Module 9: Geometry

GEOMETRY

TIME FRAME: 11 days

ENDURING UNDERSTANDINGS

Geometry allows us to measure physical quantities (like volume and surface area) as well as to compare quantities (for example, using similarity of figures).

CRMS

Geometry 5.1, 5.2, 5.3, 5.4
Connections 3.3, 3.4
Algebra 7.2, 7.3

AT THE END OF THE CHAPTER STUDENTS WILL KNOW AND BE ABLE TO:

1. Understand the difference between volume and surface area.
2. Use the Pythagorean Theorem to find an unknown side length of a right triangle.
3. Calculate the distance between two points in the xy-plane.
4. Recognize the equation of a circle centered at the origin in the xy-plane.
5. Use similarity to determine unknown side lengths of triangles.

PRE-REQUISITE KNOWLEDGE/SKILLS:

1. Ability to perform measurements with rulers.
2. Ability to solve an algebraic equation by substitution.

PRE-ASSESSMENT: None.

ACTIVITIES:

1. Gro-Beast Activity
2. Volume and Surface Area Worksheet
3. Pythagorean Theorem Worksheet
4. From Triangles to Circles Worksheet
5. Maximizing Volume of a Cone Activity
6. Forming a Tetrahedron Activity
7. Similar Triangle/Proportion Worksheet

POST-ASSESSMENT: None.

RESOURCES:

Gro-Beasts

Paper and Scissors (for the cone and tetrahedron activities)

DAILY PLAN

Day 1
- A Quick Review of Geometric Terms (paper folding activity)
- Gro-Beast project introduction and discussion[*]

Days 2-3
- **Volume and Surface Area** Worksheet
- **Pythagorean Theorem** Worksheet

Day 4-5
- **From Triangles to Circles** Worksheet

Day 6
- Activity: **Maximizing the Volume of a Cone**

Day 7-8
- Introduction to Right Triangle Trigonometry (direct instruction and problem solving practice)

Day 79
- Finish **Gro-Beast Activity**[*]

Day 10
- **Forming a Tetrahedron** activity
- **Similar Figures, Unit Analysis and Proportion** Worksheet

Day 11
- Review/Catch-up

* - This activity requires special timing, because you must be able to collect a week's worth of data without an interruption due to a weekend or holiday. You will have to decide for yourself what day to actually begin the project, but you can explain it in outline to the students on day one. If you have to begin it on a different day, then Day 7 can be moved somewhere else on the schedule, as necessary.

A QUICK REVIEW OF GEOMETRICAL TERMS

Give each student an 8 inch diameter circle in either construction paper or waxed paper (something that folds well!). Lead the class through the following folds asking them what the shapes are that they create. This is a great way to review what characteristics these shapes should have.

1. Find the **center** of the circle and mark it with a dot. Talk about how to do this (using **diameters**).
2. Fold one edge of the circle so that the edge just touches the center. You have created a **chord**. (Unfold so that they can see this.)
3. Fold the edge back to the center. Fold another edge of the circle so that the first and second folds meet at a point on the edge of the circle. Do the same with the remaining side of the circle to form an **equilateral triangle**. Talk about the measurement of each **angle**. If they have protractors, they can check!
4. Fold one triangle **vertex** to the **midpoint** of the opposite side to form a **trapezoid**. Talk about the **area** of the trapezoid compared to the area of the equilateral triangle.
5. Fold a second vertex of the equilateral triangle to the midpoint of the opposite side to form a **rhombus**. (Talk about **parallelogram** vs. rhombus.)
6. Fold the last vertex to the midpoint to form a **smaller equilateral triangle**. You could talk about area again or look at the angle measures. This is also a good time to mention **similarity**.
7. Letting the three folded smaller triangles 'stand up' has created a **3 dimensional tetrahedron**. Talk **volume**!
8. Unfold the sides completely to return to the large equilateral triangle. Fold one vertex to the center of the circle to form another **trapezoid**.
9. Fold another vertex to the center of the circle to form a **pentagon** (not **regular**).
10. Fold the last vertex to the center of the circle to form a **regular hexagon**.
11. Allowing the small triangles just created to form a 'lid', gently squeeze the shape to form a **3 dimensional frustum**. You can create wonderful 3 dimensional icosahedrons by gluing 20 of these frustums together on their sides.

GRO-BEASTS
TEACHER RESOURCE

The goal of the Gro-Beast project is for students to gain a better understanding of measurement and the corresponding units for various types of measurement. In addition, there is a review of plotting points, calculating slopes and determining intercepts. The idea of slope is extended to look at the rate of change of slopes over the 6 days of measurement. Essentially the students are encouraged to think about the second derivative (hence moving beyond the concept of linearity).

In terms of supplies, you will need to borrow a few items from the chemistry, biology or physics instructors in your school. You will need to borrow several sizes of graduated cylinders (beakers with milliliter labels) and a good measuring balance scale with units in grams. Other supplies needed include string for measuring perimeter, strong zip lock bags (quart size will do for the small beasts, gallon is necessary for the alligators!) and rulers. All the necessary graph paper is supplied in the project itself. Gro-Beasts are available from Educational Innovations, Inc. www.teachersource.com.

Timing and scheduling is very important for this project. Each group is given a dinosaur to measure. They will measure the dry beast on Friday when you know that you will meet in class for a full 5 days the next week. After measuring for the first time, have the students put the beast in a bag labeled with their group name. As the instructor, you will need to take these home so that Sunday morning you can fill each bag up with water just so that the beast is submerged. Filtered water works best, but tap water will also suffice.

On Monday and every day after until Friday, the students will measure their growing beasts. They may need to add water to the bags as the beast grows. It helps to have several stations around the room so that students can take turns using the beakers and the scale. The measurements they will take are: length, height, width (they need to agree as a group what how to measure these concepts and stick with that agreement for the entire time!), perimeter, area, weight and volume. They will calculate density from their measurements.

A few comments about measuring: Most students have had very little practice with using a scale and a beaker for measuring so it is good to walk them through the first measurements. The area measurement is the trickiest since they have to estimate how much of a square is covered. It is important to let the students know that although they may want to slightly dry off the beast before laying in on paper to trace for area, **they don't want to squeeze it.** Also remind the students that they need to fit all six days of area tracing on one sheet of paper so start in a corner! With the volume, you want to be certain to have a small cylinder for the first couple of days as the dinosaurs do not initially displace very much water. If the dinosaur floats the when dry, have the students gently push it with a pencil until it is barely submerged. Tell the students to have two

people check each others measurements AND make sure that everyone gets the chance to do the measuring.

When grading the projects: the important issue is that the students understand the importance of correct measurements and labels. Hopefully they will enjoy the project! Understanding the theoretical ratios compared to the experimental ratios ($x:x$ for perimeter, $x:x^2$ for area and $x:x^3$ for volume) is important. You might point out that in scary movies where there are bugs as tall as buildings, these animals could not really exist since the volume in their legs would be the cube but the strength is proportional to the cross sectional area (squared). Consequently the bugs would collapse! (This is also true for extremely tall humans!)

The following pages should be copied and distributed to students. If you find it easier to print an electronic version to make copies, you can download it from the Project TIME discussion forum on Google Groups. The file is named "Gro-Beast Handout.pdf".

IMPORTANT SUGGESTION

You should consider keeping one of the Gro-Beasts long term: you can use the data for it's continued growth as an example of something that can be modeled by the a logistic equation, which will be covered in the module on Continuous Functions.

FANTASTIC JURASSIC

Gro-Beasts are small animal figurines that are made from a polymer (sodium polyacrylate). Your responsibilities include completing a Gro-Beast lab in which a Gro-Beast will be observed over a period of 5 days. Changes in length, height, width, perimeter, area, mass, volume, and density will be recorded over the 5-day period, and the results graphed and evaluated. Note that this means you will have to measure them 6 times, day 0 to day 5.

Objective:

Demonstrate skills in working with measurement, ratios and proportion, linear graphs, intercepts, and slope.

