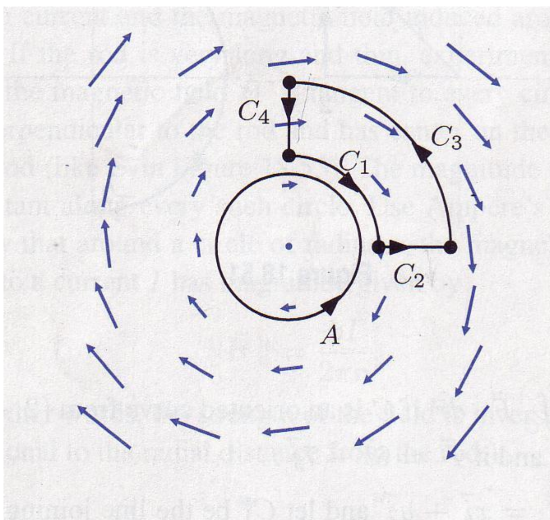
Math 283: Test 5 **Name:___________________________________**

Please show all of your work in a neat, logical fashion. Credit is based on the amount of correct work you show, not just on the final answer. If the problem requires you to use a specific technique, you must use that technique to receive credit for your answer.

1.

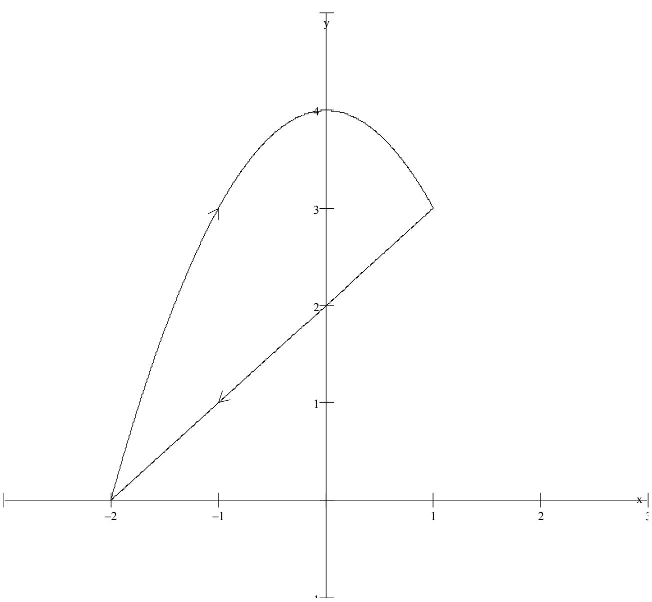


(a) Determine whether the given line integral is positive, negative or zero:

$$\int_C \vec{F} \bullet d\vec{r}\ , \text{ where } C = C_1 + C_2 + C_3 + C_4$$

(b) Is $\vec{F}$ a conservative vector field? Use your result from part (a) to explain your answer.

2. For the vector field $\vec{F} = (x+y)\hat{i} + x^2\hat{j}$ evaluate $\int_C \vec{F} \bullet d\vec{r}$ by using Green's Theorem. The oriented path C is shown in the graph below (the orientation is clockwise, if you can't see the arrows). The curved portion of the graph is $y = 4 - x^2$



3. Given the vector field $\vec{F} = -y\sin(x)\,\hat{i} + \cos(x)\,\hat{j}$

a.) Can the curl test be applied to this vector field? Why or why not? Find $\text{curl}(\vec{F})$

b.) State three facts about the vector field that you can deduce after applying the curl test.

1.

2.

3.

c) Use the Fundamental Theorem of Calculus for Line Integrals to evaluate $\int_C \vec{F} \bullet d\vec{r}$,

where C is a path that goes from (0, 1) to $\left(\frac{\pi}{2}, 5\right)$

4. Determine the value of $\vec{A}$ for each given surface S, then find the flux for the constant vector field $\vec{F} = 2\vec{i} + 5\vec{j} + \vec{k}$, through the surface.

a) S is the rectangle in the xy-plane, $0 \le x \le 3$, $-1 \le y \le 5$, oriented upward.

b) S is a disk of radius 2 in the plane $y = 5$, centered on (0, 5, 0), oriented in the positive y direction.

c) S is a square of side 1 in the plane x + 2y + 3z = 6 , oriented upward.

