

**Instructions:** 120 minutes. Closed book. You are permitted use of a financial calculator and two sheets (8½" x 11") of notes. Answer *all* questions. Each question carries the number of points indicated. Total points is 100. Show all work on the exam paper itself. If you do not have or can not calculate a number that you need, make an assumption and proceed with the rest of the question. *No credit will be given for illegible, unsupported or ambiguous answers.*

**Practice Final Exam Questions**

- I. [28 points] XYZ Co pays monthly dividends and its current dividend per share is \$0.25. XYZ’s dividend per share is expected to grow at the same rate per month indefinitely and its current share price is \$20. The riskless rate has always been 0.8% per month and will remain at 0.8%. A market model regression of XYZ’s monthly stock return on the market’s monthly return has a slope coefficient of 1.2. The expected monthly return on the market portfolio is 1.3% and its standard deviation is 0.9%.
- A. Suppose the CAPM holds.
1. What is the intercept of the market model regression of XYZ’s monthly stock return on the market’s monthly return?
  2. What is the expected monthly return on XYZ’s equity?
  3. What is expected dividend growth rate for XYZ?
- B. Suppose instead that the ICAPM holds and investors care about  $\{E[R_p(t)], \sigma[R_p(t)], TERM(t)\}$  where  $TERM(t)$  is the difference between the yield on a long term bond and the yield on a short term bond at the end of month  $t$ . The following information is available (the table also summarizes some information already presented):

$i$	$E[R_i(t)]$	$\beta_{i,M}$	$\beta_{i,TERM}$
M	1.3%	?	0.5
XYZ	?	1.2	0.7
QV	1.1%	0.9	0.2

where:

$\beta_{i,M}$  is the slope coefficient from the regression of asset  $i$ ’s monthly return on the market portfolio’s monthly return:

$$R_i(t) = \alpha_{i,M} + \beta_{i,M} R_M(t) + e_{i,M}(t); \text{ and,}$$

$\beta_{i,TERM}$  is the slope coefficient from the regression of asset  $i$ ’s monthly return in month  $t$  on  $TERM(t)$  (a time series regression):

$$R_i(t) = \alpha_{i,TERM} + \beta_{i,TERM} TERM(t) + e_{i,TERM}(t).$$

1. What is  $\beta_{M,M}$ ?
2. What is the expected return on XYZ?

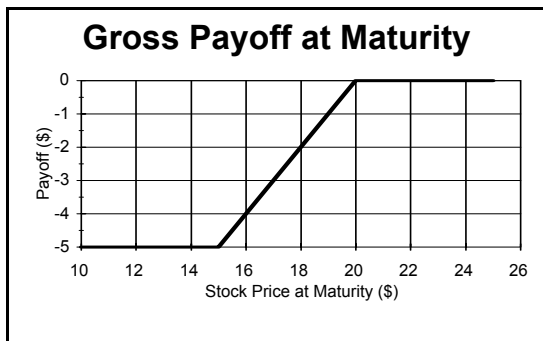
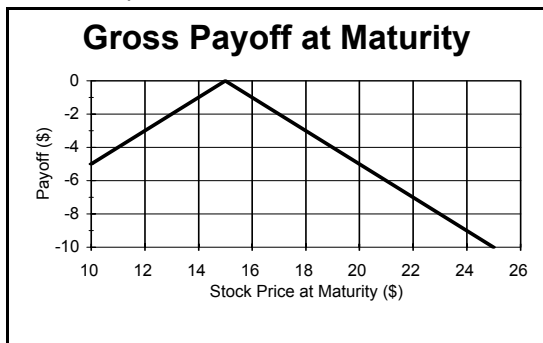
3. What is the intercept of the market model regression of XYZ’s monthly return on the market’s monthly return (i.e., what is  $\alpha_{XYZ,M}$ ?)
4. Is there any reason for not using the constant growth dividend discount model in this setting?

II. [20 points]

- A. If forward-spot parity holds, what is the one-year forward price of the French Franc, assuming that the current exchange rate is \$.20/Franc, the French one-year interest rate is 8% (expressed as an EAR) and the U.S. one-year rate is 6% (also expressed as an EAR)?
- B. The one-year dividend yield on the S&P is 4%, the current one-year risk-free rate is 6% (expressed as an EAR) and the one-year forward price on the S&P index is 460. The dividends are paid at the end of 12 months.
  1. If forward-spot parity holds, what is the current level of the S&P index?
  2. If the current S&P index is 10 points higher than you calculated in the previous part, describe a risk-free arbitrage and compute the cash flows on it.

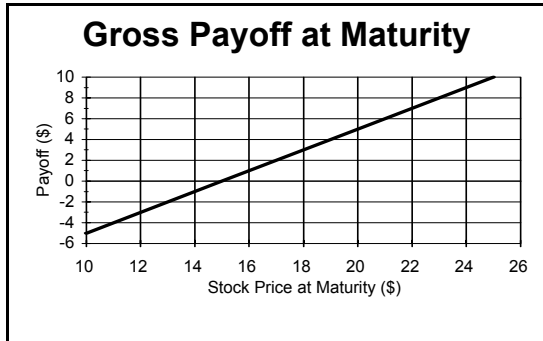
III. [16 points] Describe a portfolio of puts, calls (with their exercise prices), bonds (with par value) and shares of stock that would have the indicated gross payoffs (i.e., terminal value, ignoring the amount paid/received to set up the portfolio). Each put and each call must have 1 share of stock as the underlying, be European and expire at the terminal date. The bonds must be discount bonds which mature at the terminal date.

A.

