NAME Key

EXAM 2/90 POINTS POSSIBLE

YOUR GRAPHING CALCULATOR SHOULD ONLY BE USED TO CHECK YOUR RESULTS. CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANISED, EASY TO READ FASHION.

1. (5 POINTS) Let $f(x)=\frac{1}{2}x^2+1x-10$.

   a. (2 POINTS) Determine, without graphing, whether the given polynomial function has a maximum value or minimum value. Explain.

   $\boxed{f \text{ has a minimum since } a=\frac{1}{2}>0.}$

   b. (3 POINTS) Find the minimum value or maximum value without using your calculator. Do not use your graphing calculator and show all work.

   $-\frac{b}{2a}=-\frac{1}{\cancel{2}(\cancel{\frac{1}{2}})}=-1 \rightarrow f(-1)=\frac{1}{2}(-1)^2+(-1)-10$

   $f(-1)=\frac{1}{2}-11\frac{2}{2}$

   $f(-1)=-21/2$

   The minimum is $-\frac{21}{2}$.

2. (3 POINTS) Determine whether the function is a polynomial function. If it is, state the degree. If it is not, explain why not.

   $f(x)=x^7-\frac{3}{x^4}+2$

   $f(x)=x^7-3x^{-4}+2$

   NO, since the middle term has a negative exponent.

3. (8 POINTS) Determine whether the function is linear or nonlinear. If it is linear, determine the equation of the line.

x	y
-2	6
-1	4
0	2
1	0
2	-2

The change in x is always 1, and the change in y is always -2. So the slope is $\frac{-2}{+1}=-2$. Using the y-int, we have the $\boxed{\text{linear function } f(x)=-2x+2.}$

4. (10 POINTS) Solve the following inequality. Do not use your graphing calculator and show all work.

$$\frac{x-5}{x^2-81} \geq 0$$

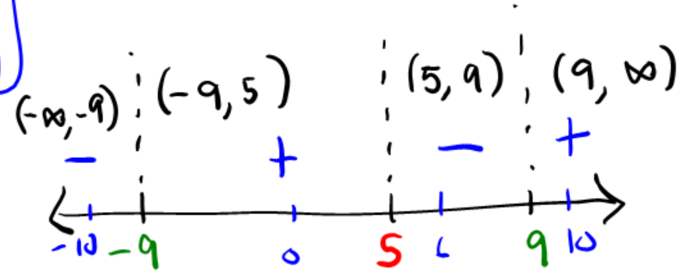
Consider $f(x) = \frac{x-5}{x^2-81} = \frac{x-5}{(x+9)(x-9)}$

$0 = x - 5$

$x = 5$

Vertical asymptotes at $x = \pm 9$

$\boxed{(-9,5] \cup (9,\infty)}$



$f(-10) = \frac{(-10)-5}{(-10)^2-81} = \frac{-15}{19} < 0$

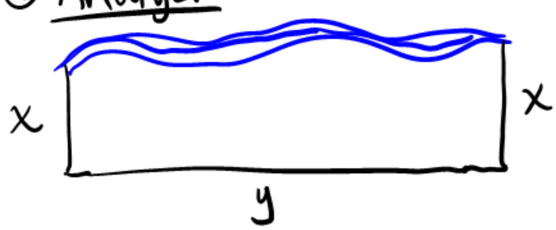
$f(0) = \frac{(0)-5}{(0)^2-81} = \frac{-5}{-81} > 0$

$f(6) = \frac{(6)-5}{(6)^2-81} = \frac{1}{-45} < 0$

$f(10) = \frac{(10)-5}{(10)^2-81} = \frac{5}{19} > 0$

5. (10 POINTS) Farmer Ed has 3,500 meters of fencing, and wants to enclose a rectangular plot that borders on a river. If Farmer Ed does not fence the side along the river, what is the largest area that can be enclosed? Use a logical, easy to follow, problem solving process, and show all work for full credit.

① Analysis



$2x + y = 3500$

$y = 3500 - 2x$

② Primary Equation

$A(x,y) = xy$

③ Reduce Primary to one variable

$A(x) = x(3500 - 2x)$

$A(x) = 3500x - 2x^2$

④ Find maximum

$-\frac{b}{2a} = -\frac{3500}{2(-2)} = 875$

$A(875) = 3500(875) - 2(875)^2$

$A(875) = 1,531,250$

⑤ Conclusion

The maximum area that can be enclosed is 1,531,250 sq. feet.

6. (10 POINTS) Consider The function $f(x)=\dfrac{x^3-x^2-5x+5}{x^2-1}$.

$$f(x)=\frac{x^2(x-1)-5(x-1)}{(x+1)(x-1)}$$

$$f(x)=\frac{(x^2-5)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

$$f(x)=\frac{x^2-5}{x+1},\quad x\neq 1$$

a. (2 POINTS) What is the domain of f?

$x\neq 1$, $x+1\neq 0$

$x\neq -1$

$$\boxed{(-\infty,-1)\cup(-1,1)\cup(1,\infty)}$$

b. (4 POINTS) Find the vertical asymptote(s) of f. If there are no vertical asymptotes, write "none".

$$\boxed{x=-1}$$

c. (2 POINTS) Find the horizontal asymptote(s) of f. If there are no horizontal asymptotes, write "none".

$$\boxed{\text{NONE}}$$

d. (2 POINTS) Find the oblique (slant) asymptote of f. If there is no oblique asymptote, write "none".

$$\begin{array}{r}
x-1 \\
x+1\,\big)\,\overline{x^2+0x-5} \\
-(x^2+1x) \\
\hline
-x-5 \\
-(-x-1) \\
\hline
-4
\end{array}$$

$$\boxed{y=x-1}$$

7. (4 POINTS) Use the rational zeros theorem to list the potential rational zeros of the polynomial function.
$f(x)=6x^6-5x^2-12x+3$

possibilities for p: $\pm1, \pm3$

possibilities for q: $\pm1, \pm2, \pm3, \pm6$

$$\boxed{\text{potential rational zeros: } \pm1, \pm\tfrac{1}{2}, \pm\tfrac{1}{3}, \pm\tfrac{1}{6}, \pm3, \pm\tfrac{3}{2}}$$

8. (8 POINTS) Form a polynomial $f(x)$ with real coefficients having the given degree and zero.

