10

Factors, powers and standard form

This chapter is about multiples, factors, powers, roots and standard form.

In the Chinese calendar two separate cycles interact. There are 10 heavenly stems and 12 zodiac animals. You can use lowest common multiples to work out when a certain year will come round again.

Objectives

This chapter will show you how to

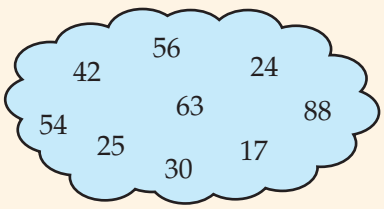
- **find multiples and factors** E
- **recognise and use prime numbers** E
- **recall the cubes of 2, 3, 4, 5 and 10** E
- **use the terms positive and negative square root** D
- **calculate common factors, highest common factors and lowest common multiples** C
- **write a number as a product of prime factors** C
- **use index laws for multiplication and division of integer powers** C B
- **write numbers in standard index form** B
- **calculate using standard index form** B

Before you start this chapter

Put your calculator away!

1 Which of the numbers in the cloud are divisible by

a 6
b 9
c 7
d 4



2 Work out

a 6^2
b $(-1)^2$
c $(-9)^2$
d 11^2

3 Work out

a 22×100
b 0.46×1000
c $22.8 \div 10$
d $17 \div 1000$

10.1 Multiples

Why learn this

Astronomers use lowest common multiples to calculate when the Sun and Moon will be aligned in an eclipse.

Objectives

E Solve problems involving multiples

C Find lowest common multiples

Keywords

multiple, lowest common multiple (LCM), common multiple

Skills check

1 Without using a calculator, work out

a 4×8 **b** 9×7

c 12×5 **d** 6×8

2 Write down the next two terms in each sequence.

a 4, 8, 12, 16, 20, ... **b** 27, 36, 45, 54, 63, ...

Multiples

When a number is multiplied by a whole number the answer is a **multiple** of the first number. The multiples of a number are the answers in its times table.

The multiples of 5 are 5, 10, 15, 20, 25, ...

All the multiples of a number are divisible by that number.

'Is divisible by ...' means 'can be divided exactly by ...'

Example 1

E

a Write down the multiples of 7 between 50 and 60.

b Is 84 a multiple of 3? Give a reason for your answer.

a $7 \times 8 = 56$

56 is the only multiple of 7 between 50 and 60.

Use trial and improvement.
$7 \times 7 = 49$
$7 \times 8 = 56$
$7 \times 9 = 63$

b Yes. $84 \div 3 = 28$

If a number is divisible by 3 then it is a multiple of 3.

Exercise 10A

E

1 Write down three multiples of 9 that are larger than 100.

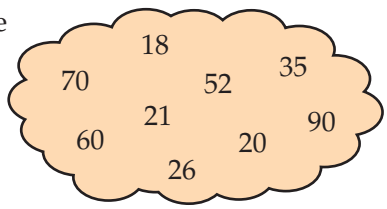
2 Write down a multiple of 12 between 80 and 90.

3 Is 412 a multiple of 3? Give a reason for your answer.

4 Write down all the numbers from the cloud which are

a multiples of 10

b multiples of 7.



E

AO2

5 Mince pies come in boxes of 8. Beth has bought some boxes of mince pies for a party. Beth counts the mince pies and says there are 130.
Tom counts them and says there are only 128.
Who is correct? Give a reason for your answer.

6 Nisha has written down a multiple of 8. She says that if she adds a zero to the end of her number it will still be a multiple of 8.
Is Nisha correct? Give a reason for your answer.

Lowest common multiples

The **lowest common multiple (LCM)** of two numbers is the smallest number that is a multiple of both numbers.

The multiples of 3 are 3, 6, 9, **12**, 15, 18, 21, **24**, ...

The multiples of 4 are 4, 8, **12**, 16, 20, **24**, ...

The **common multiples** of 3 and 4 are 12, 24, ...

So the lowest common multiple of 3 and 4 is 12.

The lowest common multiple is also known as the least common multiple.

Another way to find LCMs is covered in Section 10.5.

C

Example 2

What is the lowest common multiple of 6 and 8?

Write down the multiples of both numbers and circle the common multiples.

Multiples of 6: 6, 12, 18, (24), 30, 36, 42, (48), ...

Multiples of 8: 8, 16, (24), 32, 40, (48), ...

The common multiples are 24, 48, ...

The lowest common multiple is 24.

$6 \times 8 = 48$ is definitely a common multiple of 6 and 8 so you can stop at 48.

Exercise 10B

D

1 **a** Write down the first ten multiples of 6.
b Write down the first ten multiples of 9.
c What is the LCM of 6 and 9?

LCM stands for lowest common multiple.

2 Write down two common multiples of
a 2 and 3 **b** 4 and 5 **c** 2 and 10 **d** 9 and 3

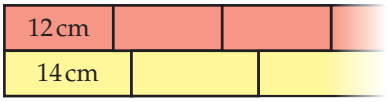
C

3 Work out the lowest common multiple of
a 5 and 6 **b** 8 and 10 **c** 2 and 5
d 10 and 15 **e** 12 and 15 **f** 20 and 30

4 Shazia says that you can find the LCM of two numbers by multiplying them together. Give an example to show that Shazia is wrong.