Materials:

The following is a list of materials needed to complete the Gro-Beast Lab.

> Gro-Beast
> Plastic Bag or Tupperware type container
> Overflow Device using 2 liter pop bottle (optional: for determining the volume).
> Balance (for determining mass)
> String (for measuring perimeter)
> Metric Rulers (for measuring length, height, and width
> COLD water (hot water will dissolve the Gro-Beast!)
> Graduated cylinder (for measuring volume)
> Graph Paper (provided in the packet)

Tasks:

- Complete the Gro-Beast lab worksheets. Be sure to record the changes in your Gro-Beast over the course of 5 consecutive days. Day 0 measurements should be the values before the dinosaurs are put in the water bag to start growing. Each day the measurements must be taken and recorded.

- Neatly sketch the six graphs on the papers provided. Be sure to label the appropriate axes and calculate the slopes for the graph in the space provided.

- Answer a set of questions based on the data you have collected.

Gro-Beast Lab
Procedure

1. Trace your Gro-Beast on the graph paper provided.

2. Measure the **length** of your Gro-Beast to the nearest centimeter and record it in the data table.

3. Measure and record the **height** of your Gro-Beast.

4. Measure and record the **width**.

5. The distance around the outside of an object is known as its **perimeter**. To measure the perimeter, take a piece of string or wire and place it on the outline of your Gro-Beast on your graph paper. How much string did it take to cover the outline of the Gro-Beast? Record this in the perimeter row of your data table.

6. The projected **area** is the amount of space an object takes up on a flat surface (in two dimensions). Measure this by counting the squares that the Gro-Beast covers on the graph paper. You may need to estimate and add any partially covered squares.
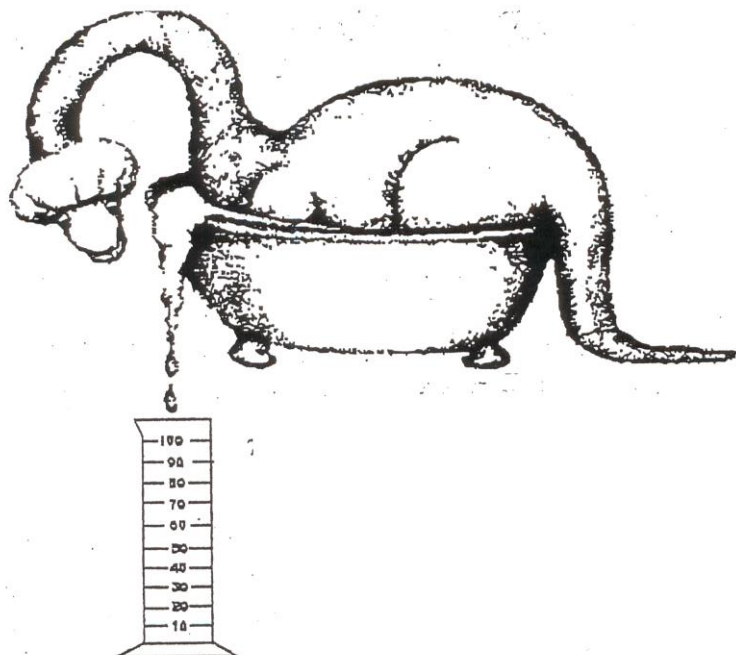

7. **Volume** is the amount of space that an object takes up in three dimensions. You can observe this when you sit down in a full bathtub and water spills out over the sides. The volume that spills out is the same as the volume of your body. Using an overflow can or a graduated cylinder, you can determine the total volume of your Gro-Beast and record it on your data table.

8. The amount of matter that an object has is known as its **mass**. Things that have a lot of matter are heavier than objects with less matter. Determine the mass of your Gro-Beast by weighing it on a balance.

9. **Density** is an object's mass divided by its volume.

Density = Mass ÷ Volume

Density is a difficult concept for some people to understand. However, you probably already understand how to apply this concept. A dense object will be heavier than an object of equal size that is less dense. Which of the following dinosaurs is more dense?
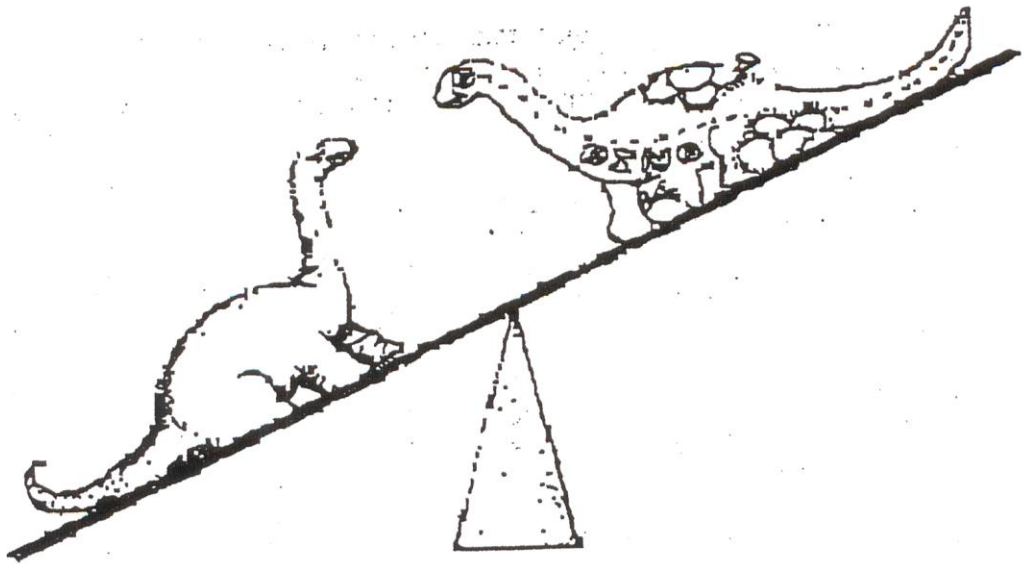
10. Divide the mass of your Gro-Beast by its volume and record it on your data table.

11. Water has a density of 1 gram per milliliter (g/ml). Is your Gro-Beast more or less dense than water? Once again, you can check your answer by putting your Gro-Beast in some water. If it sinks it is denser than water. If it floats it is less dense.
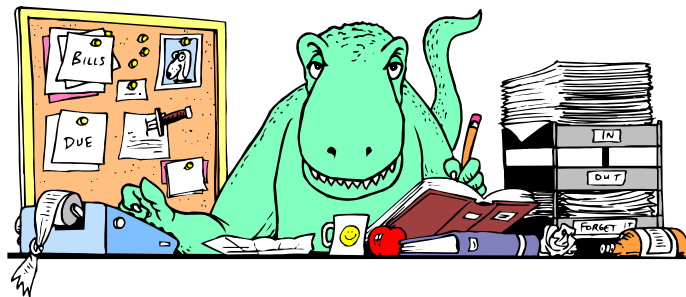
Gro-Beast Lab
Measuring and Graphing the Physical Changes of A Growing Toy Dinosaur

In this lab you will investigate how the physical measurements of length, height, width, perimeter, area, volume, mass, and density are related. Being able to use these forms of measurement is on the essential to making accurate observations of the world around us. For example, the relationship between an animal's projected **surface area** and **volume** changes, as an animal becomes larger. These physical measurements in turn have an effect **metabolism** of the animal. Animals that have a lot of volume such as whales or polar bears lose heat more slowly and have slower rates of **metabolism** than smaller animals. Small animals such as mice or hummingbirds that have more surface area to volume have much faster heartbeats and rates of metabolism. Remember that density is mass divided by volume.

Your dinosaur Gro-Beast will grow when you place it in water. You will measure, record, and graph the changes that occur over the next several days. It is important that you measure accurately.

