Sinusoidal Functions Unit 8
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Time 13 hours

In Mathematics 1201, students used the primary trigonometric ratios and the Pythagorean theorem to solve right triangle problems (M4). In Mathematics 2201, they solved problems that involved the cosine law and the sine law, excluding the ambiguous case (G3). In this unit, students describe the characteristics of sinusoidal functions by analyzing the graph and its corresponding equation: $y = a\sin b(x - c) + d$ and $y = a\cos b(x - c) + d$.

They should also determine the equation of the sinusoidal regression function that models a set of data.

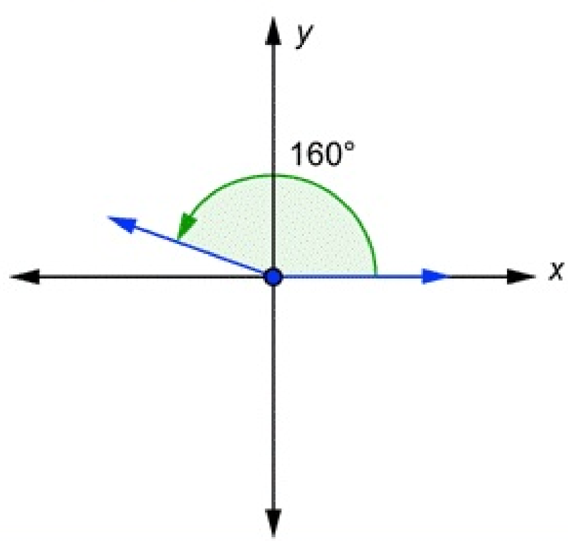
The unit circle and Radian Measure:

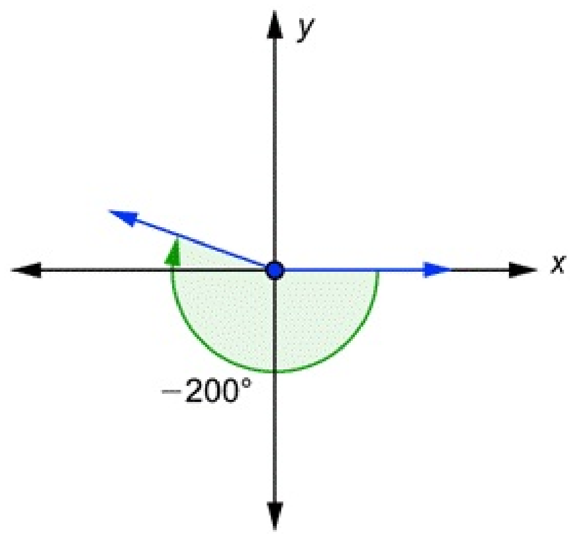
1) Angles are measured in degrees or radians (radius)

Angles in Standard Position

Angles are considered in standard positive on the x-y plane if the vertex is at the origin...initial (starting) arm is on the positive part of the X-axis and rotated in the +ve or -ve direction

Example	
Problem	**Draw a 160° angle in standard position.**
Answer	The angle is positive, so you start at the *x*-axis and go 160° counterclockwise.

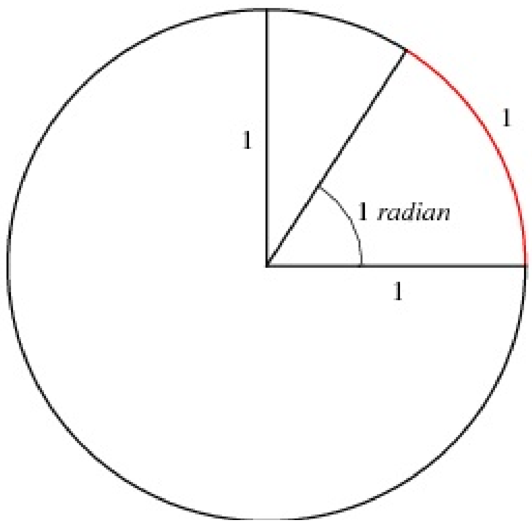


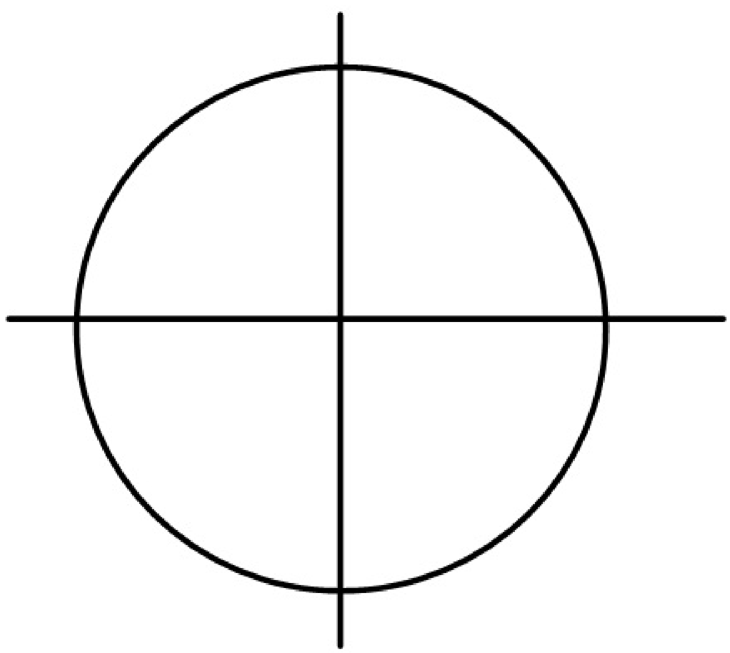
Example	
Problem	**Draw a $-200°$ angle in standard position.**
Answer	The angle is negative, so you start at the x-axis and go 200° clockwise. Remember that 180° is a straight line. That will bring you to the negative x-axis, and then you have to go 20° farther. 