5 Carla and Guy have each got the same number of CDs.
Carla has arranged her CDs into 8 equal piles.
Guy has arranged his CDs into 12 equal piles.
What is the smallest number of CDs they could each have?

6 Fred is building a wall. He uses red bricks which are 12 cm long and yellow bricks which are 14 cm long. The bricks must line up at the start and end of the wall.



Work out the shortest length of wall that Fred could build.

10.2 Factors and primes

Why learn this

Banks use prime numbers to encrypt their websites and prevent fraud.

Objectives

- E Solve problems involving factors
- E Recognise two-digit prime numbers
- C Find highest common factors

Keywords

factor, prime number, prime factor, highest common factor (HCF), common factor,

Skills check

1 Copy and complete these.

a $4 \times \square = 24$ **b** $\square \times 8 = 56$

c $12 \times \square = 84$ **d** $\square \times 7 = 91$

2 Write down six different numbers that are divisible by 20.

Factors

A **factor** of a number is a whole number that divides into it exactly.
The factors of a number always include 1 and the number itself.

The factors of 6 are 1, 2, 3 and 6.

Factors come in pairs. You can use factor pairs to help you find factors.

Factor pairs of 6:
1 × 6 = 6
2 × 3 = 6

Example 3

E

Write down all the factors of 20.

$1 \times 20 = 20$
$2 \times 10 = 20$
$4 \times 5 = 20$

Use factor pairs to help you find all the factors.
You have to include 1 and 20 in your list of factors.

The factors of 20 are 1, 2, 4, 5, 10 and 20.

Exercise 10C

E

1 Write down all the factors of

a 10 **b** 18 **c** 24 **d** 30

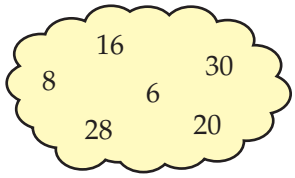
2 John says that the factors of 12 are 2, 3, 4 and 6.
Is he correct? Give a reason for your answer.

3 Choose a number from the cloud which is

a a factor of 36 **b** a multiple of 7

c a factor of 40 *and* a multiple of 10

d a factor of 24 *and* a factor of 16.



E

4 Mike says that every number has an even number of factors.
Is he correct? Give a reason for your answer.

AO2

Prime numbers

A number with exactly two factors is called a **prime number.** The factors are always 1 and the number itself.

The first few prime numbers are 2, 3, 5, 7, 11, ...

A prime number which is a factor of another number is called a **prime factor.**

1 is *not* a prime number. It only has one factor.
2 is the only even prime number.

E

Example 4

a Is 27 a prime number? Give a reason for your answer.

b Write down two prime numbers which add up to 18.

a $3 \times 9 = 27$
27 is not a prime number.

27 has four factors: 1, 3, 9, 27.

b 11 and 7

There is more than one answer to part **b**.
$11 + 7 = 18$
$13 + 5 = 18$

Exercise 10D

E

1 Write down a multiplication fact to show that each number is *not* a prime number.

a 15 **b** 21 **c** 63 **d** 121 **e** 33 **f** 91

2 Write down all the prime numbers between 30 and 50.

3 Write down two prime numbers which add up to 16.

4 Write down all the prime factors of

a 12 **b** 30 **c** 70 **d** 44

5 Show that 20 can be written as the sum of two prime numbers in two different ways.

E

AO2

Highest common factors

The **highest common factor (HCF)** of two numbers is the largest number that is a factor of both numbers.

The factors of 12 are **1**, **2**, 3, **4**, 6 and 12.

The factors of 16 are **1**, **2**, **4**, 8 and 16.

The **common factors** of 12 and 16 are 1, 2 and 4.

So the highest common factor of 12 and 16 is 4.

Another way to find HCFs is covered in Section 10.5.

Example 5

C

What is the highest common factor of 20 and 30?

Write down the factors of both numbers and circle the common factors.

Factors of 20: (1), (2), 4, (5), (10), 20

Factors of 30: (1), (2), 3, (5), 6, (10), 15, 30

The common factors are 1, 2, 5 and 10.

The highest common factor is 10.

Remember to include 1 and the number itself in your list of factors.

Exercise 10E

D

1 **a** Write down the factors of 12.

b Write down the factors of 8.

c What is the HCF of 12 and 8?

HCF stands for highest common factor.

C

2 Work out the highest common factor of

a 15 and 25 **b** 14 and 12 **c** 21 and 15

d 24 and 20 **e** 8 and 10 **f** 8 and 16

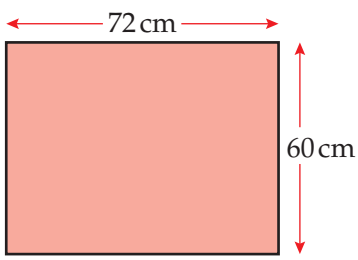
3 Write down two numbers larger than 10 with an HCF of 8.

C

4 Lydia is making Christmas decorations. She needs to cut identical squares out of a rectangular sheet of paper.

a What is the largest square size Lydia can use without wasting any paper?

b How many of these squares will Lydia be able to cut from this sheet of paper?



