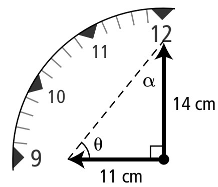
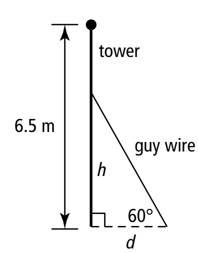
# Chapter 3 Right Triangle Trigonometry

## 3.1 The Tangent Ratio

1. **a)** $c$
   **b)** $a$
   **c)** $b$
2. **a)** $\tan A = \frac{a}{b}$ **b)** $\tan B = \frac{b}{a}$
3. **a)** 0.83
   **b)** 1.26
   **c)** 18
4. 5.31 m
5. **a)** $\tan X = \frac{\text{opposite}}{\text{adjacent}}$
   $\tan X = \frac{1.85}{1.6}$
   $\tan X = 1.15625$
   Use the inverse function on a calculator to apply the tangent ratio in reverse:
   $\tan X = 1.15625$
   $X = \tan^{-1}(1.15625)$
   $X = 49.1446\ldots°$
   $\angle X$ is 49.1°, to the nearest tenth of a degree.
   **b)** $\tan Y = \frac{\text{opposite}}{\text{adjacent}}$
   $\tan Y = \frac{1.6}{1.85}$
   $\tan Y = 0.86486$
   Use the inverse function on a calculator to apply the tangent ratio in reverse:
   $\tan Y = 0.86486$
   $Y = \tan^{-1}(0.86486)$
   $Y = 40.8552\ldots°$
   $\angle Y$ is 40.9°, to the nearest tenth of a degree.
6. $\angle A = 52°$; $\angle B = 38°$; $c = 4.06$ m; Side $c$, which is opposite the right angle, is the hypotenuse. Because the lengths of the other two sides ($a$ and $b$) are known, the length of side $c$ can be determined by substituting the values for $a$ and $b$ in the equation $a^2 + b^2 = c^2$ and solving for $c$.
7. 293 cm
8. **a)** 12.9 m
   **b)** 14.8°
9. 3.59 m
10. 200 cm, or 2 m
11. Organize the information and sketch a diagram to illustrate the problem.




   **a)** Form the tangent ratio for $\angle\alpha$ from the diagram.
   $\tan \alpha = \frac{11}{14}$
   $\alpha = \tan^{-1}\left(\frac{11}{14}\right)$
   $\alpha = 38.2°$
   The angle formed between the line and the minute hand is 38.2°.
   **b)** Form the tangent ratio for $\angle\theta$.
   $\tan \theta = \frac{14}{11}$
   $\theta = \tan^{-1}\left(\frac{14}{11}\right)$
   $\theta = 51.8°$
   The angle between the line and the hour hand is 51.8°.
12. 359 m
13. The plane travelled 13 186 m.
14. **a)** 6.1 m **b)** 7.6 m
15. 24.3°
16. **a)** Organize the information and sketch a diagram to illustrate the problem.




   First, find the value of $h$, the height in metres up the pole where the wires should be attached.
   $6.5 \text{ m} \times \frac{2}{3} = 4.33 \text{ m}$

Next, use a tangent ratio to find the distance, $d$, in metres, that each wire should extend from the base of the tower.

$\tan 60° = \frac{4.33}{d}$

$d = \frac{4.33}{\tan 60°}$

$d = 2.5$ m

Since Ramon needs 2.5 m *on each side* of the tower, the available width of 4.2 m is not enough.

**b)** Since the available width is 4.2 m, the maximum value of $d$ is $\frac{4.2 \text{ m}}{2}$, or 2.1 m.

To find the corresponding value of $h$ when $d$ equals 2.1 m, use a tangent ratio:

$\tan 60° = \frac{h}{2.1}$

$(2.1) \tan 60° = h$

$3.64 \text{ m} = h$

Since $h$ represents only two thirds of the tower's height, the maximum possible tower height is

$3.64 \times \frac{3}{2} = 5.46$ m

**17. a)** $a = 2b$ or $a = \frac{b}{2}$; $b = 2a$ or $b = \frac{a}{2}$

**b)** $a = \frac{b}{2}$ or $a = 2b$; $b = \frac{a}{2}$ or $b = 2a$

**c)** The values or ratios in parts a) and b) are the same. This is because the values are simply switched from numerator to denominator, or vice versa.