Sketch 130°, 210°, -300°, 300° in standard position

8-1 Continued Radian Measure of an Angle

Radian Measure: another form of angle measure whereby the length of the arc the angle forms is written in terms of the radius of the circle





Why 2π radians = 360°?

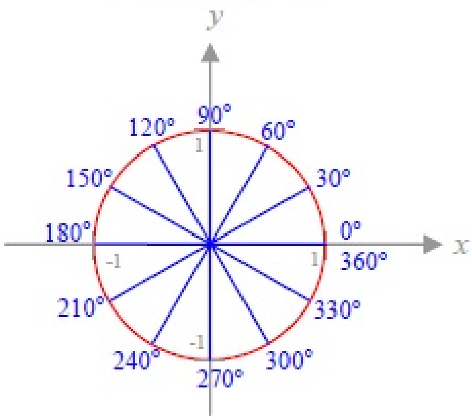
C= $2\pi r = 2\pi(1) = 2\pi = 360°$

this means the arc length on a circle with radius of 1 unit (the unit circle) is 2π= 6.28 radians

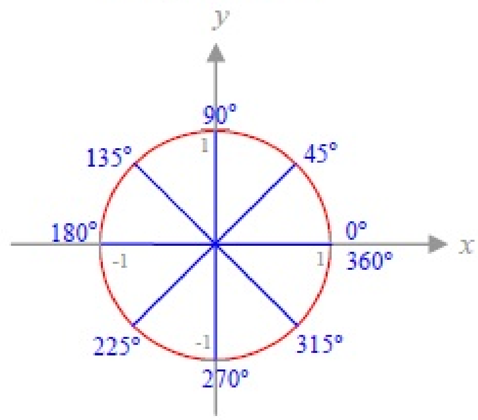
(show)

Special Rotations on the unit circle

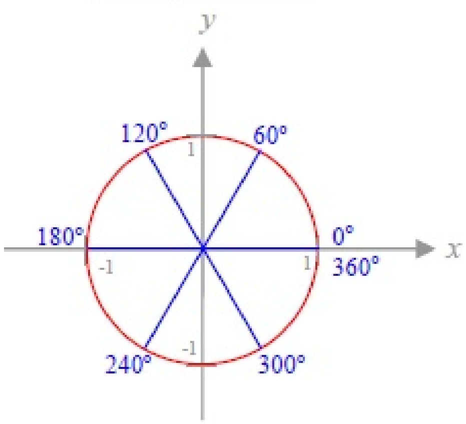
Multiples of 30°:



Multiples of 45°:



Multiples of 60°:



8-1 ctd Radian Measure May 1 2014--- April 27 2015

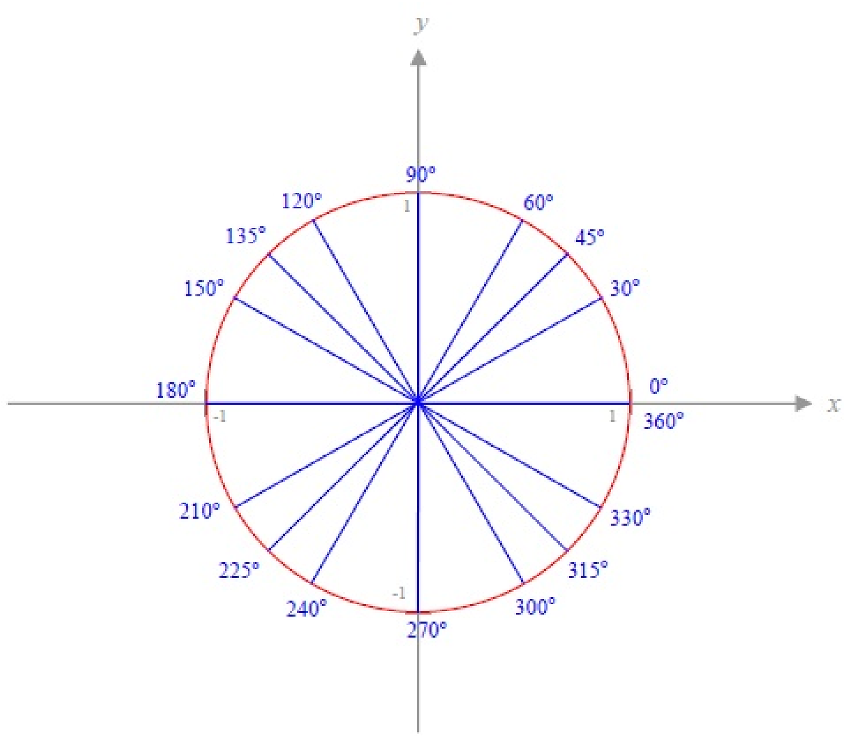
How to write angles in Radian Measure

if an angle is written in terms of π we say it is in radian measure. This means that the length of the arc the angle forms is written terms of the radius of the circle.

IE How to convert that $90° = \frac{\pi}{2} = 1.57$ radians?