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AO2

10.3 Squares, cubes and roots

Keywords
square root, positive square root, negative square root, cube root

Why learn this
This topic often comes up in the exam.

Objectives
- E Calculate squares and cubes
- E Calculate square roots and cube roots
- D Understand the difference between positive and negative square roots
- C Evaluate expressions involving squares, cubes and roots

Skills check

1 Work out
a 2^2 **b** 9^2 **c** 4^2 **d** 3^2

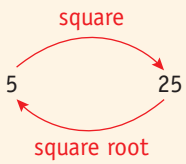
2 Work out
a -3×-3 **b** -6×-6 **c** -1×-1 **d** -12×-12

3 What is the largest two-digit square number?

Squares and square roots

To square a number you multiply it by itself.
The inverse of squaring is finding the **square root**.

Every positive number has two square roots. 5 is the **positive square root** of 25 and -5 is the **negative square root** of 25.



$5^2 = 5 \times 5 = 25$
$(-5)^2 = -5 \times -5 = 25$

The symbol $\sqrt{\ }$ is used to represent the positive square root of a number. You write $\sqrt{25} = 5$.

You can write the negative square root of 25 as $-\sqrt{25}$.

You need to know the squares of integers up to 15 and their corresponding square roots.

Cubes and cube roots

The inverse operation of cubing is finding the **cube root**. Every number has exactly one cube root. The symbol $\sqrt[3]{\ }$ is used to represent the cube root of a number.

$4^3 = 64$
$\sqrt[3]{64} = 4$

You need to know the cubes of 1, 2, 3, 4, 5 and 10 and their corresponding cube roots.

D C

Example 6

a Write down the negative square root of 81. **b** Work out $\sqrt[3]{5^2 + 2}$.

a $\sqrt{81} = 9$ so the negative square root of 81 is -9.

$(-9)^2 = 81$

b $\sqrt[3]{5^2 + 2} = \sqrt[3]{25 + 2}$
$= \sqrt[3]{27}$
$= 3$

The cube root sign is like a bracket. You have to work out the value underneath the cube root first.

Exercise 10F

1 Work out

a nine squared **b** five cubed

c the positive square root of 36 **d** the cube root of 8.

2 Work out

a 7^2 **b** 10^2 **c** 5^3 **d** 10^3

3 Work out

a $\sqrt{36}$ **b** $\sqrt{49}$ **c** $\sqrt{121}$ **d** $\sqrt{64}$

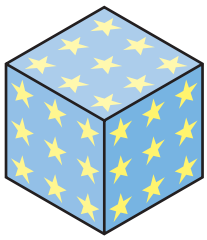
4 Work out

a $\sqrt[3]{125}$ **b** $\sqrt[3]{1}$ **c** $\sqrt[3]{1000}$ **d** $\sqrt[3]{27}$

5 Copy and complete the table.

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x^2			9							100					

6 The diagram shows a cube decorated with star-shaped stickers.
Each face of the cube has the same number of stickers.
How many stickers are there on the whole cube?



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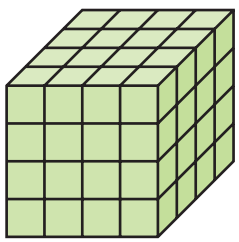
7 Andre is paving his patio. His patio is a square with side length 13 m. He uses 1 m square paving slabs. The paving slabs come in boxes of 20. Each box costs £45.

a How much will it cost Andre to buy the paving slabs for his patio?

b How many paving slabs will he have left over?

8 Carla has a bag of 100 small cubes.
She uses her cubes to make
this larger cube.
How many small cubes does she have left?



9 Show that it is possible to write 50 as the sum of two square numbers in two different ways.

10 Work out the negative square root of

a 49 **b** 25 **c** 4 **d** 9

11 Write down two possible values that would make this statement true.

$\square^2 + 9 = 45$

12 Work out

a $\sqrt{2^2 + 5}$ **b** $\sqrt{6^2 + 8^2}$ **c** $\sqrt{5^2 - 4^2}$ **d** $\sqrt{13^2 - 12^2}$

13 Work out

a $\sqrt{5^3 - 5^2}$ **b** $\sqrt{2^3 + 2^3}$ **c** $\sqrt[3]{11^2 + 2^2}$ **d** $\sqrt[3]{3^2 - 1}$

E E AO2 D D AO2 C

C

14 Estimate the answers to these calculations by rounding each value to the nearest whole number.

a $6.7^2 + 3.1^2$ **b** $5.04^3 - 3.28^3$ **c** $\sqrt{19.8 - 4.1}$ **d** $\sqrt[3]{2.1^2 + 1.7^2}$

C

AO3

15 Amber and Simon have each made a 10 cm square pattern using 1 cm square tiles. Amber gives some tiles to Simon. They are both able to arrange their tiles exactly into square patterns.
How many tiles did Amber give to Simon?

10.4 Indices

Why learn this

The formula for radioactive decay uses indices. Scientists use this formula to work out the age of fossils.

