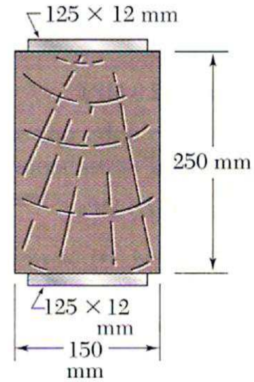
4.44 Wooden beams and steel plate are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	14 GPa	200 GPa
Allowable stress	14 MPa	150 MPa

4.44

Transform the section into wood:

$n = E_s/E_w = 200/14 = 14.2857$

Width of steel plate in transformed section $= n \times 125 = 1785.7143$ mm

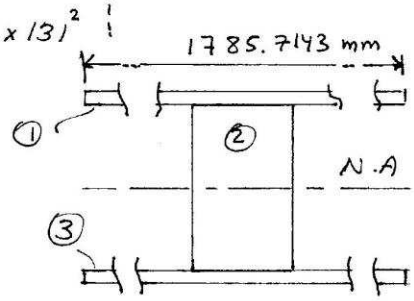
For the transformed section:

$I_1 = I_3 = (1785.7143 \times 12^3)/12 + 1785.7143 \times 12 \times 131^2$

$= 367.9929 \times 10^6\ mm^4$

$I_2 = \frac{1}{12} \times 150 \times 250^3 = 195.3125 \times 10^6\ mm^4$

$\therefore I = I_1 + I_2 + I_3 = 931.2982 \times 10^6\ mm^4$



For wood:

$|\sigma| = \left|\frac{My}{I}\right| \Rightarrow M = \frac{\sigma I}{y} = \frac{(14 \times 10^6)(931.2982 \times 10^{-6})}{0.125}$

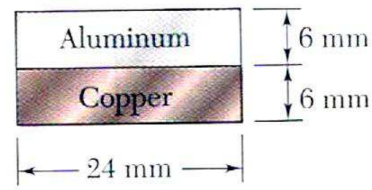
$= 104305$ N.m

For steel:

$|\sigma| = n\left|\frac{My}{I}\right| \Rightarrow M = \frac{\sigma I}{ny} = \frac{(150 \times 10^6)(931.2982 \times 10^{-6})}{(14.2857)(0.137)} = 71377$ N.m

Choose the smaller value $\therefore \boxed{M = 71.38\ kN.m}$

4.45 A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N.m, determine the maximum stress in *(a)* the aluminum strip, *(b)* the copper strip.

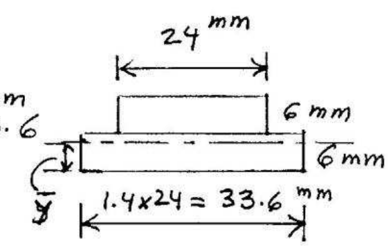


4.45

Transform the copper into aluminum

$n = 105/75 = 1.4$

Width of transformed part $= 1.4 \times 24 = 33.6^{mm}$



$$\bar{y} = \frac{24 \times 6 \times 9 + 33.6 \times 6 \times 3}{24 \times 6 + 33.6 \times 6} = 5.5 \text{ mm}$$

$$I = \frac{1}{12} 24 \times 6^3 + 24 \times 6 \times 3.5^2 + \frac{1}{12} 33.6 \times 6^3 + 33.6 \times 6 \times 2.5^2$$

$$= 2196 + 1864.8 = 4060.8 \text{ mm}^4$$

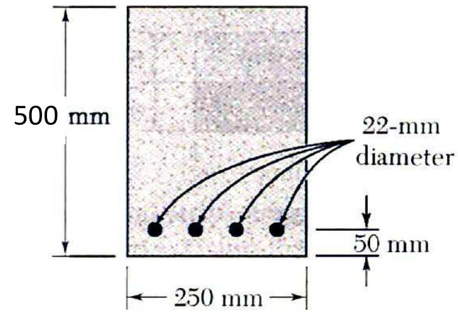
a) Max. stress in aluminum $= -\dfrac{35 \times 0.0065}{4060.8 \times 10^{-12}} \times 10^{-6} = \boxed{-56.02 \text{ MPa.}}$

b) Max. stress in copper $= -\dfrac{1.4 \times 35 \times -0.0055}{4060.8 \times 10^{-12}} \times 10^{-6} = \boxed{66.37 \text{ MPa}}$

4.49 For the composite bar of Prob. 4.45, determine the radius of curvature caused by the couple of moment 35 N.m.

4.49 $\rho = \dfrac{E_{al} I}{M} = \dfrac{(75 \times 10^{9})(4060.8 \times 10^{-12})}{35} = \boxed{8.70 \text{ m}}$

4.56 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN.m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine *(a)* the stress in the steel, *(b)* the maximum stress in the concrete.