Degree 4; zeros: $\sqrt{3},\ 2+i$

If $x=\sqrt{3}$ is a zero, $x=-\sqrt{3}$ is a zero.
If $x=2+i$ is a zero, $x=2-i$ is a zero.

$x=-\sqrt{3}$ or $x=\sqrt{3}$ or $x=2-i$ or $x=2+i$

$x+\sqrt{3}=0$ or $x-\sqrt{3}=0$ or $x-(2-i)=0$ or $x-(2+i)=0$

$0=(x+\sqrt{3})(x-\sqrt{3})[x-(2-i)][x-(2+i)]$

$f(x)=[x^2-(\sqrt{3})^2][x^2-(2+i)x-(2-i)x+(2-i)(2+i)]$

$f(x)=(x^2-3)[x^2-2x-ix-2x+ix+(2)^2-(i)^2]$

$f(x)=(x^2-3)[x^2-4x+4-(-1)]$

$f(x)=(x^2-3)(x^2-4x+5)$

$f(x)=x^4-4x^3+5x^2-3x^2+12x-15$

$\boxed{f(x)=x^4-4x^3+2x^2+12x-15}$

9. (8 POINTS) Find the real solutions of the following equation without using your graphing calculator.

$2x^4 + 5x^3 + 11x^2 + 20x + 12 = 0$.

potential rational zeros: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrrr} 1 & 2 & 5 & 11 & 20 & 12 \\ & & 2 & 7 & 18 & 38 \\ \hline & 2 & 7 & 18 & 38 & 50 \end{array}$$

NO

$$\begin{array}{r|rrrrr} -1 & 2 & 5 & 11 & 20 & 12 \\ & & -2 & -3 & -8 & -12 \\ \hline & 2 & 3 & 8 & 12 & 0 \end{array}$$

yes!

$$2x^4 + 5x^3 + 11x^2 + 20x + 12 = 0$$
$$(x+1)(2x^3 + 3x^2 + 8x + 12) = 0$$
$$(x+1)[x^2(2x+3) + 4(2x+3)] = 0$$
$$(x+1)(2x+3)(x^2+4) = 0$$

$x+1 = 0$ or $2x+3 = 0$ or $x^2+4 = 0$

$x = -1$ $\quad 2x = -3$ $\quad x^2 = -4$

$x = -3/2$ $\quad x = \pm 2i$

imaginary

$\boxed{\text{The real zeros are } -1 \text{ and } -\frac{3}{2}.}$

10. (24 POINTS) Consider the polynomial function $f(x)=(x+5)^2(x-6)^2$

a. (2 POINTS) Determine the end behavior of the graph of the function:

The graph of f behaves like y = x^4 for large values of x .

b. (4 POINTS) Find the x-intercept(s), if any. Do not use your graphing calculator and show all work.

$0=(x+5)^2(x-6)^2 \rightarrow 0=(x+5)^2$ or $0=(x-6)^2$

$0=x+5$ $\quad$ $0=x-6$

$x=-5$ $\quad$ $x=6$

$\boxed{x=-5,6 \quad (-5,0),(6,0)}$

c. (2 POINTS) Find the y-intercept, if any.

$f(0)=(0+5)^2(0-6)^2=(25)(36)=900$

$\boxed{y=900 \quad (0,900)}$

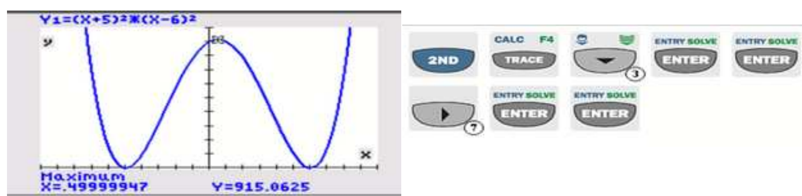
d. (4 POINTS) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis.

i. The zero(s) of f are $x=-5,6$.

ii. The zeros each have a zero of multiplicity 2, so the graph of f touches the x-axis at x = -5 and x = 6.

e. (2 POINTS) Use your graphing calculator to approximate the turning point(s) of the graph. Round the coordinates to 2 decimal places.

$\boxed{(-5,0),(0.50,915.06),(6,0)}$



f. (2 POINTS) Find the domain of the function. You may use either interval or set-builder notation.

$\boxed{(-\infty,\infty)}$ or $\boxed{\{x \mid x \in \mathbb{R}\}}$

g. (2 POINTS) Find the range of the function. You may use either interval or set-builder notation.

$\boxed{[0,\infty)}$ or $\boxed{\{y \mid y \in \mathbb{R},\ y \geq 0\}}$

h. (6 POINTS) Use the graph to determine where the function is increasing or decreasing. Give your results in interval notation.

i. On which interval(s) is the function increasing? Round the coordinates to 2 decimal places.

$\boxed{(-5,0.50)\cup(6,\infty)}$

ii. On which interval(s) is the function decreasing? Round the coordinates to 2 decimal places.

$\boxed{(-\infty,-5)\cup(0.50,6)}$