Signature _____ **Name** _____

Student ID _____

Midterm
CSE 21
Fall 2012

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Page 2	_____	**(15 points)**
Page 3	_____	**(21 points)**
Page 4	_____	**(13 points)**
Page 5	_____	**(9 points)**
Page 6	_____	**(7 points)**
Total	_____	**(85 points)**

(80 points = 100%)
(5 points Extra Credit = >6%)

This exam is to be taken by yourself with closed books, closed notes, no electronic devices.
You are allowed one side of an 8.5"x11" sheet of paper handwritten by you.

Calculate the first 6 terms for the triangular numbers ($n = 1$, $n = 2$, $n = 3$, ... , $n = 6$). Then to the right calculate the sequence of differences between these terms. 1st triangular number is 1.

n		Sequences of differences		
1	_____ = _____			
		_____		
2	_____ = _____		_____	
		_____		_____
3	_____ = _____		_____	
		_____		_____
4	_____ = _____		_____	
		_____		_____
5	_____ = _____		_____	
		_____		
6	_____ = _____			

* n = 1
* * n = 2
* * * n = 3
* * * * n = 4
etc.

Write the recurrence relation for the triangular numbers $T(n)$.

$$T(n) = \begin{cases} \_\_\_\_\_ & \text{if } n \_\_\_\_\_ \\ \_\_\_\_\_ & \text{if } n \_\_\_\_\_ \end{cases}$$

Based on the sequence of differences (above) what is a good guess for the closed-form solution to the recurrence relation above?

$f(n) =$ _____

A) $n^4 - n^2 + 2$	B) $2n + n$	C) $2^n + n$
D) $4n - 2n$	E) $n^3 - n$	F) $(n^2 + n)/2$

Why?:

Verify this with a proof by induction. Prove $T(n) = f(n)$ for all n _____ .

Proof (Induction on n):

_____: If $n =$ ____, the recurrence relation says $T(\_\_) =$ ____,

and the closed-form solution says $f(\_\_) =$ _____ = ____, so $T(\_\_) = f(\_\_)$.

_____: Suppose as inductive hypothesis that $T(k\text{-}1) =$ _____ for some k ____.

_____: Using the recurrence relation, $T(k) =$ _____, by 2nd part of RR

= _____, by IHOP

= _____

= _____

So, by induction, $T(n) = f(n) =$ _____ for all n _____ (as desired).

Which general recursive decompositions discussed in class are most appropriate for the following algorithms:

Sum as defined _____
Sum(L) = x + Sum(L') [where L = L',x]
|L'| = |L| - 1

Sum as defined _____
Sum(L) = Sum(X) + Sum(Y) [where L = (X,Y)]
|X| = |Y| (+/- 1)

You may want to draw Venn diagrams (on your scratch paper) to help you answer the following few questions.

Professor Polly Morphism has 25 students in her Computer Architecture class, 15 students in her Discrete Math class, and 20 students in her Compilers class.

Assuming that there are no students who are taking more than one class from her, how many students does she have? _____

Now assume there are 5 students who take only Computer Architecture and Discrete Math at the same time and 7 students who take only Discrete Math and Compilers at the same and 6 students who take only Computer Architecture and Compilers at the same time. No student takes all three classes at the same time.

how many total students does she have? _____
how many students are only in her Compilers class but not in her other classes? _____
how many students are only in her Computer Architecture class but not in her other classes? _____
and how many students are only in her Discrete Math class but not in her other classes? _____

Now instead of no student taking all three classes, what if the above question was modified to now specify that 2 of the students take all three classes at the same time. How many total students does she have? _____
(The students who take only two classes described above remains the same.)

To save time, the CSE 21 TAs/Tutors grade only 2 randomly chosen problems out of 10 on your homework, for a total of 2 points (1 point each). Suppose you answered 7 problems correctly.

Compute the probability that all 2 randomly chosen problems are correct. _____

Compute the probability that all 2 randomly chosen problems are incorrect. _____

Compute the expected value of the number of points (your score) on your homework.

_____

How many different strings can be formed by rearranging the letters in SUCCESSLESSNESS, using all the letters?

_____

What is the probability of rolling a 4, 5, 6, 7, 8, 9, 10, or 11 (sum of two fair 6-sided dice will be a 4 - 11)?

_____ (give answer as a non-reduced fraction)

How many different strings of length 14 can be formed from a set of 26 refrigerator magnets A-Z?