Objectives

E Understand and use index notation in calculations

B Understand and use negative powers and numbers to the power of 1 or 0

Keywords

index notation, index, indices, base, power

Skills check

1 Work out

a $3 \times 3 \times 3$ **b** $2 \times 2 \times 2 \times 2$ **c** 8×8 **d** $5 \times 5 \times 5 \times 5$

2 Work out

a $\frac{6 \times 6 \times 6}{6}$ **b** $\sqrt{4^3}$ **c** $\frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$

Positive and negative indices

You can write numbers using **index notation**.

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

This number is called the **index**. The plural of index is **indices**.

This number is called the **base**.

You write 2^6. You say 'two to the **power** of six'.

This sequence of numbers halves each time.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$2^3 = 2 \times 2 \times 2 = 8$

$2^2 = 2 \times 2 = 4$

The next power in this sequence is 2^1. You can continue the pattern by halving.

$2^1 = 2$

Any number to the power of 1 is the number itself.

$2^0 = 1$

Any number to the power of 0 is equal to 1.

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

You can write 2^{-2} as $\frac{1}{2^2}$ and 2^{-3} as $\frac{1}{2^3}$.
If the index is negative, first change it to positive.
Then write the number as the denominator of a fraction.

Example 7

Work out

a 3^4 **b** 4^{-2}

a $3^4 = 3 \times 3 \times 3 \times 3$
$= 81$

b $4^{-2} = \frac{1}{4^2}$
$= \frac{1}{16}$

Work out $3 \times 3 \times 3$ first, then multiply the answer by 3.

You can give your answer as a fraction.

Exercise 10G

1 Work out

a 3^5 **b** 4^4 **c** 5^6 **d** 10^5 **e** 6^3 **f** 7^3

2 Work out

a $2^4 \times 5$ **b** $4^3 \times 3$ **c** $2^3 \times 3^3$ **d** $5^5 \div 5$ **e** $6^3 \div 3^2$ **f** $4^3 \div 2^5$

3 Write these numbers in order of size, starting with the smallest.

$2^2 \times 3^2$ 3^3 2^4 1^8 $\sqrt{225}$

4 This is a famous riddle.

> As I was going to St Ives I met a man with seven wives. Every wife had seven sacks, and every sack had seven cats. Every cat had seven kittens. Kittens, cats, sacks and wives, how many were going to St Ives?

a What is the answer to the riddle?

b Use index notation to write down the total number of kittens.

c Work out the total number of kittens and cats.

5 Work out the value of

a 8^0 **b** 6^1 **c** 100^0 **d** 17^1 **e** 29^1 **f** 83^0

6 Write each of these as a fraction in its lowest terms.

a 4^{-1} **b** 3^{-2} **c** 7^{-1} **d** 2^{-2} **e** 2×8^{-1} **f** 3×6^{-2}

7 The formula $N = 80 \times 2^t$ is used to calculate the number of bacteria on a microscope slide after t hours.

a Use this formula to work out the number of bacteria on the microscope slide after

i 2 hours

ii 6 hours.

After 2 hours, $t = 2$.

b How many bacteria do you think were placed on the microscope slide at the beginning of the experiment? Give a reason for your answer.

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10.5 Prime factors

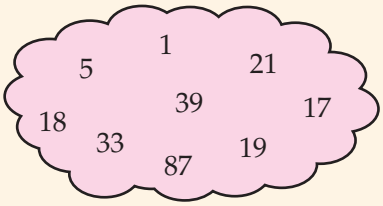
Keywords
prime factor

Why learn this
You can use prime factors to calculate lowest common multiples and highest common factors much more quickly.

Objectives
C Write a number as a product of prime factors using index notation
C Use prime factors to find HCFs and LCMs

Skills check

1 Write down all the prime numbers from the cloud.



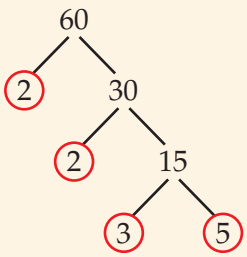
HELP Section 10.2

2 Write down a multiplication fact to show that 111 is not a prime number.

Writing a number as a product of prime factors

You can write any number as a product of **prime factors**.

You can use a factor tree to write a number as a product of prime factors.



- Split each number up into factor pairs.
- When you reach a prime number, draw a circle around it. These are the ends of the branches.
- The answer is the product of the prime numbers on the branches.

$60 = 2 \times 2 \times 3 \times 5$

You can write this using index notation as $2^2 \times 3 \times 5$.

You can also use repeated division to write a number as a product of prime factors.

2	60
2	30
3	15
5	5
	1

Divide by 2 as many times as possible.

You cannot divide 15 by 2. Try the next prime number.

Divide by each prime number as many times as possible. Stop when you reach 1.

$60 = 2^2 \times 3 \times 5$

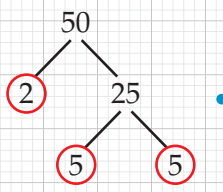
When writing a number as a product of prime factors, write the prime factors in order from smallest to largest.