Angle in Radian Measure in terms of Pi is always:

$$\theta \text{ in degrees} \bullet \frac{\pi}{180°}$$



IE: $135^\circ =$

$240^\circ =$

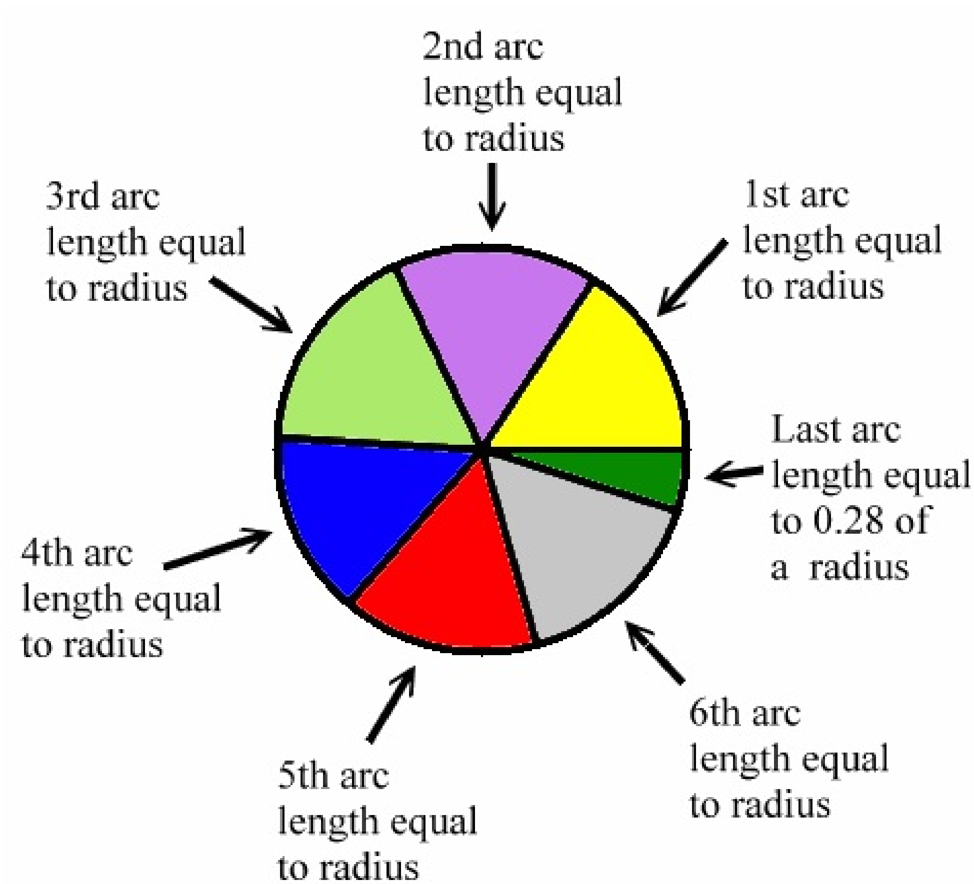
$330^\circ =$

Now since 2π(1) = 360° (1 here is the radius hence radian)