_____

How many strings of length 4 can be formed from a 17-symbol alphabet where no 2 adjacent symbols are the same?

_____

Big-Omega provides a(n) _____ bound on the growth rate of a function while big-Oh provides a(n) _____ bound on the growth rate of a function.

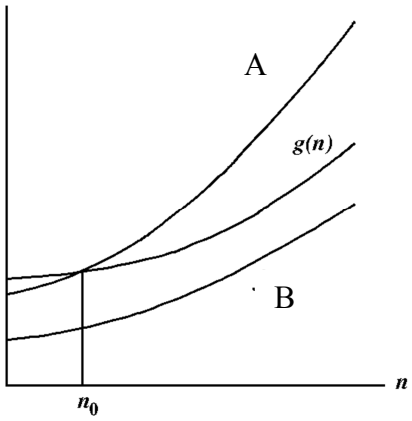
Given $K_1 f(n) \leq g(n) \leq K_2 f(n)$, we say that "$g$ is big-_____ of f" or "g is order f."

$K_1 f(n)$ represents the big-_____ of f while $K_2 f(n)$ represents the big-_____ of f.

With respect to the graph to the right,

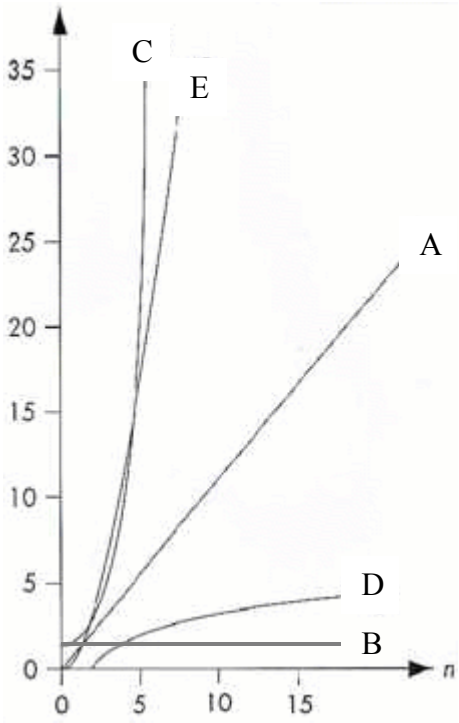
the function labeled _____ represents big-Oh of f and

the function labeled _____ represents big-Omega of f.



Match the big-Oh complexity class value to the graph. Write the letter associated with the function to the right that describes the big-Oh runtime complexity for that function.

____ $O(n^2)$
____ $O(1)$
____ $O(2^n)$
____ $O(\log_2 n)$
____ $O(n)$



Between which two plot lines would we find

$O(n \log_2 n)$ - between ____ and ____

$O(n^3)$ - between ____ and ____

What is the <u>name</u> of the complexity class for each labeled function?
(Not big-Oh – the English word name for the complexity class.)

A) _____

B) _____

C) _____

D) _____

E) _____

If you pick 10 cards out of a standard 52 card deck, at least how many of them must be the same suit (either clubs, diamonds, hearts, or spades)?

_____

How many 7-digit phone numbers are possible within an area code? Phone numbers can contain all zeros thru all nines.

_____

How many 7-digit phone numbers have all different digits (no duplicates)?

_____

How many 7-digit phone numbers contain only odd digits?

_____

How many 7-digit phone numbers have at least one even digit?

_____

How many 7-digit phone numbers start with an odd digit and end with an even digit?

_____

How many 7-digit phone numbers can be formed with at least one duplicate digit (for example, 0045689, 5886831, and 3399797)?

_____

The easiest way to solve the previous question is to use the technique called

_____

There are 13 customers and 4 cashiers. How many ways can the customers line up to the cashiers, if the order of each line does not matter.

_____

Rick has 38 comics to show in the last 7 lectures for CSE 21. If he distributes the comics evenly across the lectures, this guarantees that at least one lecture will have how many comics?

