Simplest Radical Form
In the expression $\sqrt[n]{b}$ n is the “index’ and b is the “radicand”.
Example: $\sqrt[3]{16x^4}$ the index is 3 and the radicand is $16x^4$.
Example: $\sqrt{75}$ the index is 2 and the radicand is 75.
To write and expression in simplest radical form the following conditions must be satisfied:

1. A radicand contains no prime factor to a power greater than or equal to the index.
2. The power of the radicand and the index of the radical have no common factor greater than 1.
3. No radical appears in the denominator.
4. No fraction appears within the radical.

The replacement set for all variables should be understood to be **positive** real numbers. Examples:

a) $\sqrt{75}$ violates rule 1, since $\sqrt{75} = \sqrt{3 \times 5^2}$

b) $\sqrt[3]{16x^4}$ violates rule 1 since $\sqrt[3]{16x^4} = \sqrt[3]{2^4x^4}$. In fact it violates it twice.

c) $\sqrt[4]{25x^2}$ violates rule 2.

d) $\dfrac{2}{\sqrt{5}}$ and $\dfrac{3}{2+\sqrt{5}}$ violate rule 3.

e) $\dfrac{\sqrt{x^3}}{\sqrt{y}}$ violates rule 1 and rule 3.

f) $\sqrt{\dfrac{1}{2}}, \sqrt{\dfrac{1}{x}}, \sqrt[3]{\dfrac{3}{16}}$ all violate rule 4.

To write in simplest radical form:

a) $\sqrt{75} = \sqrt{3 \times 5^2} = \sqrt{5^2} \times \sqrt{3} = 5\sqrt{3}$

b) $\sqrt[3]{16x^4} = \sqrt[3]{2^4x^4} = \sqrt[3]{2^3 \times 2 \times x^3 \times x} = \sqrt[3]{2^3x^3} \times \sqrt[3]{2x} = 2x\sqrt[3]{2x}$

c) $\sqrt[4]{25x^2} = \sqrt[4]{5^2x^2} = \sqrt{5x}$

d) $\dfrac{2}{\sqrt{5}} = \dfrac{2}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$
$\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}} = \dfrac{3(2-\sqrt{5})}{4-5} = \dfrac{6-3\sqrt{5}}{-1} = -6+3\sqrt{5}$
We say the “conjugate” of $a+\sqrt{b}$ is $a-\sqrt{b}$. Therefore the last example is often described by “mulitiply the numerator and denominator by the conjugate of the denominator”.

e) $\dfrac{\sqrt{x^3}}{\sqrt{y}} = \dfrac{\sqrt{x^3}}{\sqrt{y}} \times \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{x\sqrt{xy}}{y}$

f) $\sqrt{\dfrac{1}{2}} = \dfrac{\sqrt{1}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Another example:

$$\frac{2+\sqrt{84}}{2} = \frac{2+\sqrt{2^2 \times 3 \times 7}}{2} = \frac{2+2\sqrt{21}}{2} = \frac{2(1+\sqrt{21})}{2} = 1+\sqrt{21}$$

Write in simplest radical form:

1. $\sqrt{8}$
2. $\sqrt{50}$
3. $\sqrt{45}$
4. $\sqrt{6} \times \sqrt{12}$
5. $\sqrt{3^2 + 4^2}$
6. $\sqrt{27x^3}$
7. $\sqrt{40x^3y^5}$
8. $\sqrt[3]{40x^3y^5}$
9. $\sqrt[4]{4x^2y^6}$
10. $\dfrac{\sqrt{125}}{\sqrt{5}}$
11. $2\sqrt{xy^2} - y\sqrt{x}$
12. $\dfrac{1}{\sqrt{3}}$
13. $\dfrac{1}{\sqrt{x}}$
14. $\dfrac{\sqrt{8}}{2}$
15. $\dfrac{1}{2-\sqrt{3}}$
16. $\dfrac{x}{\sqrt{x}-1}$
17. $\dfrac{2-\sqrt{8}}{2}$
18. $\dfrac{2+\sqrt{16-12}}{2}$
19. $\dfrac{-2+\sqrt{2^2-4\times(-7)}}{2}$
20. $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$