	Day 0	Day 1	Day 2	Day 3	Day 4	Day 5
Length						
Width						
Height						
Perimeter						
Area						
Mass						
Volume						
Density						
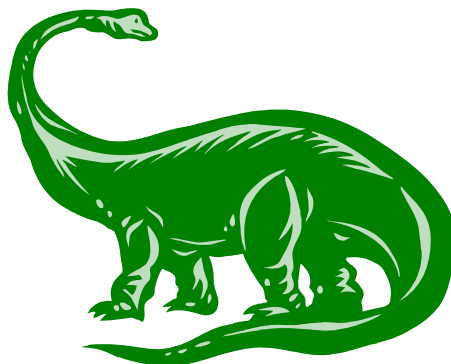
THE QUESTIONS:

1. By reading the calculated slopes on the graphs of length, width, height, and perimeter describe the slope values.
   a. Is the slope decreasing or increasing?
   b. What are the units for the slope?
   c. Do the slope values change from day zero to day five?

2. Do the measurements of length, width, height and perimeter increase at the same rate?
   a. How do you measure the rate?
   b. On what days is the rate of growth the greatest? The least?
   c. Why does the rate of growth change?
   d. Of the length, width, height and perimeter measurements, which overall slope is steepest? Why do you think this is so?

3. For the perimeter graph consider the intercepts.
   a. What is the y-intercept?
   b. What is the meaning of the y-intercept in this graph? (Think of this in real-world terms as applies to the Gro-Beast.)
   c. Is there an x-intercept?
   d. Would the x-intercept have any real meaning in the graph in this context? Why or why not?

4. For the graph of area describe the slope.
   a. What are the units for the slope?
   b. How many times larger in area does your Gro-Beast get from start to finish?

5. For the graph of mass describe the slope.
   a. What are the units for the slope?
   b. From your calculations, how many times its own weight can this material absorb?
   c. In what other ways might this material be useful? (Besides entertaining students???)

6. For the graph of volume, describe the slope.
   a. What are the units for the slope?

7. If your Gro-Beast was a real animal would its metabolism be getting faster or slower? Explain your answer based on the graph of the volume and the information given to you at the introduction to the table of data.

8. For the graph of density, describe the slope.
   a. What are the units for the slope?
   b. Water has a density of 1 gram per milliliter (g/ml). Initially is your Gro-Beast more or less dense than water? Does the answer to this question change over the course of your measurements? **(You can check your answer by putting your Gro-Beast in some water. If it sinks it is more dense than water. If it floats it is less dense.)**
   c. Is the density measurement closer to the density of water at the beginning or at the end of the project? Why is this true?
   d. How does the graph of density demonstrate what is happening to the composition of the dinosaur over the course of the project?

9. a. The values of the following ratios should be calculated.

> Length of Gro-Beast after day 4
> Length of original Gro-Beast

> Area of Gro-Beast after day 4
> Area of original Gro-Beast

> Volume of Gro-Beast after day 4
> Volume of original Gro-Beast

b. The second of these in theory should be the square of the first. The third should be the cube of the first. How do your three ratios compare to the theoretical values?

10. If you were to carry this experiment on for another day, what you predict the length, area, and volume would be? Describe how you calculated your predicted values (or show your calculations).

Overall Assessment:

Your grade will be based on the quality and correctness of the following:

> Data Collection Sheet
> Traces of Area
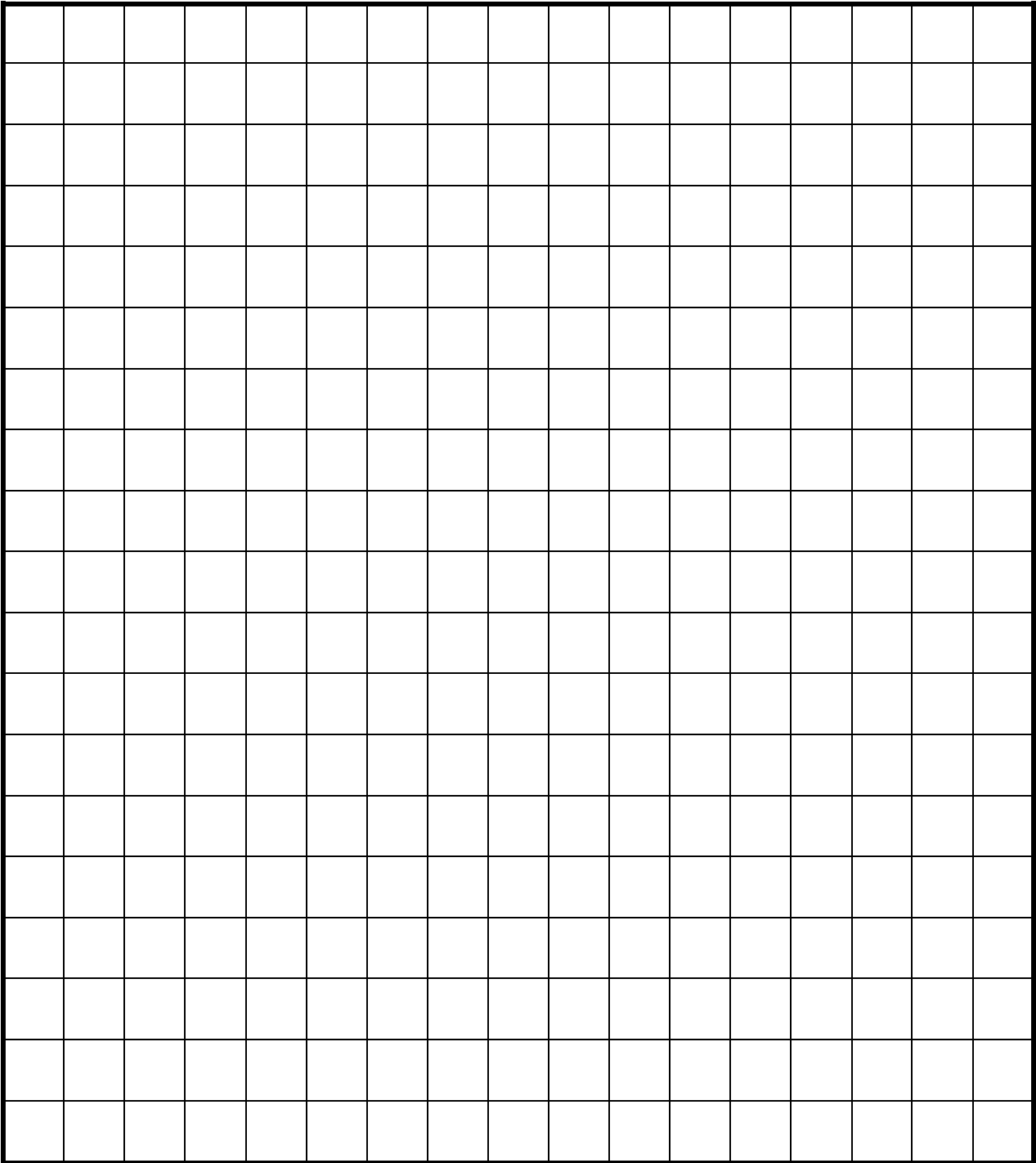> Graphs and Slope Calculation
> Answers to Questions

