09: Quadratic Equations

Key Terms

Equation: a statement that two expressions have the same value.

Quadratic equations: an equation that has the standard form $ax^2 + bx + c = 0$.

Zero product property: if $ab = 0$, the $a = 0$ or $b = 0$.

Factor: to rewrite an expression as a product.

Solutions or roots of the equation: values an equation takes when the values of its domain are substituted for the variable.

Solution set: collection of all solutions to an equation.

Quadratic inequality: a quadratic equation where the equal symbol is replaced by an inequality symbol.

Perfect square trinomial: $a^2 - 2ab + b^2 = (a - b)^2$; $a^2 + 2ab + b^2 = (a + b)^2$

Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

Discriminant: the value under the radical in the quadratic formula, $b^2 - 4ac$.

Quadratic function: function in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

Parabola: the graph of a quadratic function.

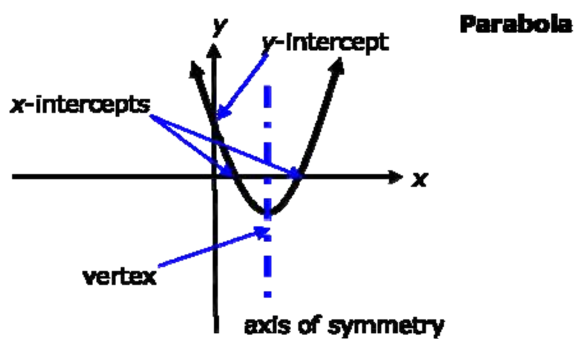
Axis of symmetry: divides parabola into two equal parts, each part is a mirror image of another.

Vertex: point where the parabola intercepts the axis of symmetry.

x-intercepts: the points where parabolas intercepts x-axis (where $y = 0$).

y-intercepts: point where the parabola intercepts the y-axis (where $x = 0$).

Quadratic Function Graph



A parabola can open downward ($a < 0$) or upward ($a > 0$).

Example: Factoring

Solve. $x^2 - x - 2 = 0$

Solution:

$(x - 2)(x + 1) = 0$	Factor left side
$x - 2 = 0$ or $x + 1 = 0$	Apply zero-product
$x = 2$ or $x = -1$	Solve equations

The solution set of this equation is {-1, 2}.

Example: Square Root Method

Solve using the square root method. $x^2 - 9 = 0$

Solution:

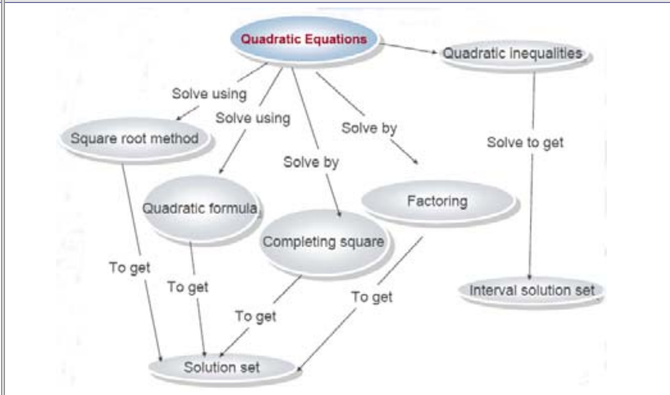
$x^2 = 9$

$x = \pm\sqrt{9}$

$x = 3$ or $x = -3$

The solution set of this equation is {-3, 3}.

Concept Map



Example: Quadratic Inequality

Find the solution set. $x^2 + 3x < 18$

Solution: Put the equation in standard form.

$x^2 + 3x - 18 < 0$

To define boundaries, change the inequality to an equality then find the solution of the equation.

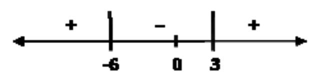
$x^2 + 3x - 18 = 0$

$(x + 6)(x - 3) = 0$

$x = -6$ or 3

Denote the test intervals: $(-\infty, -6)$, $(-6, 3)$, $(3, \infty)$.

Find the sign, positive or negative, in each interval using test values.



The solution set of the inequality is the interval (-6, 3).

Example: Quadratic Formula

Solve using the quadratic formula. $2x^2 - 3x + 1 = 0$

Solution: Identify a, b, and c of the quadratic equation, then use the quadratic formula to solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{1}}{2(2)}$$

$$= 1 \text{ and } \frac{1}{2}$$

The solution set of this equation is {½, 1}.

Example: Complete the Square

Solve by completing the square. $4x^2 - 12x = -5$

Solution:

$4x^2 - 12x = -5$

$4x^2 - 2(3)(2x) + 9 = -5 + 9$

$(2x - 3)^2 = 4$

Apply the square root method.

$2x - 3 = \sqrt{4} = 2$ or $2x - 3 = -\sqrt{4} = -2$

Solve the equations.

$2x - 3 = 2$		$2x - 3 = -2$
$2x - 3 + 3 = 2 + 3$		$2x - 3 + 3 = -2 + 3$
$2x = 5$	or	$2x = 1$
$x = \frac{5}{2}$		$x = \frac{1}{2}$

The solution set of this equation is $\left\{\frac{1}{2}, \frac{5}{2}\right\}$.

How to Use This Cheat Sheet: These are the key concepts related this topic. Try to read through it carefully twice then rewriite it out on a blank sheet of paper. Review it again before the exam.