2)(x^3)}{8}$

8) **Solve:** $\ln(y+2)-\ln(y-7)=\ln 4$

$\ln\dfrac{(y+2)}{y-7}=\ln 4$

$\dfrac{y+2}{y-7}=4$

$y+2=4(y-7)$

$y+2=4y-28$

$30=3y$

$\boxed{y=10}$

9) **Solve:** $\log_4(z+3)+\log_4(z-3)=1$

$\log_4 (z+3)(z-3)=1$

$4^1=(z+3)(z-3)$

$4=z^2-9$

$\boxed{13=z^2}$

$z=\pm\sqrt{13}$

However... only $+\sqrt{13}$ works b/c a log must be positive and $-\sqrt{13}+3$ is <0

$\boxed{\sqrt{13}}$

10) **Solve:** $3^{a+2}=5$

$(a+2)\ln 3=\ln 5$

$a\ln 3+2\ln 3=\ln 5$

$\dfrac{a\ln 3}{\ln 3}=\boxed{\dfrac{\ln 5-2\ln 3}{\ln 3}}$

OR

$\boxed{\approx -.5350}$

11) **(calculator) Solve:** $e^{2x+3}=7$

$\ln e^{2x+3}=\ln 7$

$2x+3=\ln 7$

$x=\boxed{\dfrac{\ln 7-3}{2}=-.5270}$

12) **(calculator) Solve:** $\log_{10} x=2.437$

$10^{2.437}=x$

$\boxed{x=273.5269}$

13) **(calculator) Solve:** $3^x=2x+3$

easiest way: graph $y_1=3^x$

$y_2=2x+3$ and find point of intersection

$(-1.3916, .2168)$

$(1.6856, 6.3711)$

14) **(calculator) Solve:** $2^x=-3x+4$

Same as 13

$(0.7663, 1.7010)$

15) **(calculator) Solve:** $\ln_e x=1.967$

$e^{1.967}=x$

$\boxed{7.1492}$

16) (calculator) How much would you need to invest in an account earning 3.25% interest compounded weekly to have \$15,000 after 8 years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $\quad$ $15000 = P\left(1+\frac{.0325}{52}\right)^{52(8)}$

$15000 = P(1.2968)$ $\quad$ $\boxed{P = \$11566.71}$

17) Consider the function defined by $f(x)=\sqrt{4-x}$. Find $f^{-1}(x)$. Find the domain and range for f(x) and for $f^{-1}(x)$.

To find inverse, swap x & y var and solve for y. The $\overset{f(x)\ f^{-1}(x)}{D \to R}$ and $\overset{f(x)\ f^{-1}(x)}{R \to D}$

	$f(x)$	$f^{-1}(x)$
D:	$(-\infty, 4]$	$[0, \infty)$
R:	$[0, \infty)$	$(-\infty, 4]$

To find inverse

$x = \sqrt{4-y}$

$x^2 = 4-y$

$y = 4-x^2$

$\boxed{f^{-1}(x) = 4-x^2}$

18) Find the inverse of the function $f(x)=x^2+1,\ x \le 0$. Find the domain and range for f(x) and for $f^{-1}(x)$.

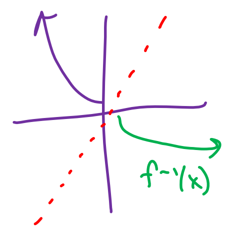
	$f(x)$	$f^{-1}(x)$
D	$(-\infty, 0]$	$[1, \infty)$
R	$[1, \infty)$	$(-\infty, 0]$

$f^{-1}(x)$

$x = y^2+1$

$x-1 = y^2$

$y = \pm\sqrt{x-1}$



$f^{-1}(x) = -\sqrt{x-1}$

keep the negative one because that is the one that is symm with the line $y=x$

19) (calculator) Find the future value if \$3200 is invested at 4.35% compounded monthly for 7 years.

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 3200\left(1+\frac{.0435}{12}\right)^{12(7)}$

$A = 3200(1.3552)$

$\boxed{= \$4336.64}$

20) (calculator) Find the future value of \$1750 compounded continuously for 5 years at 3.25%.

$A = Pe^{rt}$

$1750e^{.0325(5)}$

$A = \boxed{\$2058.78}$