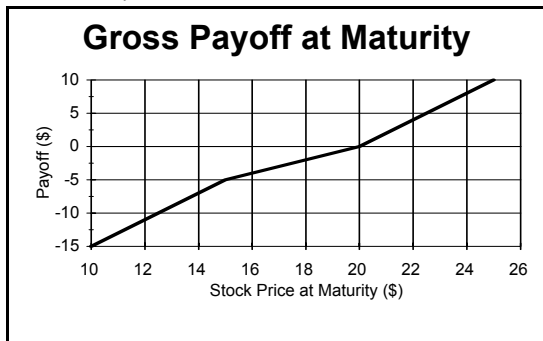


B.

C.



D.



IV. [16 points] The common stock of Sternco is currently trading at \$40 per share (up from \$25 at the beginning of the year). Sternco is currently “in play” as a takeover target and is not expected to pay any dividends for the next 6 months. You believe that if management is successful at repelling all offers, the stock will fall significantly, but if they are unsuccessful, the stock will rise. You want to profit from either outcome. The risk-free rate is 10% (continuously compounded annual rate) and a 6-month call option with an exercise price of \$40 is selling at \$4.

- A. A dealer offers you a 6-month European put option with an exercise price of \$40. What is a fair price for this option?
- B. Propose a strategy to take advantage of your beliefs that uses one or more of the following instruments: the stock, 6-month call options with an exercise price of \$40, 6-month European put options with an exercise price of \$40 and discount bonds maturing in 6 months.
- C. One month ago the firm trying to take over Sternco publicly announced its intentions. This announcement caused Sternco’s stock price to increase and the volatility of Sternco’s stock return to increase. What would have happened to the price of

European calls on Sternco's stock on the announcement date?

- V. [5 points] If forward-spot parity holds, what is the current spot price of the English Pound (expressed as U.S. dollars per English pound), assuming that the one year forward rate is \$1.80/£, the English one-year interest rate is 7% (expressed as an EAR) and the U.S. one-year rate is 9% (also expressed as an EAR)?

**Solutions**

I.

A.

1.  $\alpha_{XYZ,M} = (1 - \beta_{XYZ,M}) R_f = (1 - 1.2) \times 0.8\% = -0.16\%$ .
2. XYZ lies on the SML:

$$E[R_{XYZ}(t)] = R_f + \beta_{XYZ,M} \{E[R_M(t)] - R_f\} = 0.8\% + 1.2 \{1.3\% - 0.8\%\} = 1.4\%.$$

3. Use the constant growth DDM:

$$P_{XYZ}(0) = D_{XYZ}(0) [1+g] / \{E[R_{XYZ}(t)] - g\}$$

$$20 = 0.25 [1+g] / \{0.014 - g\}$$

$$20 \{0.014 - g\} = 0.25 [1+g]$$

$$0.28 - 20g = 0.25 + 0.25g$$

$$g = 0.03/20.25 = 0.00148 = 0.148\% \text{ per month.}$$

B.

1. By definition,  $\beta_{M,M} = 1$ .
2. In an ICAPM world, the following holds for any asset i:

$$i: E[R_i(t)] = R_f + \beta_{i,M} \lambda_M + \beta_{i,TERM} \lambda_{TERM}$$

Thus,

$$M: 1.3\% = 0.8\% + 1 \times \lambda_M + 0.5 \times \lambda_{TERM}$$

$$QV: 1.1\% = 0.8\% + 0.9 \lambda_M + 0.2 \times \lambda_{TERM}$$

Now M implies  $\lambda_M = \{0.5\% - 0.5 \times \lambda_{TERM}\}$ . Substituting into QV gives

$$0.3\% = 0.9 \{0.5\% - 0.5 \times \lambda_{TERM}\} + 0.2 \times \lambda_{TERM}$$

$$-0.15\% = -0.25 \times \lambda_{TERM}$$

which implies  $\lambda_{TERM} = 0.6\%$  and  $\lambda_M = 0.2\%$ . So finally

$$XYZ: E[R_{XYZ}(t)] = 0.8\% + 1.2 \times 0.2\% + 0.7 \times 0.6\% = 1.46\%.$$

3. Since the regression coefficients  $\alpha_{XYZ,M}$  and  $\beta_{XYZ,M}$  are chosen so that

$$E[e_{XYZ,M}(t)] = 0,$$

$$E[R_{XYZ}(t)] = \alpha_{XYZ,M} + \beta_{XYZ,M} E[R_M(t)]$$

and so

$$\alpha_{XYZ,M} = E[R_{XYZ}(t)] - \beta_{XYZ,M} E[R_M(t)] = 1.46\% - 1.2 \times 1.3\% = -0.1\%.$$

4. There may be a reason for not using the constant growth DDM. If investors

care about TERM(t) because it is correlated with expected returns in month t+1 then expected returns are varying through time. In contrast, the growth DDM assumes expected return on equity is constant through time.

II.

A. Covered interest rate parity says:

$$S^{S/f}(0) = F_T^{S/f}(0) [1+y^{*f}_1(0)] / [1+y^{*s}_1(0)]$$

which implies

$$F_T^{S/f}(0) = S^{S/f}(0) [1+y^{*s}_1(0)] / [1+y^{*f}_1(0)] = 0.2 \{1.06/1.08\} = 0.1963.$$

B.