## 3.2 The Sine and Cosine Ratios

**1. a)** $\sin A = \frac{a}{c}$ **b)** $\cos A = \frac{b}{c}$

**c)** $\sin B = \frac{b}{c}$ **d)** $\cos B = \frac{a}{c}$

**2. a)** $\cos A = \frac{12}{17}$ **b)** $\sin A = \frac{10}{15}$

**c)** $\sin B = \frac{1.9}{2.4}$ **d)** $\cos B = \frac{2.6}{3.9}$

**e)** $a = \frac{75}{6} = \frac{25}{2}$ **f)** $a = \frac{9}{3} = 3$

**3. a)** $\angle R = 48.4°$ **b)** $\angle S = 41.6°$

**4. a)** 54.3° **b)** 56.7°
**c)** 12.6° **d)** 87.2°

**5. a)** $x = 1.85$ cm **b)** $x = 4.39$ m

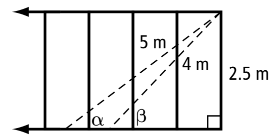
**6. a)** $z = 72.48$ mm **b)** $z = 9.70$ m

**7.** The angle is 61° and the backing piece must have a height of 155 mm.

**8. a)** 6.3 ft **b)** 54.9°

**9.** 34 m

**10.** First, label the given diagram.




Then, form appropriate ratios:

$\sin \alpha = \frac{2.5 \text{ m}}{5 \text{ m}}$ $\sin \beta = \frac{2.5 \text{ m}}{4 \text{ m}}$

Solve for the angles:

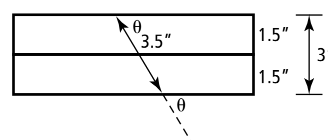
$\alpha = \sin^{-1}\left(\frac{2.5}{5}\right)$ $\beta = \sin^{-1}\left(\frac{2.5}{4}\right)$

$\alpha = 30°$ $\beta = 39°$

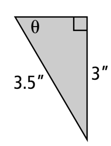
The 5-m rod forms an angle of 30° with the floor. The 4-m rod forms an angle of 39° with the floor.

**11.** The boat ramp must be 4.8 m in length.

**12.** First, modify and label the given diagram.




Then, create a simplified diagram.




Select the appropriate trigonometric ratio and solve for θ:

$\sin \theta = \frac{3}{3.5}$

$\theta = \sin^{-1}\left(\frac{3}{3.5}\right)$

$\theta = 59°$

The screws should be driven in at an angle of 59° or less to prevent the points from sticking through.

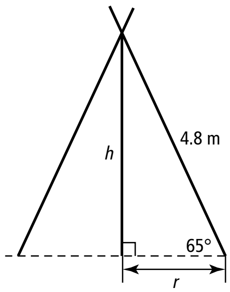
**13. a)** The water gun can spray the middle 7.2 m of the opposite side.

**b)** 2.8 m (1.4 m at each end)

**c)** 7.8 m

**14.** CD = 14.7 cm

**15.** Consider the teepee as a cone.




Use a ratio to find the radius of the base.

$\cos 65° = \frac{r}{4.8 \text{ m}}$

$(4.8 \text{ m}) \times \cos 65° = r$

$2.03 \text{ m} = r$

Since the radius is one half of the diameter, multiply the value by 2:

diameter = 2 × radius

diameter = 2 × 2.03 m

diameter = 4.06 m, or 4.1 m, to the nearest tenth of a metre

The diameter of the teepee is 4.1 m.

**16. a)**

| θ | tan θ | sin θ | cos θ |
|---|---|---|---|
| 15° | 0.2679 | 0.2588 | 0.9659 |
| 30° | 0.5774 | 0.5 | 0.8660 |
| 45° | 1 | 0.7071 | 0.7071 |
| 60° | 1.7321 | 0.8660 | 0.5 |
| 75° | 3.7321 | 0.9659 | 0.2588 |

**b)** As the values of the tangent and sine increase, the value of the cosine decreases.

**c)** The sine and cosine generate the same values, but in the opposite order.

## 3.3 Solving Right Triangles

**1. a)** ∠B = 23°; $a$ = 5.2; $c$ = 5.6

**b)** ∠B = 42°; $b$ = 12.9; $c$ = 19.2

**c)** ∠Z = 49.0°; ∠Y = 41.0°; $x$ = 2.3

**d)** Use the Pythagorean theorem to calculate the length, $x$, of side YZ:

$(YZ)^2 = (XY)^2 + (XZ)^2$

$x^2 = (1.75)^2 + (1.52)^2$

$x^2 = 3.0625 + 2.3104$

$x^2 = 5.3729$

$x = \sqrt{5.3729}$

$x$ = 2.317…, or 2.3 units, to the nearest tenth of a unit

**2. a)** ∠A, ∠C

**b)** ∠B, ∠D

**c)** ∠A = ∠B, ∠C = ∠D

**3. a)** Both are correct. The name of the angle changes when the point of view changes.

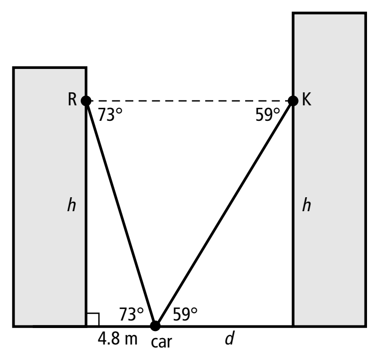
**b)** The angle of elevation from one point to a second point is equal to the angle of depression from the second point to the first point.