TRACES OF PROJECTED AREA
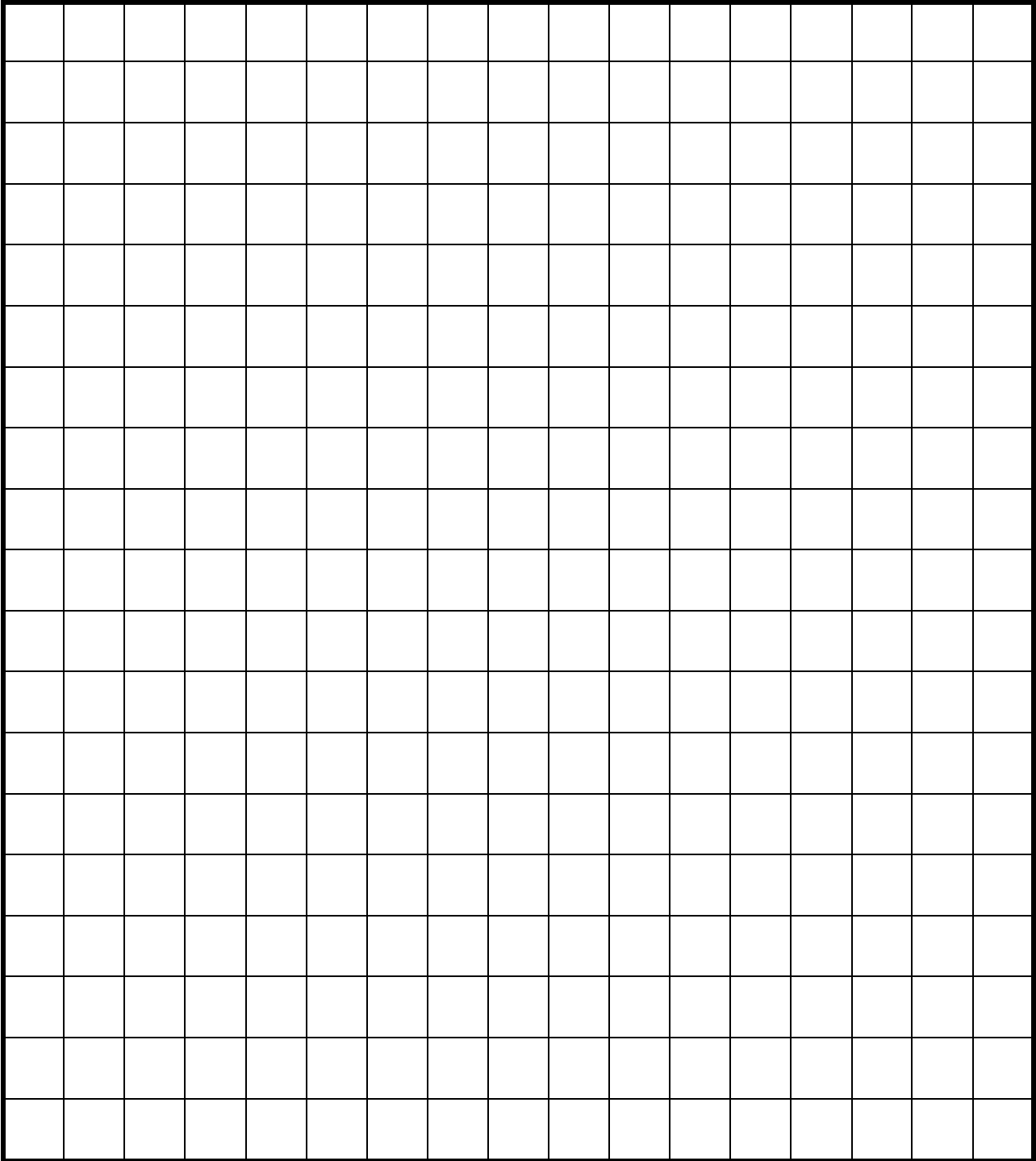
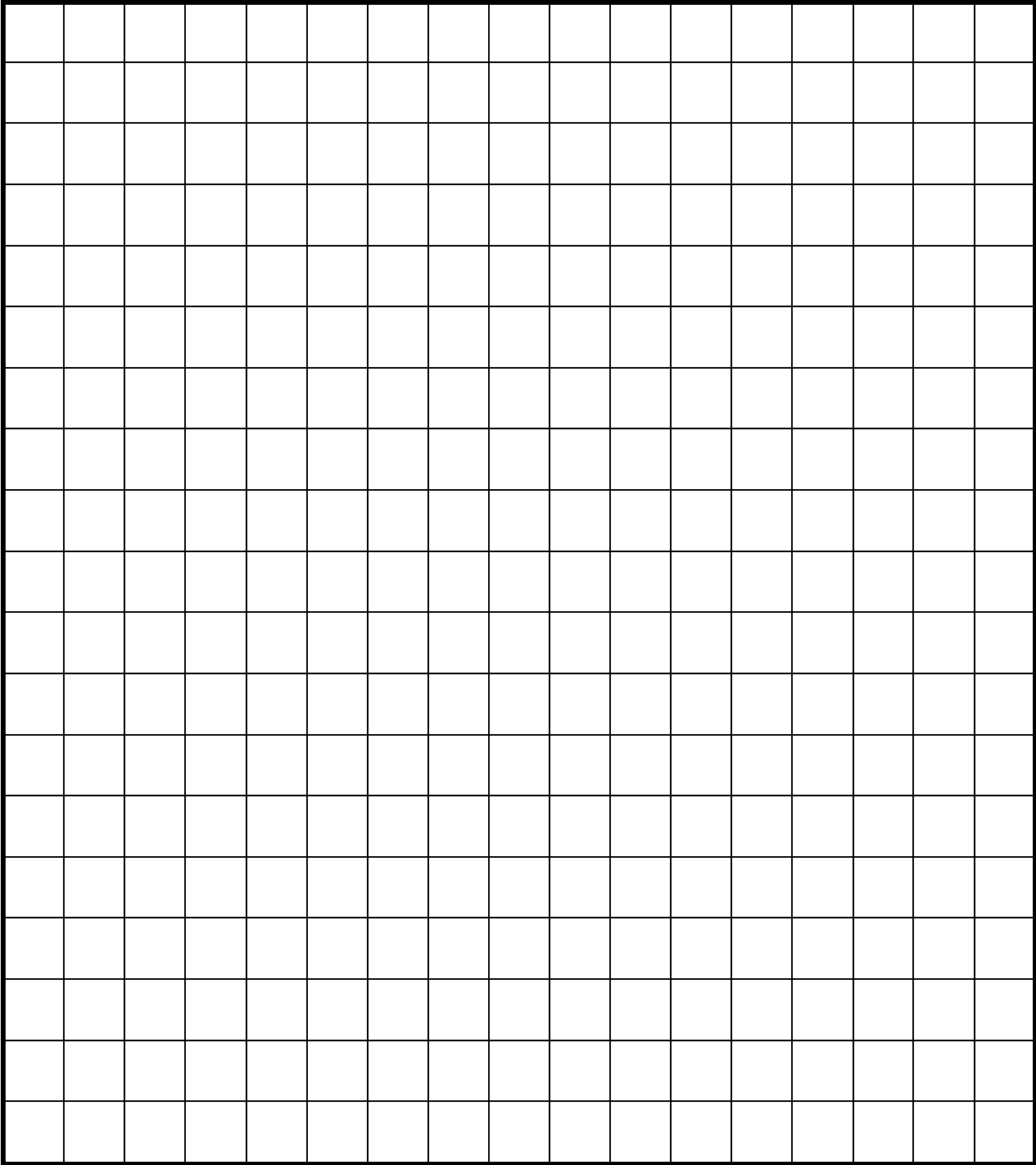**(Each square is 1 cm^2.)**

LENGTH   HEIGHT   WIDTH

Day interval	Day 0 – 1	Day 1 – 2	Day 2 – 3	Day 3 – 4	Day 5 - 6
Slope (with units)					

PERIMETER

Day interval	Day 0 – 1	Day 1 – 2	Day 2 – 3	Day 3 – 4	Day 5 - 6
Slope (with units)					

PROJECTED AREA

Day interval	Day 0 – 1	Day 1 – 2	Day 2 – 3	Day 3 – 4	Day 5 - 6
Slope (with units)					