$$\text{Therefore 1 Radian} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

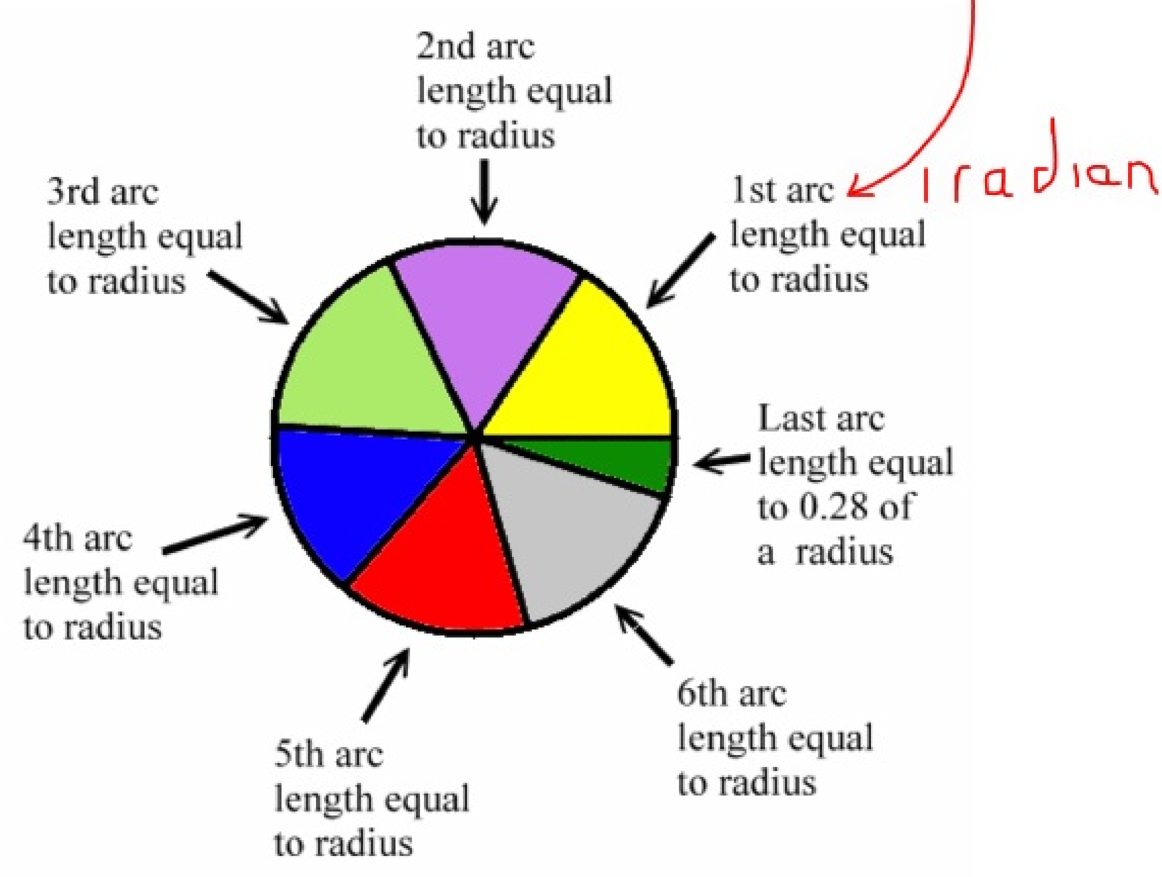
Hence 1 radian is approximately 57°

2 radians is_______



Hence 1 radian is approximately 57°

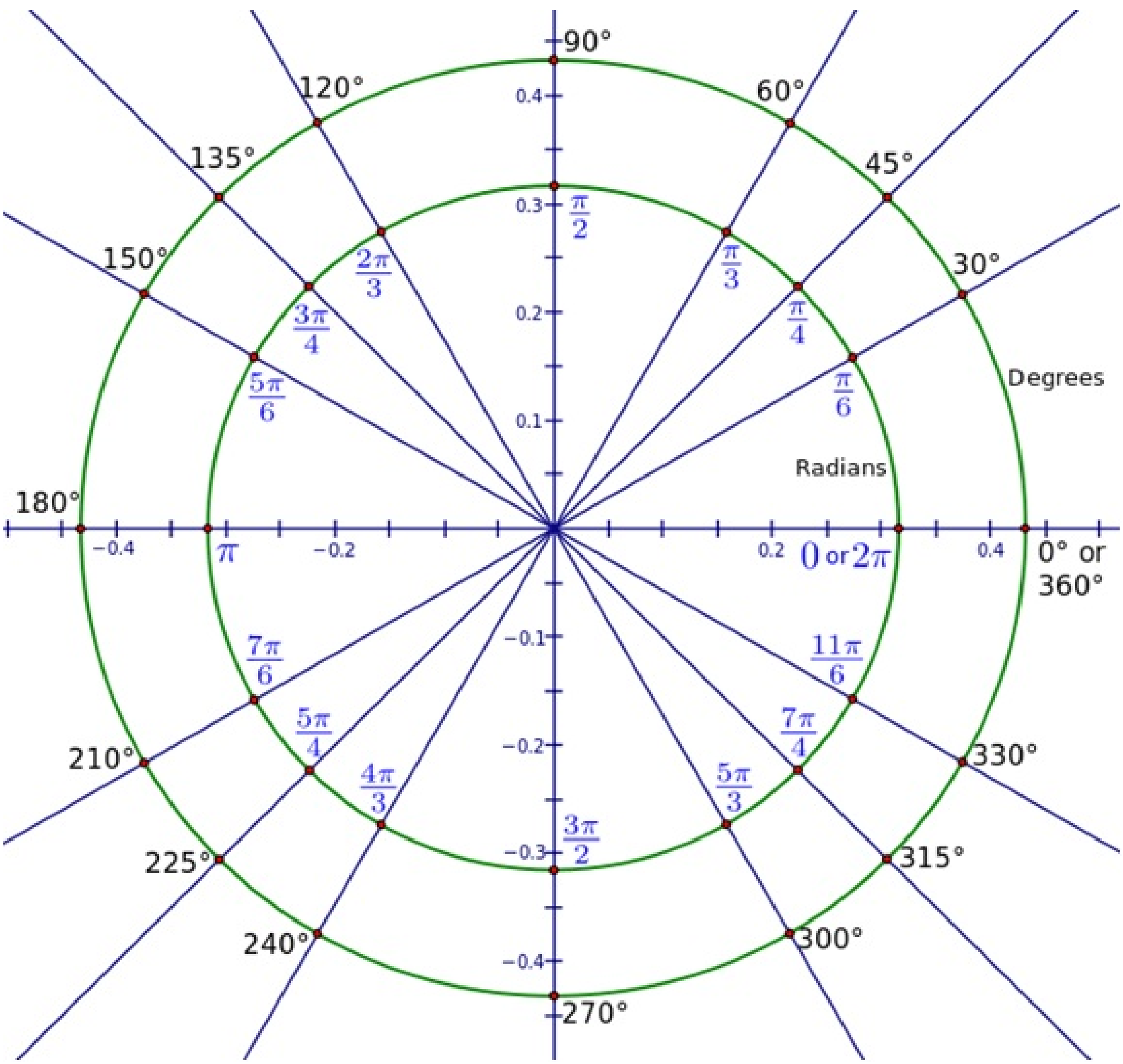
2 radians is_______



Formula for converting Radian Measure in terms of Pi or radians into degree measure:

$$\text{Angle in Degree Measure} = \text{Angle in Radian} \bullet \frac{180^\circ}{\pi}$$

Ex)



IE $60° =$ $\frac{5\pi}{3} =$

$300\ ° =$

$120° =$

$135° =$

Ask students to approximate the value for each conversion:

(i) 0.4 radians to degrees

(ii) 4.5 radians to degrees

(iii) 150° to radians

(iv) 470° to radians

A student approximated 45° to be 0.8 in radian measure. Ask students to use this measure to estimate the missing values.