Example 8

Write each number as a product of prime factors using index notation.

a 50 **b** 1960

a



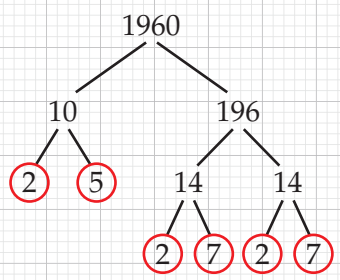
$50 = 2 \times 5 \times 5$

$= 2 \times 5^2$

You could also use repeated division.

2	50
5	25
5	5
	1

b



$1960 = 2 \times 2 \times 2 \times 5 \times 7 \times 7$

$= 2^3 \times 5 \times 7^2$

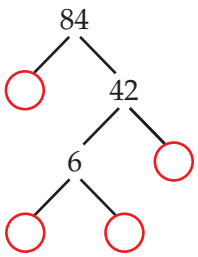
You could also use repeated division.

2	1960
2	980
2	490
5	245
7	49
7	7
	1

Exercise 10H

1 **a** Copy and complete this factor tree.

b Write 84 as a product of prime factors using index notation.



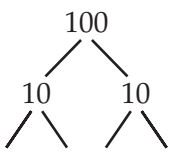
2 Write each number as a product of prime factors using index notation.

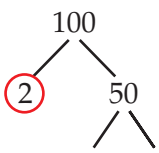
a 20 **b** 63 **c** 64 **d** 45 **e** 110 **f** 81

3 Write each number as a product of prime factors using index notation.

a 156 **b** 1980 **c** 7700 **d** 608 **e** 2025 **f** 980

4 **a** Copy and complete these two factor trees for 100.





b Jamie says that the prime factors will be different depending on which factor pair you choose first.
Do you agree with him?
Demonstrate your answer using another number.

Using prime factors to find the HCF

- Write each number as the product of prime factors.
- If a prime number is in *both* lists, circle the *lowest* power.
- Multiply these to find the HCF.

If a prime number is only in one of the lists you can't include it in your HCF.

Using prime factors to find the LCM

- Write each number as the product of prime factors.
- Circle the *highest* power of each prime number.
- Multiply these to find the LCM.

C

Example 9

Work out

a the HCF of 180 and 168 **b** the LCM of 24 and 60.

a $180 = (2^2) \times 3^2 \times 5$

$168 = 2^3 \times (3) \times 7$

The HCF of 180 and 168 is $2^2 \times 3 = 12$.

The prime numbers in both lists are 2 and 3. The lowest power of 2 is 2^2. The lowest power of 3 is 3^1.

b $24 = (2^3) \times 3$

$60 = 2^2 \times (3) \times (5)$

The LCM of 24 and 60 is $2^3 \times 3 \times 5 = 120$.

You only need to circle the highest power of 2.

3 is a factor of 24 and 60. You only need to circle it once.

Exercise 10I

C

1 **a** Write 90 as a product of prime factors.
b Write 165 as a product of prime factors.
c Find the HCF of 90 and 165.

2 **a** Write 42 as a product of prime factors.
b Write 30 as a product of prime factors.
c Find the LCM of 42 and 30.

3 Work out the highest common factor of each pair of numbers.

a 32 and 56 **b** 80 and 72 **c** 27 and 45
d 100 and 75 **e** 48 and 64 **f** 60 and 160

4 Work out the largest whole number that will divide exactly into 264 and 150.

5 Work out the lowest common multiple of each pair of numbers.

a 18 and 20 **b** 6 and 32 **c** 27 and 15
d 9 and 75 **e** 60 and 80 **f** 14 and 21

6 Work out the smallest number that is a multiple of 90 *and* a multiple of 105.

7 Work out the highest common factor of 2016 and 1512.

8 Tarik and Archie have the same amount of money.
Tarik's money is all in 20p pieces and Archie's money is all in 50p pieces.
What is the smallest amount of money that they could each have?

9 Amy is investigating the relationship between the LCM and the HCF.

a Work out the HCF of 18 and 30.

b Amy says that she can find the LCM of 18 and 30 using the rule LCM $= \frac{18 \times 30}{\text{HCF}}$.
Show working to check that Amy's rule works.

c Show that Amy's rule will also work for 16 and 40.

10 David has a pack of playing cards with some missing.
He arranges his playing cards into 15 rows of equal length. He then rearranges his playing cards into 9 rows of equal length.
How many cards are missing from David's pack?

A normal pack of playing cards contains 52 cards.

C AO2

C AO3

10.6 Laws of indices and standard form

Why learn this

You can use standard form to write very large and very small numbers. The mass of a water molecule is about 3×10^{-26} kg.

Objectives

C B Use laws of indices to multiply and divide numbers written in index notation

B Carry out calculations with numbers given in standard form

Keywords

laws of indices, standard form

Skills check

1 Write each of these using index notation.

a $2 \times 2 \times 2$ **b** $7 \times 7 \times 7 \times 7$

c $8 \times 8 \times 8$ **d** $3 \times 3 \times 3 \times 3 \times 3$

2 Work out the value of

a 2^5 **b** 3^4 **c** 10^3 **d** 2^7

Laws of indices

To *multiply* powers of the same number you *add* the indices.

$$4^2 \times 4^5 = (4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4) = 4^7$$

You can multiply in any order so the brackets don't matter.