4.56 $\quad n = E_s/E_c = 200/25 = 8$

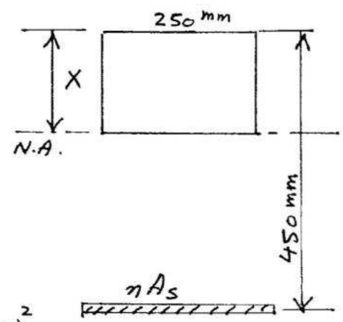
$A_s = 4\left[\frac{\pi}{4}\, 22^2\right] = 1520.53 \text{ mm}^2$

To locate the neutral axis :

$(250)(x)\left(\frac{x}{2}\right) - (8)(1520.53)(450 - x) = 0$

$\therefore\ 125\,x^2 + 12\,164.25\,x - 5.4739 \times 10^6 = 0$

$\therefore\ x = 166.19 \text{ mm} \qquad ; 450 - x = 283.81 \text{ mm}$



$I = \frac{1}{3}(250)(166.19)^3 + (8)(1520.53)(450 - 166.19)^2$

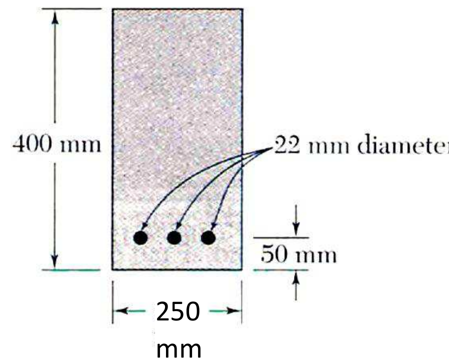
$= 1.3623 \times 10^9 \text{ mm}^4$

a) Stress in steel, $\sigma_s = -\dfrac{(8)(175 \times 10^3)(-0.28381)}{1.3623 \times 10^{-3}} \times 10^{-6} = \boxed{291.7 \text{ MPa}}$

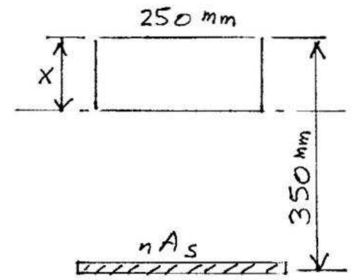
b) Max. stress in concrete, $\sigma_c = -\dfrac{(175 \times 10^3)(166.19 \times 10^{-3})}{1.3623 \times 10^{-3}} \times 10^{-6}$

$= \boxed{-21.3 \text{ MPa}}$

4.59 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9 MPa for the concrete and 140 MPa for the steel, determine the largest allowable positive bending moment in the beam.



4.59



$n = E_s / E_c = 200/20 = 10$

$A_s = 3\left[\frac{\pi}{4}(22)^2\right] = 1140.40 \ mm^2$

$nA_s = 11404.0 \ mm^2$

To locate N.A. :

$(250)(x)(\frac{x}{2}) - 11404(350 - x) = 0$

$\therefore \ 125x^2 + 11404x - 3.9914 \times 10^6 = 0$

$\therefore \ x = 138.81 \ mm \quad ; \quad 350 - x = 211.19 \ mm$

$I = \frac{1}{3}(250)(138.81)^3 + 11404(211.19)^2 = 7.3152 \times 10^8 \ mm^4$

For concrete : $|\sigma| = \left|\frac{My}{I}\right| \Rightarrow M = \frac{\sigma I}{y}$

$\therefore \ M = \frac{(9 \times 10^6)(7.3152 \times 10^{-4})}{138.81 \times 10^{-3}} \times 10^{-3} = 47.43 \ kN.m$

For steel : $|\sigma| = n\left|\frac{My}{I}\right| \Rightarrow M = \frac{\sigma I}{ny}$

$\therefore \ M = \frac{(140 \times 10^6)(7.3152 \times 10^{-4})}{(10)(211.19 \times 10^{-3})} = 48.49 \ kN.m$

Choose the smallest value $\therefore$ Max. $\boxed{M = 47.43 \ kN.m}$