1. Using spot forward parity with negative carrying costs:

$$F_1(0) / [1+y^{*f}_1(0)] - C(1) / [1+y^{*f}_1(0)] = S(0)$$

$$F_1(0) - \{-D(1)\} = S(0) [1+y^{*f}_1(0)]$$

$$F_1(0) = S(0) [1+y^{*f}_1(0)] - D(1)$$

$$F_1(0) = S(0) [1+y^{*f}_1(0)] - S(0) 0.04$$

$$460 = S(0) [1+.06-.04]$$

$$\Rightarrow S=450.98$$

2. If  $S(0) = 460.98$ , then:

Action	Today	at T=1 year
sell index at 0 and close out at T=1	+460.98	-S(T)-460.98(.04)
invest in 1 year discount bonds with face value {460 + 460.98(.04)}	$-\{460 + 460.98(.04)\}/1.06$ = 451.36	{460 + 460.98(.04)}
buy 1 stock index future today with delivery at T=1	0	S(T)-460
<b>Total</b>	<b>9.62</b>	<b>0</b>

III. These are not the only correct answers.

- A. Sell 1 C(X=15) and sell 1 P(X=15).
- B. Buy 1 P(X=15) and sell 1 P(X=20).
- C. Buy 1 C(X=15) and sell 1 P(X=15).
- D. Sell 1 P(X=20), sell 1 P(X=15) and buy 2 C(X=20).

IV.

A. Use put call parity to value the put.

$$S(0) = C_{40,0.5}(0) - P_{40,0.5}(0) + 40 e^{-\frac{1}{2} \cdot 0.1}$$

$$P_{40,0.5}(0) = C_{40,0.5}(0) + 40 e^{-0.1 \times \frac{1}{2}} - S(0) = 4 + 38.05 - 40 = 2.05.$$

B. Buy 1 P(X=40) and buy 1 C(X=40).

C. Using Black Scholes pricing model, know C(0) is increasing in S(0) and is increasing in  $\sigma$ . So both the stock price increase and stock return volatility increase cause the prices of calls on the stock to increase.

V. Use the covered interest parity theorem:

$$S^{\$/\pounds}(0) = F_T^{\$/\pounds}(0)[1+y^{\pounds}_T(0)]^T / [1+y^{\$}_T(0)]^T = 1.8 \times [1.07]/[1.09] = 1.77.$$

**Practice Final Exam**

I. [15 points] VLP has just paid a dividend of \$0.95 per share. It has a plowback (retention) ratio of .4, and a current market price of \$12. Dividends are paid annually and the CAPM holds for annual returns. The expected annual return on the market is 11% and the riskless rate is 6%. VLP's stock has a Beta with respect to the market of 1.8. The expected return on book equity each year is a constant (ROE).

A. What is the expected annual return on VLP's stock?

Use the constant dividend growth model to answer the following questions.

- B. What is the expected growth rate of dividends for VLP stock?
- C. What is the expected return on book equity (ROE) for VLP?
- D. What is the book value of equity (per share) today for VLP?
- E. If you buy VLP stock today and hold it for one year, what is your expected

holding period return?

- II. [7 points] WT Co has assets worth \$15M and has equity with a market value of \$10M. The Beta of WT’s assets (with respect to the market portfolio) is 1.5 while the Beta of WT’s equity (with respect to the market portfolio) is 2.
- What is the market value of WT’s debt?
  - What is the Beta of WT’s debt (with respect to the market portfolio)?
  - Is WT’s debt riskless?
- III. [12 points] Let  $DEF(\text{Jan})$  be the difference in the yield on a long term low-grade corporate bond and the yield on a long term government bond at the end of January and let  $R_M(\text{Jan})$  be the January return on the market portfolio. Suppose each individual cares about  $\{E[R_p(\text{Jan})], \sigma[R_p(\text{Jan})], \sigma[R_p(\text{Jan}), DEF(\text{Jan})]\}$  when forming his/her portfolio  $p$  for January. The following additional information is available:

$i$	$E[R_i(\text{Jan})]$	$\beta^*_{i,M}$	$\beta^*_{i,DEF}$
Pink	1.73%	1.3	0.25
Grey	1.34%	0.9	0.1

where  $\beta^*_{i,M}$  and  $\beta^*_{i,DEF}$  are regression coefficients from a multiple regression (time-series) of  $R_i(t)$  on  $R_M(t)$  and  $DEF(t)$ :

$$R_i(t) = \phi_{i,0} + \beta^*_{i,M} R_M(t) + \beta^*_{i,DEF} DEF(t) + e_i(t).$$

Also know that riskless rate for January,  $R_f(\text{Jan})$ , 0.7%.

- What is the risk premium for bearing  $\beta^*_{i,M}$  risk?
  - What is the expected January return on the market portfolio  $E[R_M(\text{Jan})]$ ?
  - What is the risk premium for bearing  $\beta^*_{i,DEF}$  risk?
  - Is the market portfolio on the minimum variance frontier of the risky assets in the economy? Why or why not?
- IV. [12 points] Suppose the CAPM with no riskless borrowing or lending holds. The expected return on the market portfolio is 12% and the expected return on VW Co is 10%. VW Co has a beta with respect to the market of 0.75.
- Do all individuals hold the market portfolio? If individuals do not hold the market portfolio, describe the portfolios they do hold.
  - Does the market portfolio lie on the minimum variance frontier. Explain why or why not?
  - Do you have enough information to determine the expected return on an asset whose return is uncorrelated with the market portfolio’s return. If so, what is the

expected return on an asset whose return is uncorrelated with the market portfolio's return?