(i) 90° = ? radians

(ii) ?° = 4 radians

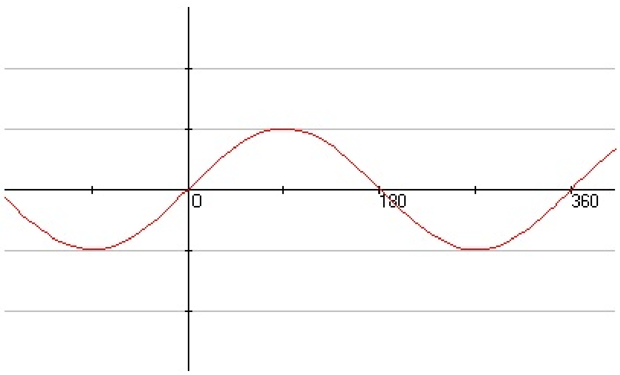
(iii) 135° = ? radians

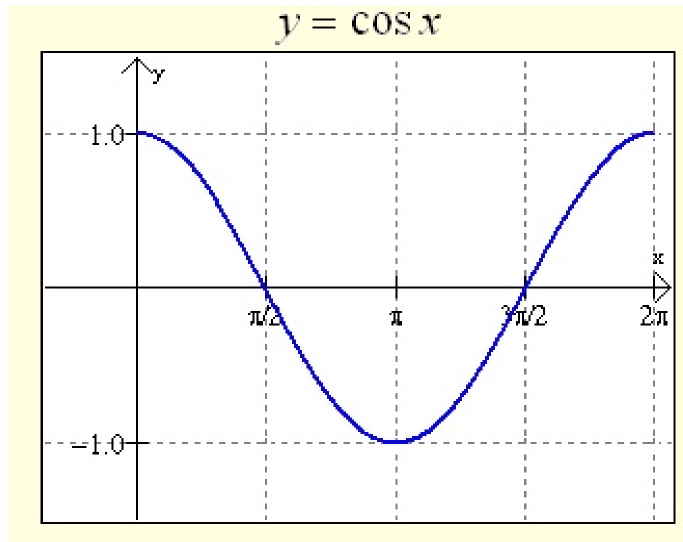
(iv) ?° = 3.2 radians

Section 8-2 See Paula's Notes...graphs of sin x and cos x

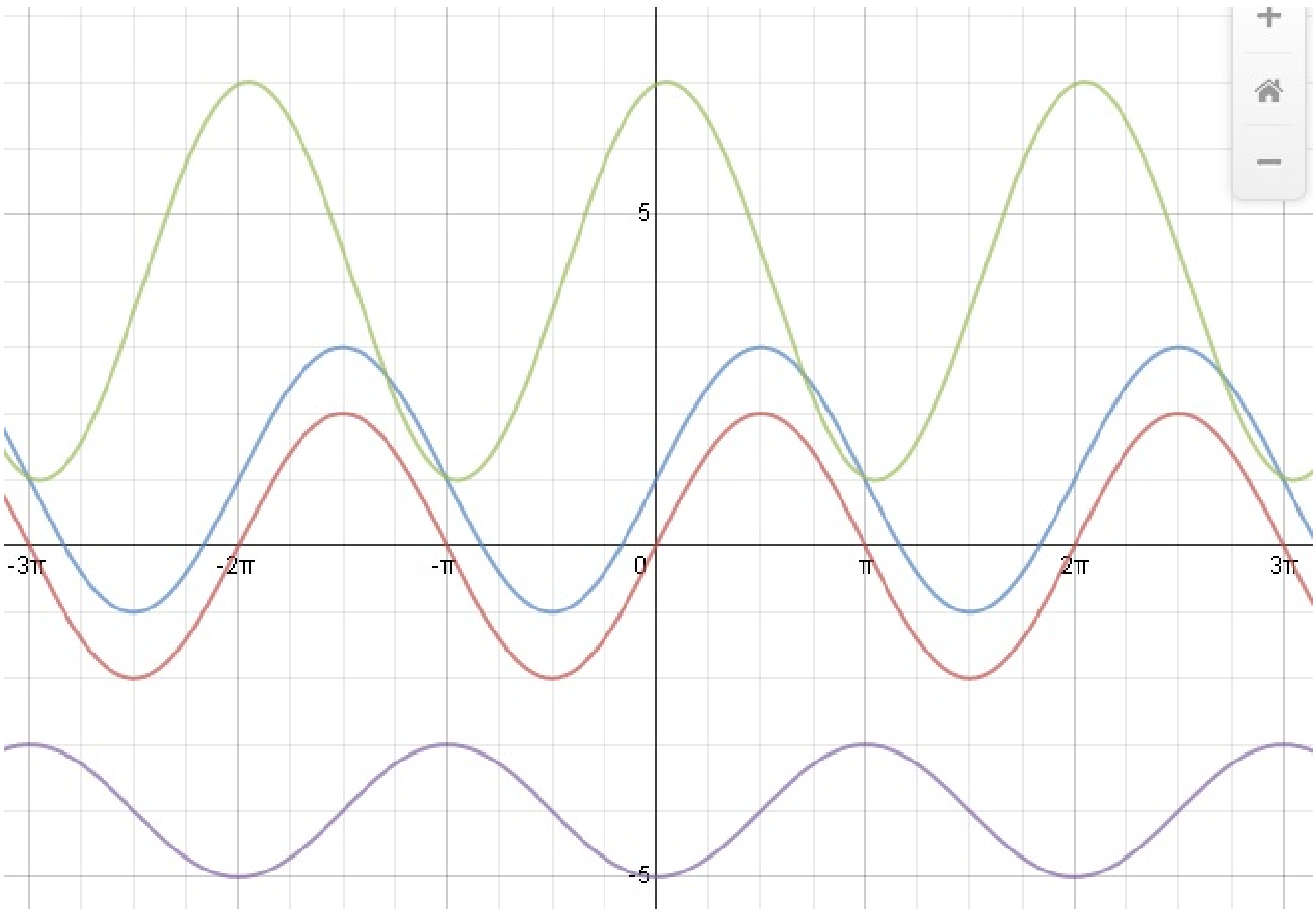
8-2 Periodic Vs. Sinusoidal Functions

Sinusoidal means it follows the path of the y = sin x or y = cos x wave





Sinusoidal Graphs: are periodic always



However, if a graph is periodic...it does not necessarily mean it is sinusoidal