5. Calculate the flux of the vector field $\vec{F} = 2\vec{r}$ through the sphere of radius 3, centered on the origin. You may either use a flux integral or reason out the answer, but the reasoning must be shown and/or explained to receive credit for your answer.

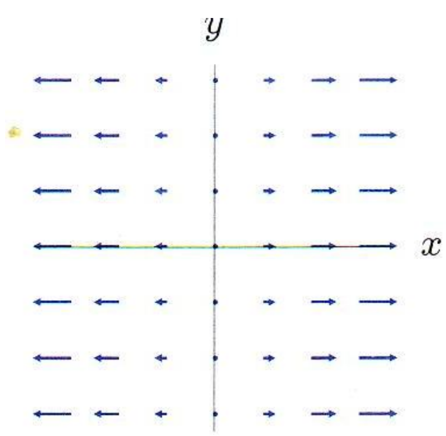
6. Given the vector field $\vec{F} = x\hat{i} + z\hat{k}$, and the surface S, where S is a plane in the first octant given by $z = -4x - 2y + 4$, $x \geq 0$, $y \geq 0$, $z \geq 0$, oriented upward, use a flux integral to find the flux of $\vec{F}$ through S**,** using a flux integral. Note that S is ONLY the plane itself.

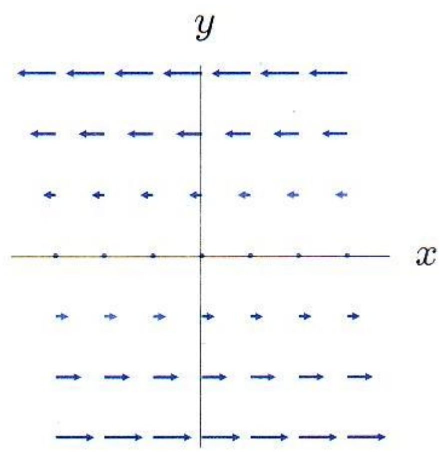
7. Given the vector field $\vec{F} = x\hat{i} + z\hat{k}$, and the closed surface S, formed by the paraboloid $z = 4 - x^2 - y^2$, and the disk $x^2 + y^2 \leq 4$, find the flux through this closed surface by **using the Divergence Theorem**. Assume S is oriented outward. (Note: once you've set up your integral involving the divergence, you are now back in Chapter 16…)

8. Each of the following graphs shows a 3D vector field whose vectors are all parallel to the xy-plane.

Determine whether div $\vec{F}(0,0,0)$ is positive, negative or zero

Divergence:________________ Divergence:________________





9. WITHOUT parameterizing the path, determine what the value of $\int_C \vec{F} \bullet d\vec{r}$ is, if C is the closed, oriented path that travels around a triangle with vertices (0,0), (5,2) and (-3,6) and $\vec{F} = y\hat{i} + x\hat{j}$.

$\int_C \vec{F} \bullet d\vec{r} =$ _____________, because____

10. For each of the following statements, state whether it is true or false.

(a) If $\vec{F} = grad(f)$ then $\vec{F}$ is path independent	True or False
(b) If S is the unit sphere, centered on the origin and oriented outward, and the flux integral $\int_S \vec{F} \bullet d\vec{A}$ is zero, then $\vec{F} = 0$	True or False
(c) The area vector $\vec{A}$ of a flat, oriented surface is perpendicular to the surface.	True or False
(d) The Divergence Theorem is used to find to find circulation.	True or False
(e) The Divergence Theorem can be applied to an open cylinder.	True or False
(f) Green's Theorem applies only to closed curves, oriented in the counterclockwise direction.	True or False