Using the **laws of indices**:

$4^2 \times 4^5 = 4^{2+5} = 4^7$

To *divide* powers of the same number you *subtract* the indices.

$$6^5 \div 6^3 = \frac{6 \times 6 \times \cancel{6} \times \cancel{6} \times \cancel{6}}{\cancel{6} \times \cancel{6} \times \cancel{6}} = \frac{6 \times 6}{1} = 6^2$$

You can cancel 6 three times at the top and at the bottom of the fraction.

Using the laws of indices:

$6^5 \div 6^3 = 6^{5-3} = 6^2$

You can use the laws of indices to understand negative powers.

$$9^3 \div 9^7 = \frac{\cancel{9} \times \cancel{9} \times \cancel{9}}{9 \times 9 \times 9 \times 9 \times \cancel{9} \times \cancel{9} \times \cancel{9}}$$

$$= \frac{1}{9 \times 9 \times 9 \times 9}$$

$$= \frac{1}{9^4}$$

Using the laws of indices:

$9^3 \div 9^7 = 9^{3-7} = 9^{-4}$

This shows that $9^{-4} = \frac{1}{9^4}$.

You will meet laws of indices again in Section 15.2.

C B

Example 10

Simplify

a $5^8 \times 5^3$ **b** $2^3 \times 2^4 \times 2$ **c** $3^2 \div 3^6$

a $5^8 \times 5^3 = 5^{8+3}$
$= 5^{11}$

When you multiply powers of the same number you add the indices.

b $2^3 \times 2^4 \times 2 = 2^{3+4+1}$
$= 2^8$

Remember that $2 = 2^1$.

c $3^2 \div 3^6 = 3^{2-6}$
$= 3^{-4}$
$= \frac{1}{3^4}$

When you divide powers of the same number you subtract the indices.

Exercise 10J

C

1 Write each of these expressions as a single power.

a $6^7 \times 6^3$ **b** $4^7 \times 4^3$ **c** 5×5^3 **d** $9^3 \times 9^3$

2 Write each of these expressions as a single power.

a $3^7 \div 3^3$ **b** $6^{10} \div 6^5$ **c** $4^{17} \div 4^{12}$ **d** $5^3 \div 5$

3 Write each of these expressions as a single power.

a $8^4 \times 8 \times 8^6$ **b** $2^2 \times 2^7 \times 2^4$ **c** $8^3 \times 8^5 \times 8$ **d** $9 \times 9^3 \times 9$

4 Write down the value of each of these expressions.

a $5^8 \div 5^5$ **b** $4^{13} \div 4^{11}$
c $2^{10} \div 2^6$ **d** $7^6 \div 7^3$

Give your answers as a whole number.

5 Alison writes that $7^{10} \div 7^2 = 7^5$.
Is Alison correct? Give a reason for your answer.

C
AO2
B

6 Write each of these expressions as a single power of 10.

a $(10^4)^2$ **b** $(10^2)^3$

7 Write each of these expressions as a single power.

a $5^2 \div 5^9$ **b** $\frac{3^2}{3^3}$ **c** $7 \div 7^5$ **d** $\frac{7^2}{7^6}$

8 Write each of these expressions as a fraction.

a $4^3 \div 4^5$ **b** $2 \div 2^4$ **c** $\frac{3^2 \times 3}{3^5}$ **d** $\frac{16}{4^4}$

9 Write down the value of

a $6^{-2} \times 6^4$ **b** $3^{-3} \times 3^6$ **c** $\frac{5^{-1} \times 5^6}{25}$ **d** $\frac{6^6 \times 6^{-1}}{6^4}$

Powers of different numbers

You can only use the laws of indices to multiply and divide powers of the same number.

To simplify powers of different numbers you need to look at each base separately.

$(2^4 \times 3^2) \times (2 \times 3^7) = (2^4 \times 2) \times (3^2 \times 3^7)$

$= 2^5 \times 3^9$

You can multiply in any order.

B

Example 11

Work out the value of

a $(5^3 \times 3^2) \times (5^{-1} \times 3)$ **b** $\frac{7^5 \times 10}{7^3 \times 10^2}$

a $(5^3 \times 3^2) \times (5^{-1} \times 3) = (5^3 \times 5^{-1}) \times (3^2 \times 3)$

$= 5^2 \times 3^3$

$= 25 \times 27$

$= 675$

Use long multiplication.

$$\begin{array}{r} 2\ 5 \\ \times\ 2\ 7 \\ \hline 1\ {}_{3}7\ 5 \\ {}_{1}5\ 0\ 0 \\ \hline 6\ 7\ 5 \end{array}$$

b $\frac{7^5 \times 10}{7^3 \times 10^2} = \frac{7^5}{7^3} \times \frac{10}{10^2}$

$= 7^2 \times 10^{-1}$

$= 49 \times \frac{1}{10}$

$= 4.9$

Multiplying by $\frac{1}{10}$ is the same as dividing by 10.