- V. [15 points] Today's price for a 6 month discount bond is 97 and today's price for a 12 month discount bond is 94. Today's forward price for 1 oz of gold to be delivered in 6 months is \$465. There are no carrying costs associated with holding gold.
- A. In the absence of arbitrage, what must be the spot price of 1 oz of gold today?
- B. Suppose that today's forward price for 1 oz of gold to be delivered in 12 months is \$475 and that you do not know the spot price of gold today (so the spot price of gold need not equal the price you calculated in part A above). However, you can buy and sell gold today. Is there an arbitrage opportunity? If so, describe it and demonstrate that it is in fact an arbitrage opportunity.
- VI. [15 points] Consider the following two strategies involving options on BG stock which does not pay dividends.
- (1) Buy a European put expiring in 6 months on BG stock with a strike price of \$80 and buy a European call expiring in 6 months on BG stock with a strike price of \$80. (A straddle.)
- (2) Buy a European put expiring in 6 months on BG stock with a strike price of \$90 and buy a European call expiring in 6 months on BG stock with a strike price of \$70.
- A. Draw the payoff (ignoring any purchase price paid or received to enter the position) in 6 months for each strategy as a function of the price of BG stock at that time. Be sure to clearly indicate the angle of any lines you draw.
- B. Which strategy would cost more? Why?
- C. Replicate strategy (1) using only: BG stock; discount bonds maturing in 6 months (state the face value of bonds bought or sold); and, European puts on BG stock that expire in 6 months and have a strike price of \$80.
- VII. [14 points] Today is the 2/15/95. The following information is available.  
*Government Bonds and Notes.*

Rate	Maturity	Price
4	Feb 96	98.84
6	Aug 97	102



*U.S. Treasury Strips.*

Maturity	Spot Price	Forward Price for delivery on 8/15/95	Forward Price for delivery on 2/15/96	Forward Price for delivery on 8/15/96
Aug 95	97			
Feb 96	?	?		
Aug 96	93	95.876	?	
Feb 97	?	93.814	95.789	97.849

- A. What is the price of the Feb 96 strip?  
 B. What is the price of the Feb 97 strip?

VIII. [10 points] The current stock price of FG Co is \$15 and the yield on a 1 year discount bond expressed as an EAR is 8%. Suppose the current price of a European put on FG Co that expires in one year and has a strike price of \$18 is \$1.50. Describe an arbitrage opportunity and show that it is in fact an arbitrage opportunity.

**Practice Final Exam Solution**

I.

- A. Using the SML:

$$E[R] = 6\% + 1.8 (11\% - 6\%) = 15\%.$$

- B. The constant growth DDM says:

$$P(0) = D(0) [1 + g] / \{E[R] - g\}$$

which implies

$$12 = 0.95 [1+g] / \{0.15 - g\}$$

$$12 \{0.15 - g\} = 0.95 + 0.95g$$

$$g = 0.85/12.95 = 0.06564.$$

- C. ROE =
- $g/b = 0.06564/0.4 = 0.16409$
- .

- D. First need to determine expected earnings in one year's time:

$$E[D(1)] = D(0) [1+g] = 0.95 [1+0.06564] = 1.012358.$$

$$E[E(1)] = E[D(1)]/(1 - b) = 1.012358/0.6 = 1.68726.$$

Then can calculate book value of equity today:

$$K(0) = E[E(1)]/ROE = 1.68726 / 0.16409 = 10.2825.$$

- E. Know that the expected holding period return on VLP's equity is 15%. So the answer is 15%.

II.

A.  $B = V - S = 15M - 10M = 5M.$

B. Know

$$\beta_V = (B/V) \beta_B + (S/V) \beta_S$$

and so

$$\beta_B = (V/B) [\beta_V - (S/V) \beta_S] = (15/5) [1.5 - (10/15) 2] = 0.5$$

- C. Since  $\beta_B \neq 0$ , the debt is not riskless.

III. Know ICAPM holds. So

$$E[R_i(\text{Jan})] = R_f(\text{Jan}) + \beta_{i,M}^* \lambda_M^* + \beta_{i,DEF}^* \lambda_{DEF}^*$$

where  $\lambda_M^* = E[R_i(\text{Jan})] - R_f(\text{Jan}).$

- A. Using the above formula for Pink and Grey:

Pink:  $1.73 = 0.7 + 1.3 \lambda_M^* + 0.25 \lambda_{DEF}^*$

Grey:  $1.34 = 0.7 + 0.9 \lambda_M^* + 0.10 \lambda_{DEF}^*$

Now Pink  $\Rightarrow \lambda_{DEF}^* = 4 (1.03\% - 1.3 \lambda_M^*)$

which can be substituted into Grey to obtain

$$1.34 = 0.7 + 0.9 \lambda_M^* + 0.10 \times 4 (1.03\% - 1.3 \lambda_M^*).$$

It follows that  $\lambda_M^* = 0.6\%$  and  $\lambda_{DEF}^* = 1\%$ .

So the risk premium for bearing  $\beta_{i,M}^*$  risk  $\lambda_M^*$  is 0.6%.

B.  $E[R_i(\text{Jan})] = \lambda_M^* + R_f(\text{Jan}) = 0.6\% + 0.7\% = 1.3\%.$

C. From above, the risk premium for bearing  $\beta_{i,DEF}^*$  risk  $\lambda_{DEF}^*$  is 1%.

D. The market portfolio is not on the minimum variance frontier for the risky assets since

$$E[R_i(\text{Jan})] = R_f(\text{Jan}) + \beta_{i,M}^* \lambda_M^* + \beta_{i,DEF}^* \lambda_{DEF}^*$$

implies

$$E[R_i(\text{Jan})] \neq E[R_{0,M}(\text{Jan})] + \beta_{i,M} \{E[R_M(\text{Jan})] - E[R_{0,M}(\text{Jan})]\}$$

where  $E[R_{0,M}(\text{Jan})]$  is the expected return on a risky asset uncorrelated with the market portfolio

and  $\beta_{i,M} = \text{cov}[R_i(\text{Jan}), R_M(\text{Jan})]/\sigma^2[R_M(\text{Jan})]$ .

IV.