VOLUME
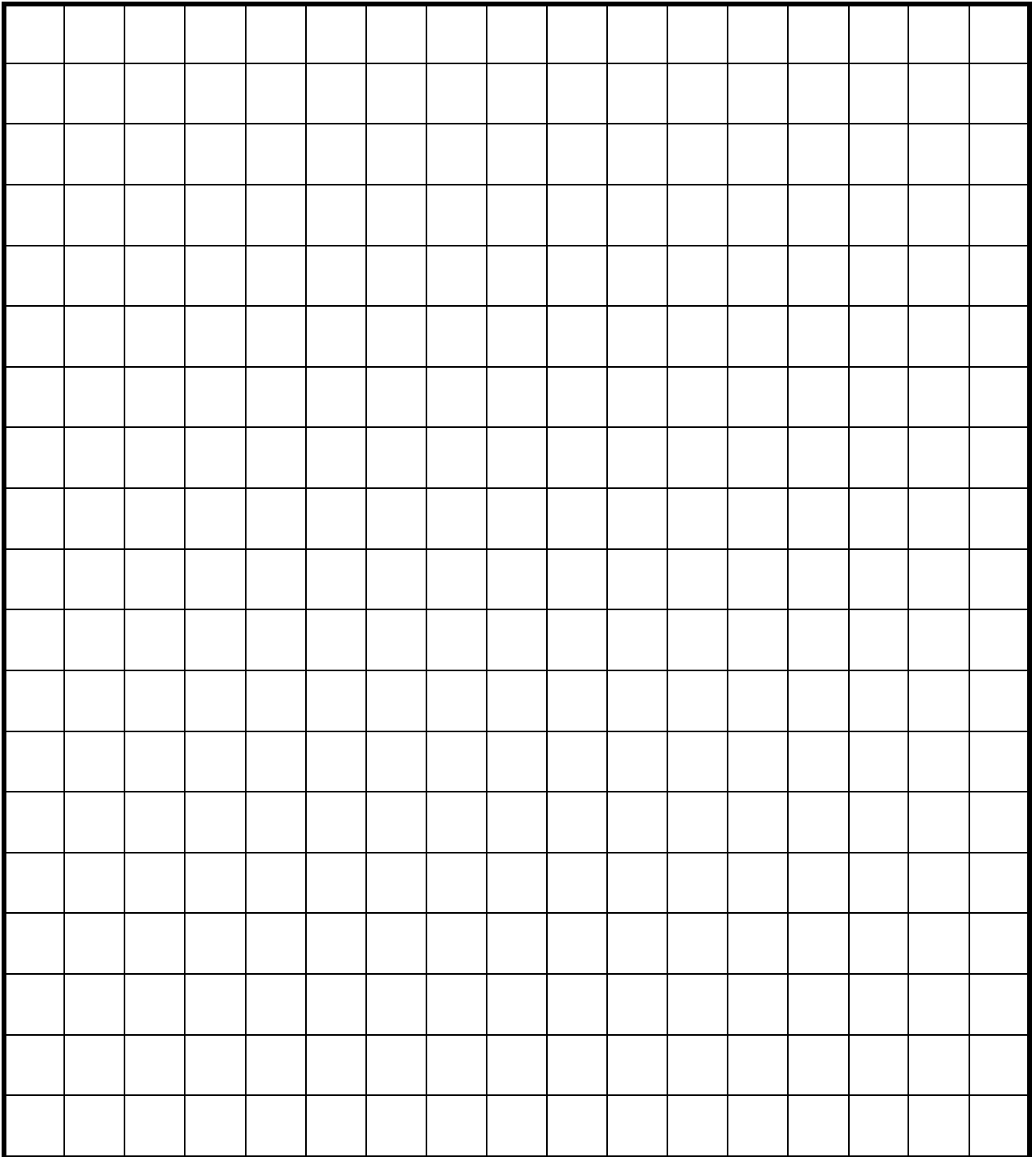
Day interval	Day 0 – 1	Day 1 – 2	Day 2 – 3	Day 3 – 4	Day 5 - 6
Slope (with units)					

MASS
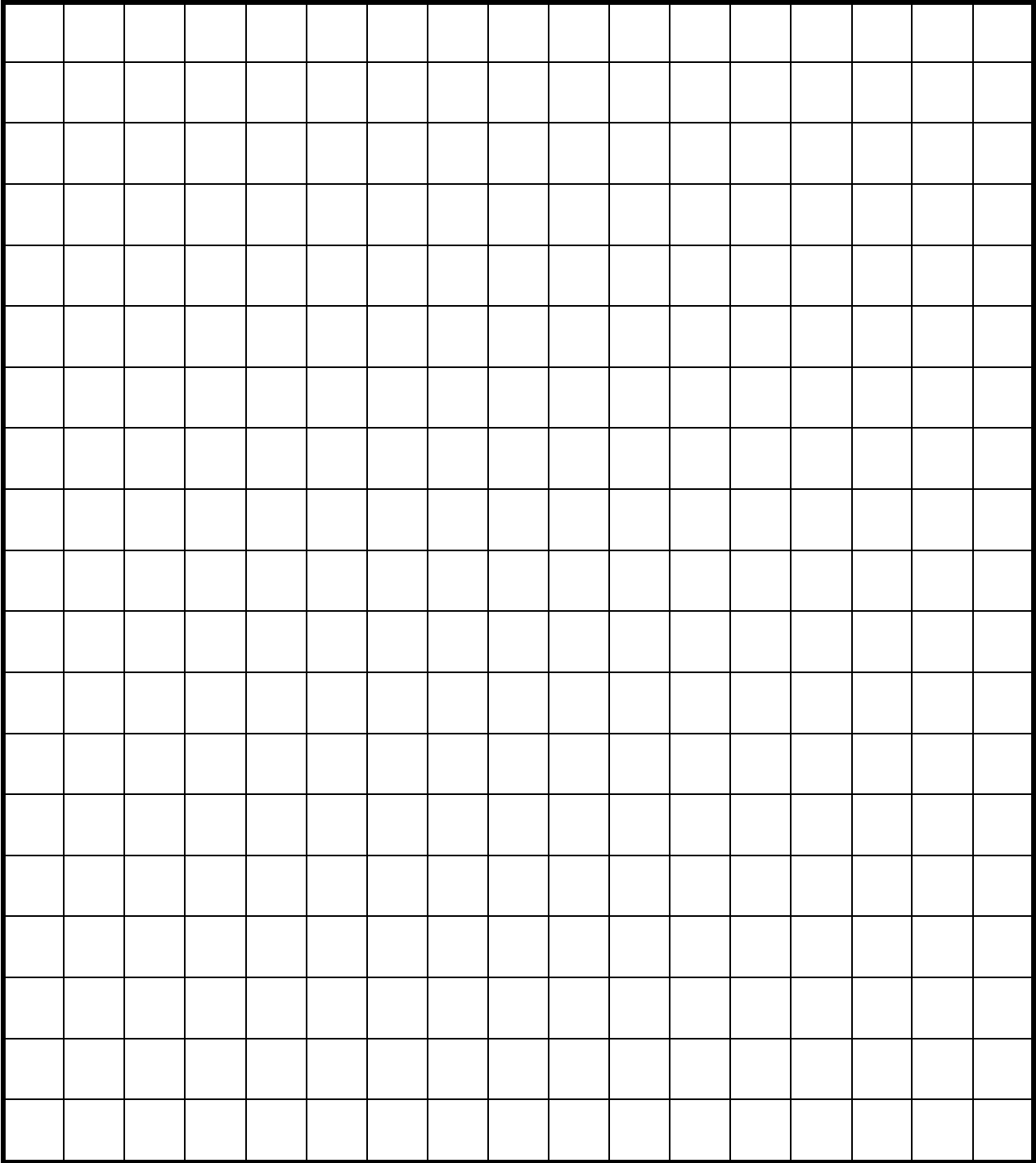
Day interval	Day 0 – 1	Day 1 – 2	Day 2 – 3	Day 3 – 4	Day 5 - 6
Slope (with units)					

DENSITY

Day interval	Day 0 – 1	Day 1 – 2	Day 2 – 3	Day 3 – 4	Day 5 - 6
Slope (with units)					

Day interval	Day 0 – 1	Day 1 – 2	Day 2 – 3	Day 3 – 4	Day 5 - 6
Slope (with units)					