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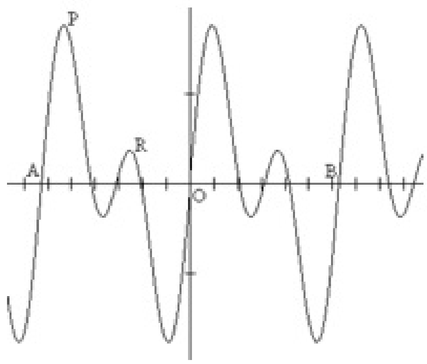
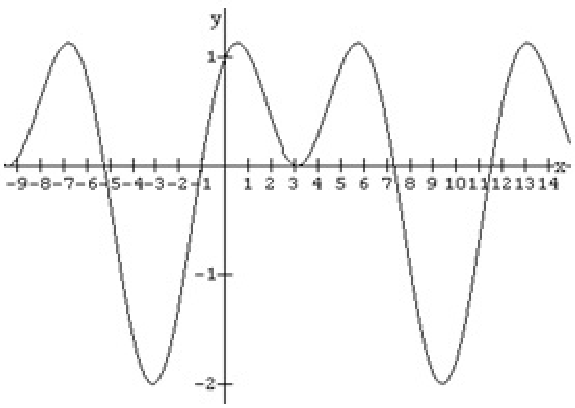
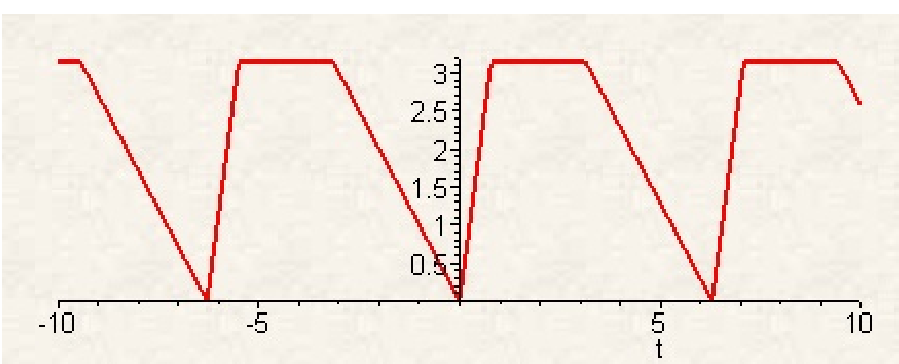


for example this graph has two different max heights in one period...therefore NOT sinusoidal but periodic



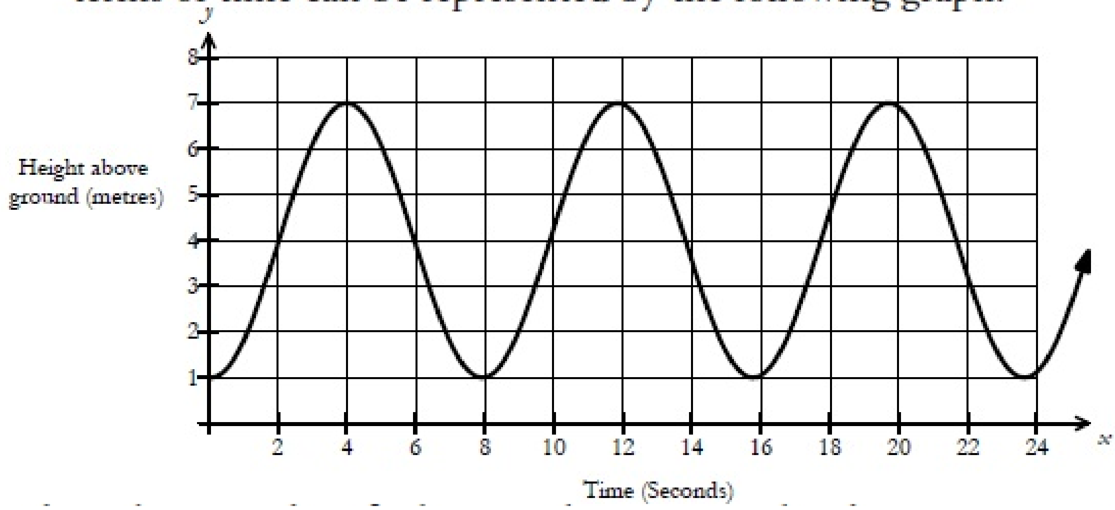
Not Sinusoidal but periodic



Periodic but NOT Sinusoidal

Applications of Sinusoidal Functions
Modeling Real World Situations

- While riding on a Ferris wheel, Mason's height above the ground in terms of time can be represented by the following graph.