- A. All individuals do not hold the market portfolio. Rather, all individuals hold portfolios on the positive sloped portion of the minimum variance frontier.
- B. The market portfolio does lie on the minimum variance frontier. The reasoning is as follows. The market portfolio is the sum of individuals' portfolios. Minimum variance mathematics says that any portfolio of minimum variance portfolios is also on the minimum variance frontier. Since all individuals hold minimum variance portfolios, it follows that the market portfolio is also on the minimum variance frontier.
- C. Since the market lies on the minimum variance frontier, the following formula holds for all assets:

$$E[R_i(\text{Jan})] = E[R_{0,M}(\text{Jan})] + \beta_{i,M} \{E[R_M(\text{Jan})] - E[R_{0,M}(\text{Jan})]\}$$

$$10\% = E[R_{0,M}(\text{Jan})] + 0.75 \{12\% - E[R_{0,M}(\text{Jan})]\}$$

Thus,

$$E[R_{0,M}(\text{Jan})] = 4\%.$$

- V. Know that the price of 6 month discount bond is related to the yield on a six month bond expressed as an EAR as follows:

$$97 = 100/[1+y^*_{0.5}(0)]^{0.5} \text{ which implies } 0.97 = 1/[1+y^*_{0.5}(0)]^{0.5}.$$

Know that the price of 12 month discount bond is related to the yield on a 12 month bond expressed as an EAR as follows:

$$94 = 100/[1+y^*_1(0)] \text{ which implies } 0.94 = 1/[1+y^*_1(0)].$$

- A. Since the carrying costs associated with gold are zero, forward spot parity says:  
 $S(0) = F_{0.5}(0)/[1+y^*_{0.5}(0)]^{0.5}$

which implies a spot price of

$$S(0) = 465 \times 0.97 = 451.05.$$

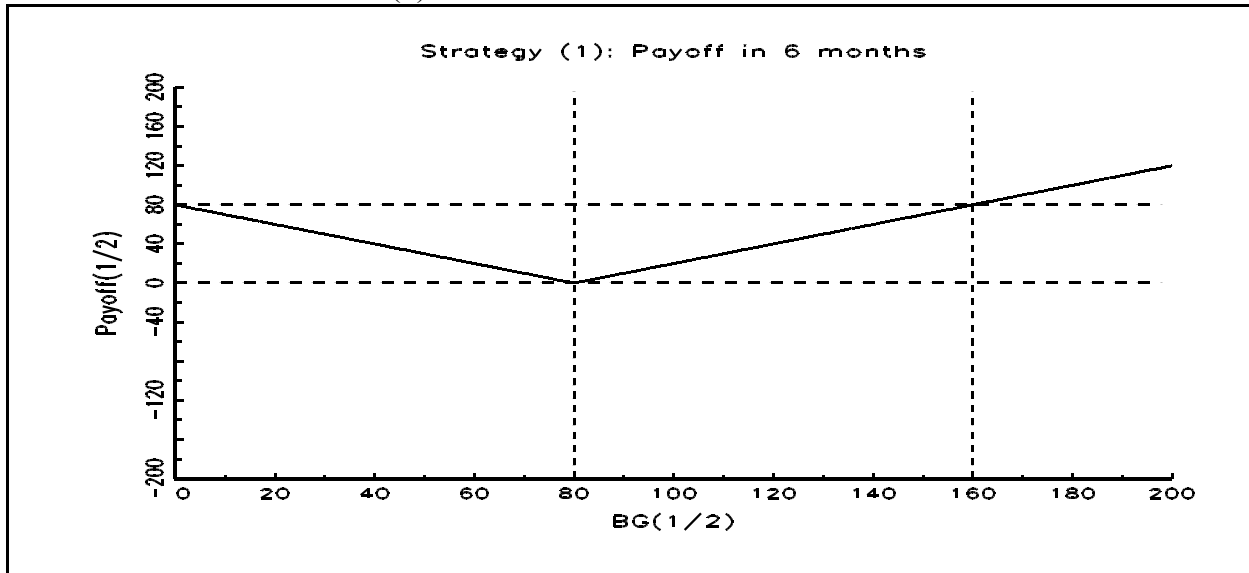
- B. Using forward spot parity, the 12 month forward price implies a spot price of  
 $S(0) = 475 \times 0.94 = 446.5$

which implies that the 6 month forward price is too high relative to the 12 month forward price. So the arbitrage involves selling the 6 month forward contract and buying the 12 month forward contract. An arbitrage position can be constructed as follows:

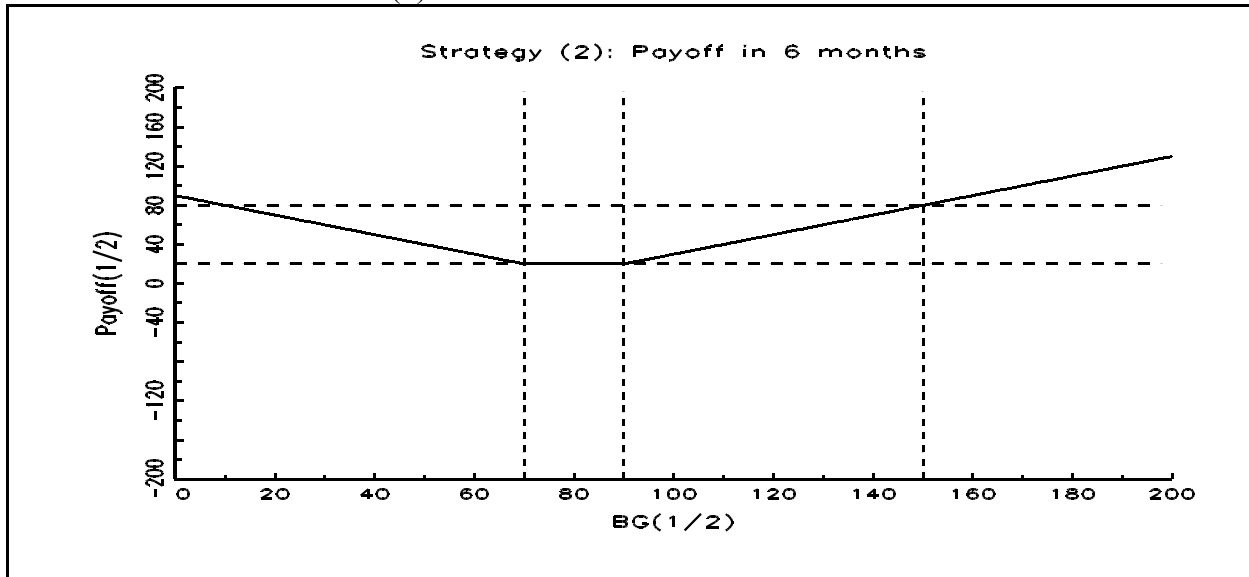
Strategy	Today	6 months	12 months
Buy 1 oz gold today and sell in 6 months	$-S(0)$	$S(1/2)$	
Sell a 6 month forward contract today	0	$465 - S(1/2)$	
Sell 6 month discount bonds today with face value of 465 today and hold to maturity	$465 \times 0.97 = 451.05$	-465	
Sell 1 oz gold today and buy in 12 months	$S(0)$		$-S(1)$
Buy a 12 month forward contract today	0		$S(1) - 475$
Buy 12 month discount bonds today with face value of 475 today and hold to maturity	$-475 \times 0.94 = -446.5$		475
Net Cash Flow	4.55	0	0

VI.

A. The payoff diagrams are:  
(1)



(2)



B. Since strategy (2) pays off more than strategy (1) irrespective of the price of the underlying in 6 months,  $BG(1/2)$ , no arbitrage implies that strategy (2) must cost more.

C. Replicating strategy:

Buy 2 European puts with strike price of 80 which expire in 6 months.

Buy 1 BG stock.

Sell 6 month discount bond with a face value of 80.

VII.

A. Can recover the yield on a six month discount bond using the Aug 95 strip:

$$y_{1/2}(\text{Feb } 95) = \{100/97 - 1\} \times 2 = 6.18557\%.$$

Can then recover the price of the Feb 96 strip using the Feb 96 note by splitting it into its two payments:

(1) The first coupon payment is paid in Aug 95. Thus, the law of one price says it can be discounted using  $y_{1/2}(\text{Feb } 95)$  to get its value on 2/15/95:

$$(4/2)/(1+0.0618557/2) = 2 \times 0.97 = 1.94.$$

(2) The value of the second and final cash flow of  $\{100 + (4/2)\} = 102$  can be calculated by subtracting the value of the Aug 95 coupon from the price of the note:

$$98.84 - 1.94 = 96.9.$$

(3) The yield on a one year discount bond can then be obtained

since 96.9 is the price on 2/15/95 of a cash flow of 102 in Feb 96:

$$y_1(\text{Feb 95}) = \{[102/96.9]^{0.5} - 1\} \times 2 = 5.1957\%$$

(4) The price of the Feb 96 strip can then be determined:

$$100/(1+y_1(\text{Feb 95})/2)^2 = 100/(1+0.051957/2)^2 = 95.$$

B. Need to use spot forward parity with the Feb 97 strip as the underlying:

1. One way:

$$S(0) = F_{0.5}(0)/[1+y^*_{0.5}(0)]^{0.5} = 93.814 \times 0.97 = 91.$$

2. A second way:

$$S(0) = F_1(0)/[1+y^*_1(0)]^1 = 95.789 \times 0.95 = 91.$$

3. A third way:

$$S(0) = F_{1.5}(0)/[1+y^*_{1.5}(0)]^{1.5} = 97.849 \times 0.93 = 91.$$

VIII. Know that the put must satisfy the following:

$$P_{18,1}(0) \geq \max\{18/[1+y^*_1(0)]^1 - S(0), 0\}.$$

But

$$P_{18,1}(0) = 1.50 < \max\{18/[1+y^*_1(0)]^1 - S(0), 0\} = 18/[1.08]^1 - 15 = 1.6667.$$

So an arbitrage opportunity exists.

Strategy	Now	1 Year	
		S(1)<18	S(1)≥18
Buy a European put option at 0 with exercise price of 18 and hold until expiration in 1 year	-1.5	[18-S(1)]	0
Buy FG stock at 0 and sell in 1 year	-15	S(1)	S(1)
Sell a 1 year discount bond with face value of 18 and close out at maturity	18/[1+0.08] =16.6667	-18	-18
Net Cash Flow	0.1667	0	S(T)-18≥0

Foundations of Finance  
 Final Exam Spring 1996  
 Prof. Anthony Lynch

Instructions: 120 minutes. Closed book. You are permitted use of a financial calculator and two sheets (8½" x 11") of notes. Answer *all* questions. Each question carries the number of points indicated. Total points is 100. Show all work on the exam paper itself. If you do not have or can not calculate a number that you need, make an assumption and proceed with the rest of the question. *No credit will be given for illegible, unsupported or ambiguous answers.*

GOOD LUCK!