ASSESSMENT RUBRIC

Concept	Points Possible	Your Points
Lab:		
Collecting data for 5 days (including calculating density units must be included)	10	_____
Graph #1 (Areas traced on the same sheet for all 6 measurements)	5	_____
Graph #2 (Length/width/height)	10	_____
Graph #3 (Perimeter)	5	_____
Graph #4 (Area)	5	_____
Graph #5 (Volume)	5	_____
Graph #6 (Mass)	5	_____
Graph #7 (Density)	5	_____
Lab subtotal:	**50**	**_____**
Questions:		
1. Length, width, height, and perimeter discussion of slopes	5	_____
2. Rate of growth discussion	5	_____
3. Perimeter intercepts discussion	5	_____
4. Area discussion	5	_____
5. Mass discussion	5	_____
6. Volume discussion	5	_____
7. Metabolism discussion	5	_____
8. Density discussion	5	_____
9. Ratio calculations (actual calculations vs. theoretical calculations)	5	_____
10. Prediction for height, area, and volume	5	_____
TOTAL POINTS	100	_____

Turn in your data after Day 5

	Day 0	Day 1	Day 2	Day 3	Day 4	Day 5
Length						
Width						
Height						
Perimeter						
Area						
Mass						
Volume						
Density						

VOLUME AND SURFACE AREA WORKSHEET
Description

ESTIMATED TIME: 20 minutes

The following activity is meant for students to work together in groups and help each other to remember the appropriate formulas for volumes and surface areas of common three-dimensional shapes.
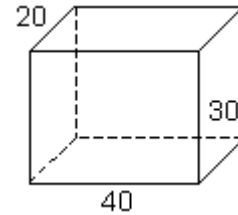
VOLUME AND SURFACE AREA

Calculate the volumes and surface areas for each of the objects below.
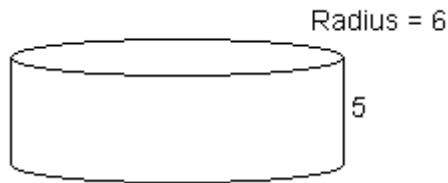
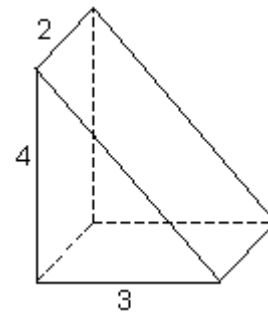**Note:** The measurements are all in centimeters (cm). Use the approximation 3.14 for Pi.
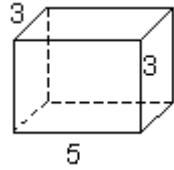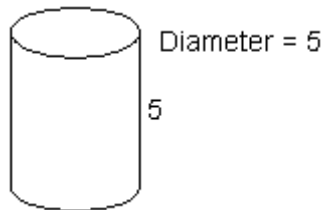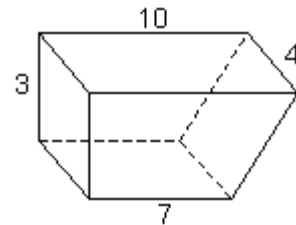

volume =
surface area=

volume =
surface area=

Radius = 6

volume =
surface area=

volume =
surface area=

Diameter = 5

volume =
surface area=

volume =
surface area=

PYTHAGOREAN THEOREM WORKSHEET
Description

The following activity introduces students to one of the geometric interpretations of the Pythagorean Theorem. After students complete this activity, the instructor should guide them toward recollecting the Pythagorean Theorem in the form $a^2 + b^2 = c^2$.

ESTIMATED TIME: 25 minutes

MATERIALS NEEDED: Centimeter Rulers
                          Graph Paper

THE PYTHAGOREAN THEOREM

Your job is to carefully draw, using a centimeter ruler, three scalene triangles – 1 obtuse, 1 acute and 1 right. Cut out an area of centimeter^2 for each side of each triangle from graph paper. Cut and paste these areas to the sides of the triangles you have drawn.
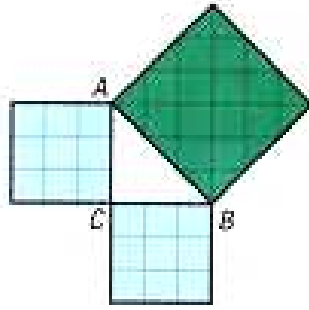
 Then, fill in the following tables:

Acute

Units	Short leg	Medium leg	Sum	Long leg
Centimeters				
	(Short leg)^2	(Med leg)^2	Sum	(Long leg)^2
centimeters^2				

Obtuse

Units	Short leg	Medium leg	Sum	Long leg
Centimeters				
	(Short leg)^2	(Med leg)^2	Sum	(Long leg)^2
centimeters^2				

Right

Units	Short leg	Medium leg	Sum	Long leg
Centimeters				
	(Short leg)^2	(Med leg)^2	Sum	(Long leg)^2
centimeters^2				

What can you conclude about the lengths of the sides of acute triangles?

What can you conclude about the lengths of the sides of obtuse triangles?

What can you conclude about the lengths of the sides of right triangles?

FROM TRIANGLES TO CIRCLES WORKSHEET

Description and Answer Key

ESTIMATED TIME: 40 minutes

This activity asks students to extend their understanding of the Pythagorean Theorem by guiding them from it's simplest application (that of finding an unknown length of a side of a right triangle), to the distance formula for two points in a Cartesian plane, to the equation for a circle in the xy-plane. Students who complete the worksheet will also get to review a bit of algebra skill when they use substitution to find the coordinates where a line and a circle intersect.

SOLUTIONS

(1) $a^2 + b^2 = c^2$

(2) The ~~square~~ of the length of the hypotenuse of a ~~right~~ triangle is equal to the ~~sum~~ of the squares of the ~~lengths~~ of the other two ~~sides.~~

(3)   (a) x=10   (b) x=5

(4) $D = \sqrt{38} \approx 6.16$

(5) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(6) The formula is missing $-x_2$ in the first set of parentheses, y_2- in the second set of parentheses, and an exponent of 2 on the last parentheses.

(7) $2 = \sqrt{x^2 + y^2}$, and the graph should be a circle of radius 2 centered at the origin

(8) $x^2 + y^2 = 9$

(9) $x = \pm 12$ (students should obtain this by substituting y=2 into the equation for the circle, $x^2 + y^2 = 16$)

From Triangle to Circles

Name: _____

In this worksheet, you review the Pythagorean Theorem applied to triangles and, in the process, derive an equation for a circle in the xy-plane.

The following figure depicts a right triangle (meaning one of the interior angles is a right angle). The numbers a, b and c refer to the lengths of the sides of the triangle.
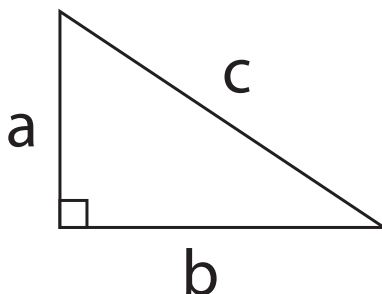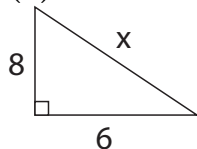

$\boxed{1}$ Write down the Pythagorean Theorem in terms of a, b and c.

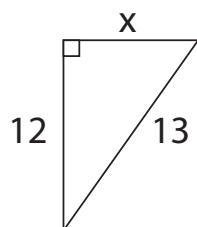$\boxed{2}$ Fill in the blanks to make a sentence that describes what the Pythagorean Theorem tells us:

The _____ of the length of the hypotenuse of a _____ triangle is equal to the _____ of the squares of the _____ of the other two _____.