1 Period = how long it takes to complete 1 full rotation
= HORIZONTAL distance between any two maxes or mins
= Horizontal distance between any three points on the midline

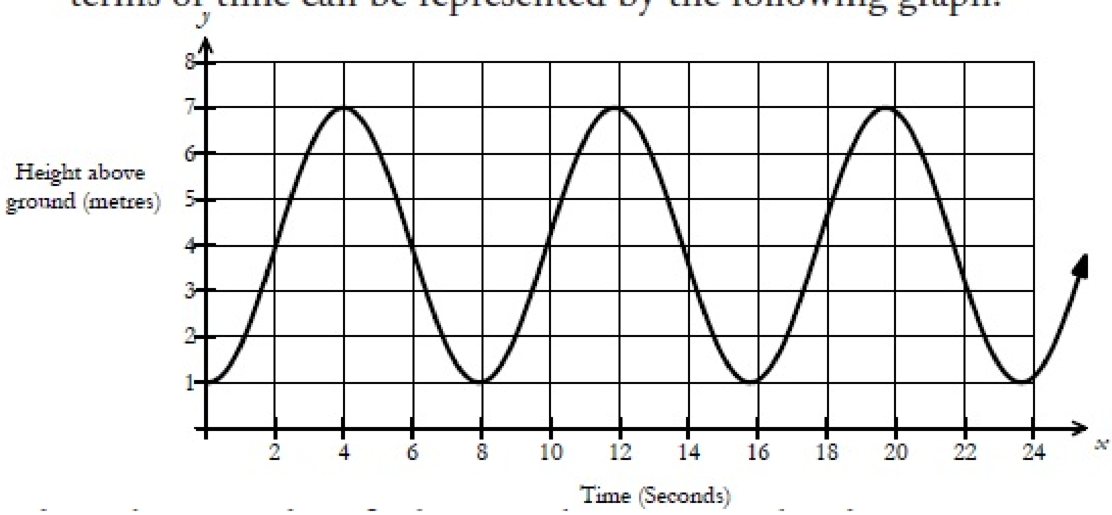
Period =

2 Amplitude: distance from the midline to a Max ir to a min.

Midline is at y =____

Amplitude is also the height of the axle of the wheel.

- While riding on a Ferris wheel, Mason's height above the ground in terms of time can be represented by the following graph.



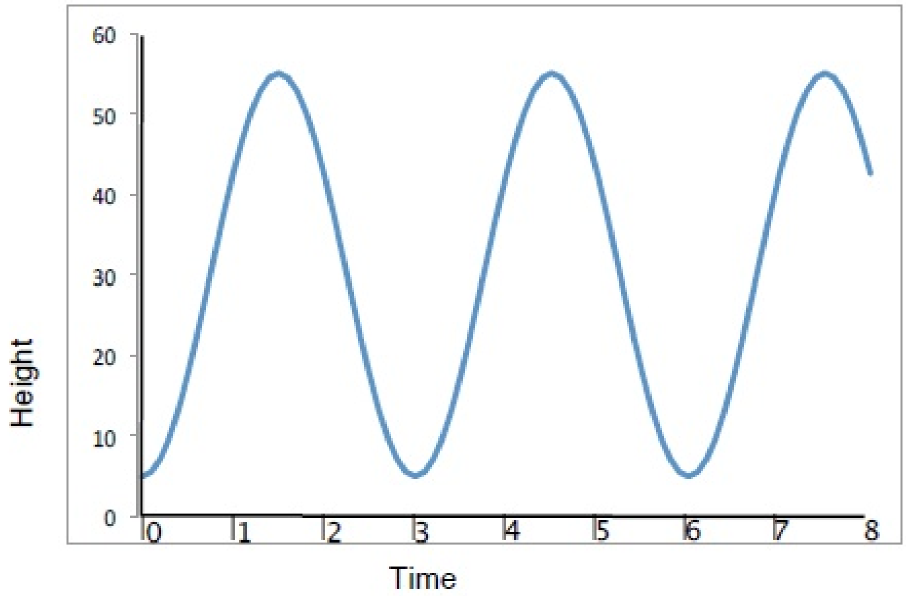
Here the axle is at y = 4 (4 meters above the ground)

Therefore Amp = distance from 4 to 7 which is 3

Alternate Method: (7-1)/2   (Max - Min)/2=6/2=3

3) Intercepts: x none why? If there were the Ferris wheel would be rolling down the street for starters and that's not a good thing with you in the chair :-)

y -intercept: y = 1 this has a meaning. It means the minimum height the Ferris wheel is above the ground. It also could mean at what height you boarded the Ferris wheel.



How long for one revolution? Period =

Height of the axle of the Ferris wheel? Midline

At what height did you board the Ferris wheel?

What was the maximum height achieved?

What is the radius of the Ferris wheel? Amp=25 m

8-3 Graphs of Sinusoidal Functions

The graph of a sinusoidal function is shown. Describe this graph by determining its range, the equation of its midline, its amplitude, and its period.

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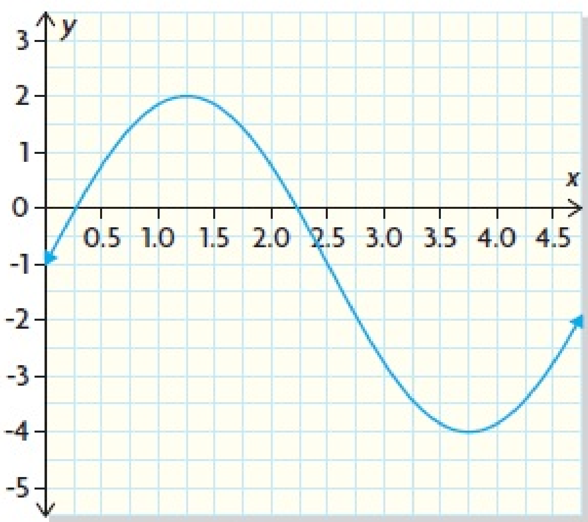

8-4 The equations and properties of Sinusoidal Functions

$y = a \bullet \sin b(x-c)+d \quad y = a\cos(x-c)+d$

1) Base graph: y = sin x studying the effect of changing the "a" in y = a sin x.

y = sin x y = 2sin x y = 4 sin x y = -2 sin x

- What happens to the amplitude if $a > 0$?
- Is the shape of the graph affected by the parameter *a*?
- How is the range affected by the parameter *a*?
- Will the value of *a* affect the cosine graph in the same way that it affects the sine graph? Why or why not?