_____

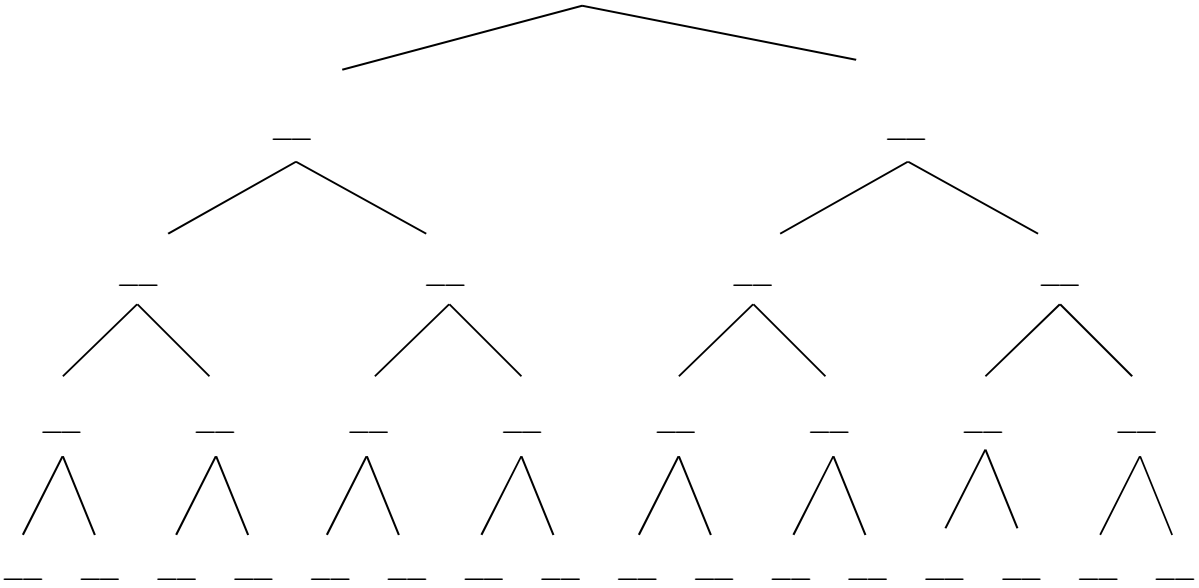
An urn contains 12 balls numbered 0-11. Seven balls are drawn from the urn in sequence, and the numbers on the balls are recorded. How many ways are there to do this, if

all seven balls are drawn at once (one handful of seven balls)? _____

each ball is replaced before the next one is drawn? _____

when each ball is drawn it is not replaced? _____

How many four-digit binary strings are there that do not contain 101 or 000? First draw a decision tree. Each slot/line should have a single 0 or 1. 0s to the left and 1s to the right at each split.



How many such four-digit binary strings that do not contain 101 or 000? _____

What is the value of P(6,3)? Your answer should be an actual number for this one. _____

What is the value of C(12,2)? Your answer should be an actual number for this one. _____

C(42,5) is the same value as C(42, ____)

C(38,1) is the same value as C(38, ____)

C(29,29) is the same value as C(29, ____)

Consider the following algorithm:

```
for ( i = 1; i <= 6; ++i )
{
  beep;

  for ( j = 1; j <= 4; ++j )
  {
    beep;
  }

  for ( k = 1; k <= 3; ++k )
  {
    for ( l = 1; l <= 5; ++l )
    {
      beep;
    }

    for ( m = 1; m <= 4; ++m )
    {
      beep;
    }
  }
}
```

How many times does the `beep` statement get executed? _____

Find a big-Theta estimate for the number of ways
to choose a set of five or fewer elements from a set of size *n*. Big-Theta(_____)

Match the person to what the person is famous for. (1/2 point each)

1) Niklaus Wirth
2) C.A.R. Hoare
3) Edsger Dijkstra
4) Donald Knuth
5) Alan Turing
6) Grace Hopper
7) John von Neumann
8) Alan Perlis
9) John McCarthy
10) John Backus

_____ Father of Lisp.

_____ First Turing Award winner.

_____ Invented garbage collection.

_____ Invented Quicksort algorithm.

_____ Invented Merge sort algorithm.

_____ Helped popularize the term "debugging."

_____ Father of Fortran and part of BNF notation

_____ Known as the father of the analysis of algorithms.

_____ null reference self-described as a billion-dollar mistake.

_____ Proved that what is now known as the halting problem is undecidable.

_____ Has a well-known conference celebrating women in computing named after.

_____ Invented the single-source shortest path algorithm and the semaphore used in operating systems.

_____ Visualized a nanosecond with a piece of wire just under a foot in length as the distance light travels in a nanosecond.

_____ Described early single-memory, stored program architecture we now commonly know as the general purpose computer.

Scratch Paper

Scratch Paper