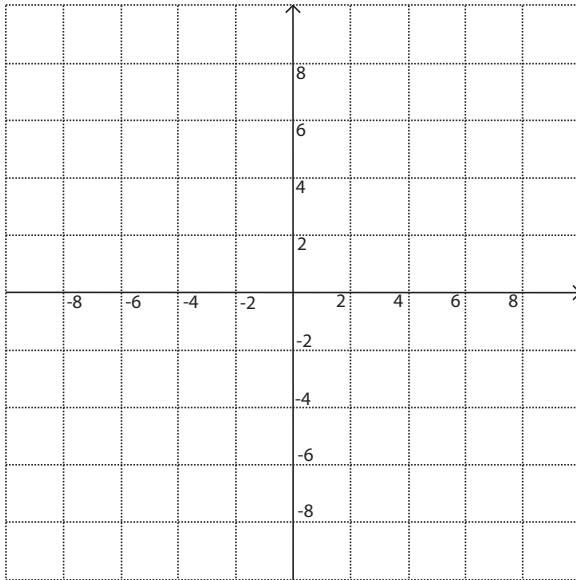$\boxed{3}$ Find the unknown side length for each of the following triangles.

(a)

(b)

4 On the coordinate plane below, plot the points $(-2, 4)$ and $(4, 6)$. Then draw a triangle, with those two points as the ends of a hypotenuse, so that you can calculate the distance between those two points. Then calculate the distance. Give an exact answer *and* a decimal approximation.

distance = _____

5 As in the previous problem, draw a triangle that allows you to use the Pythagorean Theorem to find the distance bewteen the two points shown in the figure. Then calculate the distance. *(Hint: After you draw a triangle, don't forget to label the lengths of the sides.)*
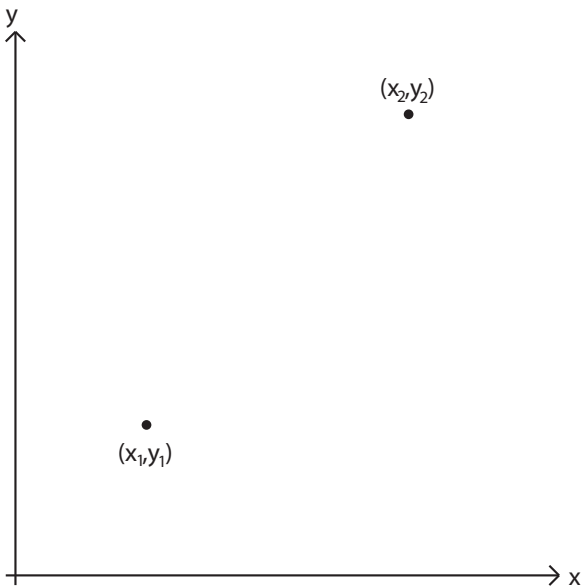
distance = _____

$\boxed{6}$ Use your answer for the previous problem to fill in the missing parts of the following formula for the distance between two points (x_1, y_1) and (x_2, y_2).

$$D = \sqrt{(x_2 \qquad)^2 + (\qquad y_1)}$$

$\boxed{7}$ Write down an equation, using your result from the previous question, that says the distance of (x, y) from the origin is 2. Then sketch a graph of all the points that satisfy this equation.

$\boxed{8}$ Find an equation for a circle, centered at the origin, with radius 3. Simplify your equation to remove any square-root symbols.

$\boxed{9}$ The circle in the picture below has a radius of 4 and a center at the origin. The line is horizontal and has the equation $y = 2$. Find the x-coordinates of the points where the line and the circle intersect.
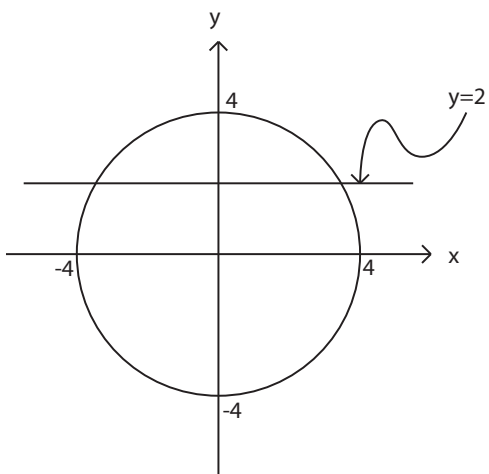
$x =$ _____ or _____

MAXIMIZING THE VOLUME OF A CONE
Description

PURPOSE: This activity has two main objectives.
- To find the maximum volume of a cone that is formed when an central angle is cut out of a circle with a 10 cm radius
- To find the relationship between the volume of a cube and a cylinder with the same base and same height

MATERIALS:

Each group will need the following
- A compass per student
- Three pieces of card stock paper per student
- Protractor per student
- A ruler
- Scissors
- A graduated cylinder
- Tape
- About 3 cups of uncooked rice
- Graphing Calculator per student
- Graph Paper

PRIOR KNOWLEDGE:

Review the Pythagorean Theorem (If students need a review activity, the website http://www.arcytech.org/java/pythagoras/ has a good discovery lesson for students and teachers). After reviewing the Pythagorean Theorem go through the mathematics of finding the height of a cone, given the slant height (10 cm) and the circumference of the base. These are the calculations that are used in the spreadsheet.

Have students draw a circle with a radius of 10 cm on several pieces of cardstock. Have them mark off central angles of 10 degrees through 180, in 10-degree intervals. In the group have students cut out different central angles and then make a cone with the remaining circle sector. It might be interesting to have students guess what size angle cutout will give the greatest volume of the cone.

Students should measure the volume of the cone with rice. This is tricky since it is hard to find the right level of rice especially when the cone has a large base and short height. It is helpful to use the ruler to level the rice. Have students keep a record of the volume of the cylinder on the spreadsheet that goes with this activity. Each student should make three different cones and measure them. Cutting out central angles of 10, 30, 50, 60, 70, 80, 90, 110, 120, 140 might be good measures.

After the students have collected the volume of different cones have them enter the volume with respect to the size of the angle that was cut out. Also have them graph the volume of the cylinder with the same size base and height on the same graph. Students should be able to see that the volume of a cone is 1/3 the volume of the cylinder. From the graph have them find the maximum volume of the cones and the corresponding angle that should be cut out to get that angle.

The excel worksheet on the following page is filled in for all values, including the volume of the cones, although the cone measurements are in red. You can obtain an electronic version of the spreadsheet from the Project TIME discussion forum on Google Groups. The file is named "Volumes of Cones.xls".

You might erase the volume column and have students fill it in after they have found the volume with rice. There is a column for the rice measurements. After finding the volume with rice and seeing that the volume of the cone is 1/3 of the cylinder, then go ahead and fill in the formula column. The graphs of the volume of the cylinder and cones with respect to the center angle cut out are on sheets 2 and 3.

ASSESSMENT

Students are to graph the data on the graphing calculator and on the graph paper marking the maximum volume of the cone. Once students have done the graphing they should compare their guess of the size the cone with the greatest volume with the actual cone and describe what they observe and why they made the guess that they did. On the graphing calculator, students should try to find the regression that best fits the data and write that answer on their hard copy.

Volume of Cones and Cylinders with the Same Base and Height

Center angle in degrees	Circumference of Base in cm	Radius of Base in cm	Height of cone in cm	Slant Height in cm	Height of Cylinder in cm	Volume of Cylinder in cm^3	Volume of Cone in cm^3 or ml	Measurements in rice
10.00	61.09	9.72	2.34	10.00	2.34	695.04	231.68	250.00
20.00	59.34	9.44	3.29	10.00	3.29	921.01	307.00	
30.00	57.60	9.17	4.00	10.00	4.00	1055.01	351.67	350.00
40.00	55.85	8.89	4.58	10.00	4.58	1137.17	379.06	
50.00	54.11	8.61	5.08	10.00	5.08	1184.37	394.79	400.00
60.00	52.36	8.33	5.53	10.00	5.53	1205.96	401.99	
70.00	50.61	8.06	5.93	10.00	5.93	1207.94	402.65	420.00
80.00	48.87	7.78	6.29	10.00	6.29	1194.52	398.17	
90.00	47.12	7.50	6.61	10.00	6.61	1168.86	389.62	350.00
100.00	45.38	7.22	6.92	10.00	6.92	1133.40	377.80	
110.00	43.63	6.94	7.20	10.00	7.20	1090.14	363.38	
120.00	41.89	6.67	7.45	10.00	7.45	1040.71	346.90	
130.00	40.14	6.39	7.69	10.00	7.69	986.50	328.83	

INTRODUCTION TO RIGHT TRIANGLE TRIGONOMETRY
Comments on Direct Instruction

This instruction described below is a discussion of the very basics of right triangle trigonometry using the since, cosine and tangent functions. If you feel you have a strong class, you may want to go deeper than this. If you have favorite worksheets or activities of your own for trigonometry, feel free to substitute those.