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y = .5 sin x

Summary:
1) increasing a increases the _____ or the vertical stretch
2) Making "a" negative will cause a _____ in the ____ axis
3) Maxes will become Mins and vice versa. (If a is negative) However points on the original midline (sinusoidal axis) will not change (yet)
4) Amplitude = $|a|$

y = 6sinx y = -6sinx Amp

2) 2^{nd} Effect Changing d

in y = sin x (base graph) to y = a•sinb(x-c)+d

Sketch y = sin (x), y = sin (x) + 3, Y = sin (x) - 2

- How does each graph change when compared to $y = \sin x$?
- How is the value of d related to the equation of the midline?
- Is the shape of the graph or the location of the graph affected by the parameter d ?
- Is the period affected by changing the value of d ?
- Will the value of d affect the cosine graph in the same way that it affects the sine graph? Why or why not?

Combine the two effects.

What will y = 2sin(x) + 3 do to the base graph y = sin(x)?

1)

2)

What will y = -3cos(x) + 2 do to the base graph y = cos(x)?
1)

2)

Changing d in y = asin[b(x+c)] + d causes the base graph to

1) moves the base graphs up or down d units depending on the sign age

2) midline to change as well

See notes 8-4 from word

8-4 Summary of all the effects

For: $y = a\sin b(x-c)+d$ or $y = a\cos b(x-c)+d$

In alphabetical order:

1) a changes the amplitude (stretches the graph vertically)

The amplitude for the base graph y = (1)sin x
Y = sin (x) is 1

IE in y = 3sin5(x-30°)+8 the new amplitude is 3
IE in y = 2sin(x-40°) -5 the new amplitude is 2
IE in y = .5sin(x) the new amplitude is .5

2) b affects the period of the base graph. It either shortens or lengthens it (speeds it up or slows it down)

Base graph: y = cos x period = 360° or 2π

IE in y = 2sin4(x+60°) new period is $\frac{360^\circ}{4} = 90^\circ$

IE in y = $y = 2\sin\frac{1}{4}(x-20^\circ)+3$ $period = \frac{360^\circ}{\frac{1}{4}} = 360^\circ x4 = 1440^\circ$

3) c I) c is positive causes base graph to move left c units (shift in x= HT)

II) c is negative cause base graph to move c units right (shift in x HT)

IE y = sin x y = 4sin(x - 60°)+3 causes y = sin x to move 60° units right. HT = 60° right

Y = cos x   y = cos (x + 90°)-6 causes y = cos x to move 90° left. HT = 90° left

4) d I) causes the base graph to move up or down vertically (shift in y)

II) also y = d is the new MIDLINE

IE y = 3sin(x-10°)+6 causes d = 6 causes the base graph to move 6 units up.

Midline is y = 6

Ex) For y = 4 cos 2(x - 45°) +7   y = a cos b(x-c)+d

a = 4,   b = 2,   c= 45°   d= 7

Amplitude=
Period =
Horizontal Translation:
Vertical Translation:
New Midline:

Ex) For y = $y = 2.5\cos\frac{1}{2}(x - 30^\circ) - 4$ repeat:

Amplitude=
Period =
Horizontal Translation:
Vertical Translation:
New Midline:

8-5 Modeling Data with sinusoidal functions

RF8.7 *Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.*

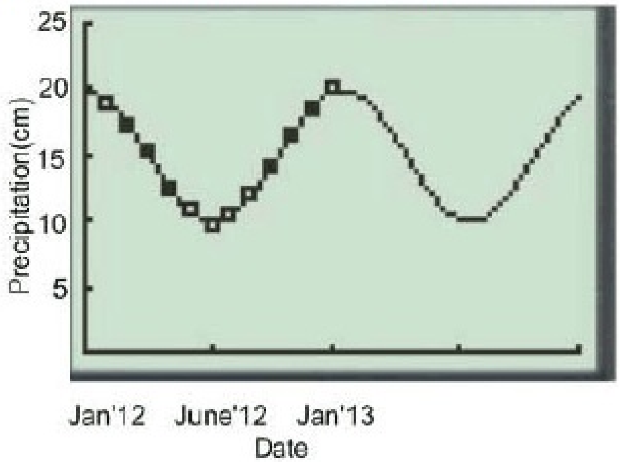
- A marine biologist recorded the tide times in the Greater St. Lawrence area for July 17th and 18th, 2012 in the chart below.

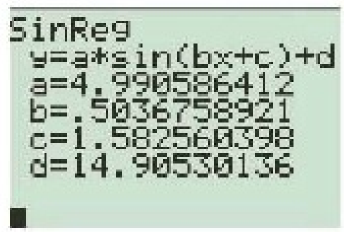
Time (Hour:Minutes)	Height (Metres)
01:50	0.8
07:55	1.5
13:35	0.7
20:38	1.9
26:25	0.8
32:40	1.6
38:15	0.6

Ask students to use a curve of best fit to estimate the water height after 15 hours of observation. Can this model be used to predict the tide height over the next 2 days? The next week? The next month? Discuss with your classmates. (RF8.6, RF8.7)

Perform curve of best fit SinReg

- Meteorologist Bryan Nodden recorded the average precipitation in Grand Falls-Windsor for 2012 and created a sinusoidal regression for the data. Ask students to use the graph to predict the amount of precipitation in August 2013. They should use the equation to predict the amount of precipitation in March 2017. Ask them to consider what some problems might be with the use of a sinusoidal regression to predict future precipitation.





End of Unit!