Exercise 10K

B

1 Work out the value of each of these.
Give your answer as a decimal number where appropriate.

a $(5^2 \times 2^2) \times (2^{-1} \times 5)$

b $(4^5 \times 10^2) \times (10 \times 4^{-2})$

c $(2^5 \times 6^3) \times (2^{-7} \times 6^{-1})$

d $(5 \times 10^2) \times (5^2 \times 10^{-3})$

e $(2 \times 3^2 \times 5^2) \times (2^2 \times 3)$

f $(5^2 \times 4^2 \times 10) \times (5^{-3} \times 4^{-1} \times 10^2)$

2 Work out the value of each of these.
Give your answer as a decimal number where appropriate.

a $\frac{3^2 \times 6^6}{3^3 \times 6^4}$

b $\frac{6^2 \times 3^5}{6^4 \times 3^3}$

c $\frac{10 \times 5^9}{10^4 \times 5^6}$

d $\frac{2 \times 10^7}{2^4 \times 10^4}$

B

AO3

3 Salma says that you can write $2^{11} \times 8^2$ as a single power of 2.
Show working to explain why Salma is correct.

Standard form

Look back at Section 8.3 to refresh your memory.

Numbers in standard form have two parts.

This part is a number greater than or equal to 1 and less than 10.

6.67×10^{-11}

This part is a power of 10.

You can use standard form to write very large or very small numbers.

$360\,000 = 3.6 \times 10^5$

Numbers larger than 10 have a positive power of 10.

$0.0056 = 5.6 \times 10^{-3}$

Numbers less than 1 have a negative power of 10.

B

Example 12

Write these numbers in order of size, starting with the smallest.

6.4×10^3 1×10^5 7.4×10^{-6} 5.2×10^3 4.3×10^5

7.4×10^{-6}
5.2×10^3
6.4×10^3
1×10^5
4.3×10^5

Look at the indices first. 7.4×10^{-6} has the lowest index, so it is the smallest number.

5.2×10^3 and 6.4×10^3 have the same index. 5.2 is smaller than 6.4, so 5.2×10^3 is smaller.

Exercise 10L

1 Write each of these numbers in standard form.

a 34 000 **b** 2600 **c** 740

d 200 000 **e** 6 030 000 **f** 22.6

2 Write each of these numbers in standard form.

a 0.0006 **b** 0.0032 **c** 0.003 09

d 0.445 **e** 0.01 **f** 0.000 009 8

3 Write each of these as a decimal number.

a 2×10^4 **b** 6.2×10^3 **c** 4.54×10^{-3}

d 2.07×10^{-1} **e** 7.9×10^7 **f** 4.551×10^{-6}

4 Write these numbers in order of size, starting with the smallest.

8.04×10^3 4.8×10^4 9.31×10^{-1} 3.2×10^{-2} 8.66×10^{-1} 1×10^7

5 In the 2001 census the population of the UK was recorded as 58 789 194.
Round this number to 3 significant figures and write your answer in standard form.

6 The table shows the masses of atoms of different elements.
Write the elements in order of mass, smallest first.

Substance	Mass of atom (kg)
carbon	2.0×10^{-26}
plutonium	4.1×10^{-25}
gold	3.3×10^{-25}
iron	9.1×10^{-26}
hydrogen	1.7×10^{-27}
tungsten	3.1×10^{-25}

7 A human hair has a diameter of 0.002 54 cm.
Write this in metres using standard form.

B

B

AO2

Calculating with numbers in standard form

To multiply or divide numbers in standard form you multiply or divide each part separately. You can use the laws of indices to convert your answer back into standard form if necessary.

To add or subtract numbers in standard form you write them as decimal numbers first. Then you add or subtract. Finally, you write your answer in standard form.

B

Example 13

Work these out. Give your answers in standard form.

a $(5 \times 10^3) \times (7 \times 10^5)$

b $\dfrac{4.5 \times 10^3}{5 \times 10^7}$

c $(3.1 \times 10^2) + (6.8 \times 10^3)$

d $(9.2 \times 10^4) - (1.6 \times 10^3)$

a $(5 \times 10^3) \times (7 \times 10^5) = (5 \times 7) \times (10^3 \times 10^5)$

$= 35 \times 10^8$

$= 3.5 \times 10^9$

35×10^8 is not in standard form. You can use the laws of indices to write it in standard form.

$35 \times 10^8 = (3.5 \times 10) \times 10^8$

$= 3.5 \times 10^9$

b $\dfrac{4.5 \times 10^3}{5 \times 10^7} = \dfrac{4.5}{5} \times \dfrac{10^3}{10^7}$

$= 0.9 \times 10^{-4}$

$= 9 \times 10^{-5}$

$0.9 \times 10^{-4} = (9 \times 10^{-1}) \times 10^{-4}$

$= 9 \times 10^{-5}$.