The instructor should begin by drawing a right triangle for the students and labeling one of the acute angles with a variable, maybe θ. Then label the sides "opposite", "adjacent" and "hypotenuse" appropriately and introduce the formulas:

$$\sin\theta = \frac{opposite}{hypotenuse} \quad \cos\theta = \frac{adjacent}{hypotenuse} \quad \tan\theta = \frac{opposite}{adjacent}$$

Note: There is no need to introduce any mnemonic devices for remembering these formulas, like the well-known "soh-cah-toa" shortcut, since this is just an introduction and students will not be expected to master these ideas at this level.

The next step would be to show how to use a calculator to find the values of sine, cosine or tangent and how to use those to calculate the lengths of unknown sides. The following examples indicate the kinds of questions students should try to answer in this activity. The instructor will probably want to draw pictures for the first few examples, and once the students get the idea, see if they can solve the problem when it is stated only in words.

- *The side opposite a 45-degree angle is 4 cm. What is the hypotenuse?*
- *The hypotenuse is 30 inches long. What is the length of a side adjacent to a 30-degree angle?*
- *The side adjacent to a 20-degree angle is 10 cm. How long is the side opposite that angle?*

If you have a group of students with strong algebra skills, you may want to end the class with a question like this one:

While standing at a certain spot, to see the top of a building in front of you, you have to raise your head to a 36-degree inclination. But if you back up 10 feet, you only have to raise your head to a 33-degree angle.

- How far away is the building?
- If your eyes are exactly 5 feet above ground level, how tall is the building?

This problem can be solved by letting r be your initial distance from the building in feet, and r+10 be your distance after you move back 10 feet. Drawing triangles will show you that

$$r \cdot \tan(36°) = (r+10) \cdot \tan(33°)$$

If you evaluate the values of $\tan(36°)$ and $\tan(33°)$ with a calculator, this gives you an algebraic equation you can solve for r. Note that to find the height of the building, you will have to take into account how far your eyes are above the ground.

Bear in mind that this is not supposed to be a detailed exposition of trigonometry. Mainly, students should get a glimpse of how these trigonometric functions are used in applications to right triangles. There is no need to discuss the periodicity of these functions, or to discuss the cosecant, secant and cotangent functions.

Try to spend as much time as possible letting students solve problems on their own by deciding what trigonometric function is appropriate for the given information and then solving for the unknown side length algebraically.

# FORMING A TETRAHEDRON
Description

ESTIMATED TIME: About 20 minutes.

PURPOSE: A quick activity that creates a three dimensional tetrahedron.
Calculate a height using the Pythagorean Theorem
Students measure surface area and volume of the most basic 3-D shape.

MATERIALS: Standard mailing envelopes for each student
Scissors
Ruler

STEPS:

- Draw the diagonals of the envelope and cut out the upper "flap" triangle.
- Fold the remaining envelope on the diagonals. Crease them front and back.
- Fold the envelope on its vertical symmetry line. Now unfold.
- Take one point and tuck it into the opposite corner.
- You now have a regular triangular pyramid or tetrahedron.
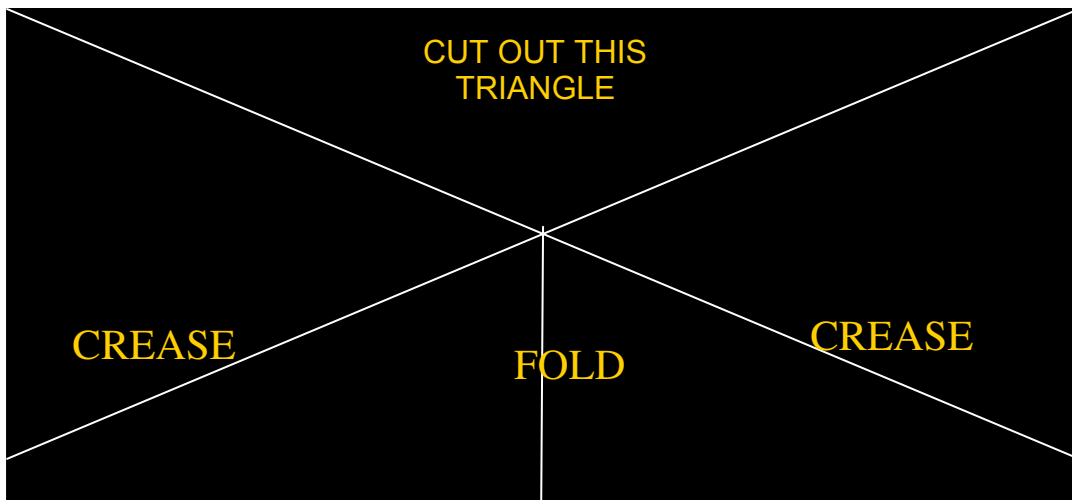
Now ask students to determine the height of the tetrahedron using the Pythagorean Theorem. They should also determine the surface area of the tetrahedron.

# SIMILAR FIGURES, UNIT ANALYSIS AND PROPORTION WORKSHEET
Description

ESTIMATED TIME: 20 minutes

This activity asks students to review the ideas of similarity and proportion.

SIMILAR FIGURES, UNIT ANALYSIS AND PROPORTION

1. You want to enlarge a small 3-in by 5-in photograph to a 12-in by 20-in copy. Assuming that the cost of photographic paper is proportional to its area, and that 3-in by 5-in reprints cost 40 cents each, how much would you expect to pay for the large copy?

2. The actor Elijah Wood, who plays Frodo Baggins in the movie of Tolkien's Lord of the Rings, is 5 ft 6 in tall, but his character is barely 4 ft tall. Put correctly, as a percentage, how much shorter is Frodo than Wood?

3. If an object is scaled linearly so that its volume grows to eight times its original volume, its surface area is scaled to _____times its original surface area.

4. In Germany the fuel efficieny of cars is measured in liters of fuel per 100km (L/100km). A typical average in a 2005 diesel compact station wagon is 7.3 L/100km. What is that in miles per gallon (mpg)?

5. Gasoline is sold in the United Staes by the US gallon and in Europe by the liter (1 US gal=231in^3; 1L=1000cm^3). What was the equivalent cost in US dollars per US gallon, for gasoline in Germany priced in euros at 1.42 euros per liter, when 1 euro = $1.25, in September, 2005?

6. In the triangle ABC shown below, A'C' is parallel to AC. Find the length y of BC'
    and the length x of A'A
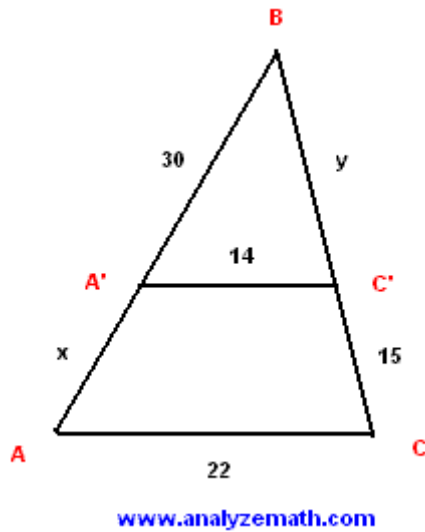
7. A research team wishes to determine the altitude of a mountain as follows: They use a light source at L, mounted on a structure of height 2 meters, to shine a beam of light through the top of a pole P' through the top of the mountain M'. The height of the pole is 20 meters. The distance between the altitude of the mountain and the pole is 1000 meters. The distance between the pole and the laser is 10 meters. We assume that the light source mount, the pole and the altitude of the mountain are in the same plane. Find the altitude h of the mountain.