Name: \_\_\_\_\_

Student #: \_\_\_\_\_

Signature: \_\_\_\_\_

- I. [13 points] Suppose the I-CAPM holds and investors care about  $\{E[R_p(t)], \sigma[R_p(t)], \sigma[R_p(t), \text{TERM}(t)]\}$  where  $\text{TERM}(t)$  is the difference between the yield on a long term bond and the yield on a short term bond at the end of month  $t$ . The riskless rate is 0.8% and the following information is available:

$i$	$E[R_i(t)]$	$\beta_{i,M}$	$\beta_{i,TERM}$
M (market)	1.3%	?	0.2
XDM	?	1.1	0.6
QL	1.0%	0.8	0.4

where  $\beta_{i,M}$  is the slope coefficient from the regression of asset  $i$ 's monthly return on the market portfolio's monthly return:  $R_i(t) = \alpha_{i,M} + \beta_{i,M} R_M(t) + e_{i,M}(t)$ ; and,  $\beta_{i,TERM}$  is the slope coefficient from the regression of asset  $i$ 's monthly return in month  $t$  on  $\text{TERM}(t)$ :  $R_i(t) = \alpha_{i,TERM} + \beta_{i,TERM} \text{TERM}(t) + e_{i,TERM}(t)$ . What is the expected return on XDM?

- II. [13 points] Suppose the CAPM with no riskless borrowing or lending holds. The expected return on the market portfolio is 15% and the expected return on LM Co is 10%. LM Co has a beta with respect to the market of 0.6.
- A. Do all individuals hold the market portfolio? If individuals do not hold the market portfolio, describe the portfolios they do hold.
  - B. Does the market portfolio lie on the minimum variance frontier. Explain why or why not?
  - C. Do you have enough information to determine the expected return on an asset whose return is uncorrelated with the market portfolio's return. If so, what is the expected return on an asset whose return is uncorrelated with the market portfolio's return?
- III. [8 points]
- A. Today's price for a 12 month discount bond (face value=100) is 95 and today's price for a 24 month discount bond (face value=100) is 90. The current spot price for 1 oz of gold is \$450. Suppose that the cost of carrying gold is zero. In the absence of arbitrage, what is today's forward price for 1 oz of gold to be delivered in 12 months?
  - B. If forward-spot parity holds, what is the one year forward price of a yen (expressed as English pounds per yen), assuming that the current spot rate is £0.0256/yen, the English one-year interest rate is 8% (expressed as an EAR) and the Japanese one-year rate is 10% (also expressed as an EAR).
- IV. [18 points] The common stock of Rosh is currently trading at \$10 per share (down from \$15 at the beginning of the year). Rosh has commenced testing of a cure for the common cold. You believe that if the testing is successful, the stock's price will rise significantly, but if it is unsuccessful, the stock's price will fall. You want to profit from either outcome while putting a lower bound on your loss if you are wrong. The risk-free rate is 6% (continuously compounded annual rate) and a 6-month European call option with an exercise price of \$6 is selling at \$7. Rosh is not expected to make any dividend payments in the next 6 months.
- A. A dealer offers you a 6-month European put option with an exercise price of \$6. What is a fair price for this option?
  - B. Propose a strategy to take advantage of your beliefs that uses one or more of the following instruments: the stock, 6-month European call options with an exercise price of \$6, 6-month European put options with an exercise price of \$6 and discount bonds maturing in 6 months.



## IV (cont)

One month ago Rosh announced that testing of the potential cure was about to commence but that its researchers were less optimistic than earlier about the likelihood of success. This announcement caused Rosh's stock price to decrease and the volatility of Rosh's stock return to increase.

- C. What would have happened to the price of European calls on Rosh's stock on the announcement date? Explain your reasoning.
- D. What would have happened to the price of European puts on Rosh's stock on the announcement date? Explain your reasoning.

- V. [16 points] A large computer manufacturer Pear has \$1M of riskless debt. The equity of Pear has a Beta with respect to the market of 1.1. Assume that the CAPM holds for annual returns and that Pear pays annual dividends. The riskless rate is 8% per annum and the expected annual return on the market is 20%. Pear has just paid a dividend of \$2 per share and Pear's dividend per share is expected to grow at 15% per year forever. It has a plowback (retention) ratio of 0.75 and the expected return on book equity each year is a constant (ROE).

- A. What is the expected annual return on Pear's stock?

Use the constant dividend growth model to answer the following questions.

- B. What is the current price of Pear stock?
- C. What is Pear's current earnings per share after interest?
- D. What is the expected return on book equity (ROE) for Pear?

- VI. [13 points] Today's price for a 12 month discount bond (face value=100) is 93 and today's price for a 6 month discount bond (face value=100) is 96. Today's forward price for delivery of IBM stock in 1 year is 132. IBM will pay a dividend of \$5 in six months. The current price of IBM stock is 125. Describe an arbitrage opportunity (if one exists) and show that it is an arbitrage opportunity.

VII. [19 points] Today is the 2/15/95. The following information is available.

*Government Notes.* (principal=100)

Rate	Maturity	Price
6	Feb 96	102.875

*U.S. Treasury Strips.* (face value=100)

Maturity	Aug 95	Feb 96	Aug 96	Feb 97
Spot Price	?	97	93	90

- A. Six months later (on the 8/15/95), the price of the Feb 96 strip is 98.
1. What is the price of the 6% Feb 96 note on the 8/15/95?
  2. What is the 6-month holding period return on the 6% Feb 96 note from 2/15/95 to 8/15/95?
- B. What is the forward price today (2/15/95) for delivery of the Feb 97 strip on 8/15/96?
- C. What is today's price of the Aug 95 strip?

Foundations of Finance  
Final Exam Spring 1996 Answer Key  
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- I. Know ICAPM holds. So  $E[R_i(t)] = R_f(t) + \beta_{i,M} \lambda_M + \beta_{i,TERM} \lambda_{TERM}$   
where  $\beta_{M,M} = 1$ .
- M:  $1.3 = 0.8 + 1 \lambda_M + 0.2 \lambda_{TERM}$   
QL:  $1.0 = 0.8 + 0.8 \lambda_M + 0.4 \lambda_{TERM}$

Now  $M \Rightarrow \lambda_M = 0.5 - 0.2 \lambda_{TERM}$  which can be substituted into QL to obtain  
 $1.0 = 0.8 + 0.8 \{0.5 - 0.2 \lambda_{TERM}\} + 0.4 \lambda_{TERM}$ .