c $(3.1 \times 10^2) + (6.8 \times 10^3) = 310 + 6800$

$= 7110$

$= 7.11 \times 10^3$

d $(9.2 \times 10^4) - (1.6 \times 10^3) = 92000 - 1600$

$= 90400$

$= 9.04 \times 10^4$

Exercise 10M

B

1 Work these out. Give your answers in standard form.

a $(2 \times 10^6) \times (4 \times 10^2)$

b $(6 \times 10^4) \times (8 \times 10^5)$

c $(2.5 \times 10^2) \times (8 \times 10^7)$

d $(9 \times 10^{-6}) \times (1 \times 10^{11})$

e $(2.5 \times 10^4) \times (2 \times 10^{-9})$

f $(3 \times 10^{-1}) \times (6.2 \times 10^{-6})$

2 Work these out. Give your answers in standard form.

a $\dfrac{8 \times 10^{12}}{4 \times 10^7}$

b $\dfrac{8.4 \times 10^5}{2 \times 10^2}$

c $\dfrac{3 \times 10^8}{6 \times 10^3}$

d $(4.2 \times 10^4) \div (2 \times 10^9)$

e $(1 \times 10^3) \div (4 \times 10^{11})$

f $(3.5 \times 10^4) \div (5 \times 10^8)$

3 Work these out. Give your answers in standard form.

a $(7 \times 10^3) + (6 \times 10^4)$

b $(1.7 \times 10^5) + (3.5 \times 10^4)$

c $(2.2 \times 10^4) - (9 \times 10^3)$

d $(8.45 \times 10^5) - (2 \times 10^4)$

e $(8 \times 10^3) + (6.9 \times 10^5)$

f $(9.4 \times 10^4) - (3 \times 10^2)$

4 A water molecule has a mass of 3×10^{-26} kg.
A glass of water contains 1.2×10^{25} molecules of water.
Work out the mass of the water in the glass. Give your answer in grams.

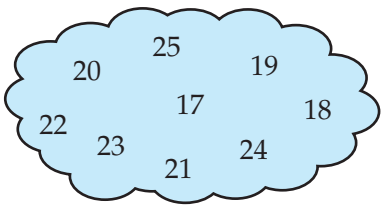
5 An adult brain has a mass of 1.4×10^{3} grams.
It contains 7×10^{10} brain neurons.
Work out the average mass of a neuron. Give your answer in grams in standard form.

B
AO2

Review exercise

E

1 Write down the numbers from the cloud which are



a multiples of 3 **[2 marks]**

b prime numbers **[2 marks]**

c factors of 100. **[2 marks]**

2 Write down

a all the factors of 48 **[2 marks]**

b all the prime factors of 48. **[1 mark]**

3 Write down a multiplication fact to show that 51 is not a prime number. **[1 mark]**

4 Write down the value of

a 5^3 **[1 mark]**

b $\sqrt{144}$ **[1 mark]**

c $\sqrt[3]{64}$ **[1 mark]**

C

5 Work out the value of $\sqrt{10^2 - 6^2}$. **[2 marks]**

6 Write 3300 as a product of prime factors using index notation. **[3 marks]**

7 Work out

a the HCF of 80 and 96 **[3 marks]**

b the LCM of 45 and 54. **[3 marks]**

8 Write each of these expressions as a single power.

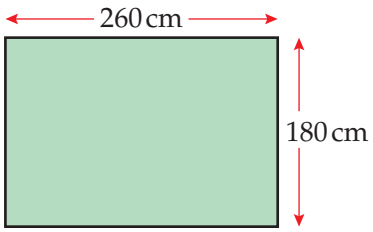
a $4^4 \times 4^2$ **[1 mark]**

b $3^8 \times 3 \times 3^5$ **[1 mark]**

c $6^{12} \div 6^5$ **[1 mark]**

C

9 Daisy is tiling her bathroom floor.



She wants to use identical square tiles to completely cover the floor with no overlap.
Work out the largest size of square tile Daisy can use. **[3 marks]**

AO2

FUNCTIONAL • FUNCTIONAL •

B

10 Work out the value of $(4^3 \times 10^{-1}) \times (4^{-4} \times 10^3)$ **[2 marks]**

11 Write each of these as a decimal number.

a 17^0 **[1 mark]**

b 2^{-1} **[1 mark]**

12 Work these out. Give your answers in standard form.

a $(6.2 \times 10^6) \times (3 \times 10^3)$ **[2 marks]**

b $(2.4 \times 10^3) \div (2 \times 10^{11})$ **[2 marks]**

B AO2

13 An ant colony contains 5×10^5 ants. The total mass of all the ants in the colony is 1.5 kg.
Work out the average mass of an ant.
Give your answer in kg in standard form. **[3 marks]**

B AO3

14 Anselm wants to take these files to a friend's house.

Filename	File size (kb)
project_ideas.doc	7.1×10^2
science1.mpg	8.34×10^5
blues_in_C.wav	9.42×10^4
me_bowling.jpg	2.41×10^3

He has a memory stick which can hold 1 000 000 kb of data.
Will all of his files fit on the memory stick? Show all of your working. **[4 marks]**

Chapter summary

In this chapter you have learned how to

- solve problems involving multiples E
- solve problems involving factors E
- recognise two-digit prime numbers E
- calculate squares and cubes E
- calculate square roots and cube roots E
- understand and use index notation in calculations E
- understand the difference between positive and negative square roots D
- find lowest common multiples C
- find highest common factors C
- evaluate expressions involving squares, cubes and roots C
- write a number as a product of prime factors using index notation C
- use prime factors to find HCFs and LCMs C
- use laws of indices to multiply and divide numbers written in index notation C B
- understand and use negative powers and numbers to the power of 1 or 0 B
- carry out calculations with numbers given in standard form B