It follows that  $\lambda_M = 0.6667\%$  and  $\lambda_{TERM} = -0.8333\%$ .

So

$$E[R_{XDM}(t)] = R_f(t) + \beta_{XDM,M} \lambda_M + \beta_{XDM,TERM} \lambda_{TERM}$$

$$= 0.8 + 0.6667 \times 1.1 + -0.8333 \times 0.6 = 1.0334\%.$$

II.

- A. All individuals do not hold the market portfolio. Rather, all individuals hold portfolios on the positive sloped portion of the minimum variance frontier.
- B. The market portfolio does lie on the minimum variance frontier. The reasoning is as follows. The market portfolio is the sum of individuals' portfolios. Minimum variance mathematics says that any portfolio of minimum variance portfolios is also on the minimum variance frontier. Since all individuals hold minimum variance portfolios, it follows that the market portfolio is also on the minimum variance frontier.
- C. Yes. Since the market lies on the minimum variance frontier, the following formula holds for all assets:  $E[R_i(t)] = E[R_{0,M}(t)] + \beta_{i,M} \{E[R_M(t)] - E[R_{0,M}(t)]\}$

$$LM: 10\% = E[R_{0,M}(t)] + 0.6 \{15\% - E[R_{0,M}(t)]\}$$

$$\text{Thus, } E[R_{0,M}(t)] = 2.5\%.$$

III.

- A. In general,  $S(0) = F_T(0) d_T(0)$ . Here,  $450 = F_1(0) 0.95$ . So  $F_1(0) = 473.68$ .
- B. In general,  $S^{\$/y}(0) = F_T^{\$/y}(0) [1+y^*yT(0)]^T / [1+y^*y_T(0)]^T$ . Here  $\text{£}0.0256/y = F_1^{\$/y}(0) 1.1/1.08$ . So  $F_1^{\$/y}(0) = \text{£}0.02513/y$ .

IV.

- A. Using put-call parity,  $S(0) = C_{X,T}(0) - P_{X,T}(0) + X e^{-rT}$ .  
So  $10 = 7 - P_{6,1/2}(0) + 6 e^{-0.06 \times 1/2}$  giving  $P_{6,1/2}(0) = 2.82$ .
- B. Buy 1 put and buy 1 call.
- C. Impact on  $C(0)$  is indeterminate since the drop in  $S(0)$  causes  $C(0)$  to drop while the increase in stock return volatility causes  $C(0)$  to increase.

D. Impact on  $P(0)$  is positive since the drop in  $S(0)$  causes  $P(0)$  to increase while the increase in stock return volatility also causes  $P(0)$  to increase.

V.

A. Using the SML,  $E[R_{IBX}] = 8\% + 1.1(20\% - 8\%) = 21.2\%$ .

B. The constant growth DDM says:

$$P^{IBX}(0) = D^{IBX}(0) [1 + g^{IBX}] / \{E[R_{IBX}] - g^{IBX}\} = 2 [1 + 0.15] / \{0.212 - 0.15\} = 37.10.$$

C.  $E^{IBX}(0) = D^{IBX}(0) / (1 - b^{IBX}) = 2 / 0.25 = 8.$

D.  $ROE^{IBX} = g^{IBX} / b^{IBX} = 0.15 / 0.75 = 0.2.$

VI. Using forward spot parity, the 12 month forward price implies a spot price of

$$\begin{aligned} S(0) &= F_T(0) d_T(0) - C(t_c) d_{tc}(0) \\ &= 132 \times 0.93 + 5 \times 0.96 = 127.56 > 125. \end{aligned}$$

which implies that the 1 year forward price is too high relative to stock price. So the arbitrage involves selling the 1 year forward contract and buying the stock. An arbitrage position can be constructed as follows:

Strategy	Today	6 months	12 months
Buy 1 IBM today and sell in 12 months	-125	5	$S(1)$
Sell a forward contract today to deliver IBM in 12 months	0	0	$132 - S(1)$
Sell 12 month discount bonds today with face value of 132 and buy at maturity	$132 \times 0.93 = 122.76$		-132
Sell 6 month discount bonds today with face value of 5 and buy at maturity	$5 \times 0.96 = 4.8$	-5	
Net Cash Flow	2.56 > 0	0	0

VII.

A.

1.  $P^{6\% \text{ Feb } 96}(8/15/95) = 103 d_{1/2}(8/15/95) = 103 \times 0.98 = 100.94.$

2.  $h^{6\% \text{ Feb } 96}(8/15/95) = \{100.94 + 3 - 102.875\} / 102.875 = 1.0352\%$  expressed as an effective 6 month return.

B.  $d_{1\frac{1}{2},2}(2/15/95) = d_2(2/15/95) / d_{1\frac{1}{2}}(2/15/95) = 0.90 / 0.93 = 0.9677.$  So the forward price today for delivery of the Feb 97 strip on 8/15/96 is 96.77.

C. Know  $P^{6\% \text{ Feb } 96}(2/15/95) = 102.875 = 3 d_{1/2}(2/15/95) + 103 d_1(2/15/95).$

and know  $d_1(2/15/95) = 0.97.$  So  $102.875 = 3 d_{1/2}(2/15/95) + 103 \times 0.97$  which gives  $d_{1/2}(2/15/95) = 0.9883.$  Thus, today's price of the 1/2-year strip is 98.83.