**4. a)** ∠C = 48°; ∠B = 52°; ∠D = 42°; $b$ = 18.3; $c$ = 23.2; $x$ = 20.3

**b)** ∠X = 27°; ∠Z = 61°; $x$ = 2.6; $y$ = 5.4; $z$ = 5.3

**5. a)** 42.1 cm **b)** 54.9°

**6.** Sketch and label a diagram with the given information to illustrate the problem.




Note that the angles of depression <u>to</u> the car equal the angles of elevation <u>from</u> the car.

Use an appropriate ratio to find $h$ first:

$\tan 73° = \frac{h}{4.8 \text{ m}}$

$(4.8 \text{ m}) \times \tan 73° = h$

$15.7 \text{ m} = h$

This is the same $h$ as for Kenneth's window.

Now we find $d$:

$\tan 59° = \frac{15.7 \text{ m}}{d}$

$d = \frac{15.7 \text{ m}}{\tan 59°}$

$d = 9.43$ m.

Add the two horizontal distances:

4.8 m + 9.43 m = 14.23 m, or 14.2 m, to the nearest tenth of a metre

The distance between the two windows is 14.2 m.

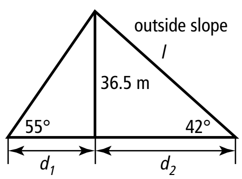
**7.** 8.6 m

8. **a)** 202 m
**b)** 714 m
**c)** 436 m

9. **a)** 98 m **b)** 50°

10. **a)** No, the diagonal length across the bottom of the box is only 118.5 cm, which is 1.5 cm less than the length of the cane.
**b)** Yes, the diagonal distance across opposite corners is 121.8 cm, which is greater than the length of the cane. Therefore, the lid can be closed.

11. Sketch a modified diagram and label it with the given information to illustrate the problem.




**a)** Width of dike $= d_1 + d_2$

Find $d_1$:

$$\tan 55° = \frac{36.5 \text{ m}}{d_1}$$
$$d_1 = \frac{36.5 \text{ m}}{\tan 55°}$$
$$d_1 = 25.56 \text{ m}$$

Find $d_2$:

$$\tan 42° = \frac{36.5 \text{ m}}{d_2}$$
$$d_2 = \frac{36.5 \text{ m}}{\tan 42°}$$
$$d_2 = 40.54 \text{ m}$$

Width = 25.56 m + 40.54 m = 66.1 m
The width of the dike is 66.1 m.

**b)** The length of the mesh is the hypotenuse of the outside triangle:

$$\sin 42° = \frac{36.5 \text{ m}}{\ell}$$
$$\ell = \frac{36.5 \text{ m}}{\sin 42°}$$
$$\ell = 54.5 \text{ m}$$

The mesh needs to be 54.5 m long.

**c)** Let the height be 39 m. Then, solve as in part a):

$$d_1 = \frac{39 \text{ m}}{\tan 55°}$$
$$d_1 = 27.31 \text{ m}$$
$$d_2 = \frac{39 \text{ m}}{\tan 42°}$$
$$d_2 = 43.31 \text{ m}$$

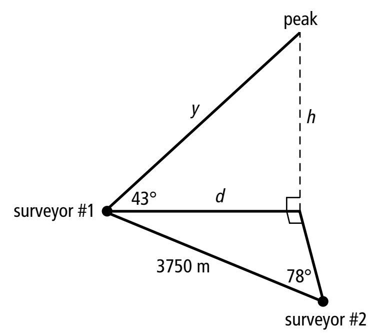
Width = 27.31 m + 43.31 m = 70.6 m
The width at the base must be 70.6 m.

**d)** Let the height equal 31 m. Then, solve as in part a):

$$d_1 = \frac{31 \text{ m}}{\tan 55°}$$
$$d_1 = 21.71 \text{ m}$$
$$d_2 = \frac{31 \text{ m}}{\tan 42°}$$
$$d_2 = 34.43 \text{ m}$$

Width = 21.71 m + 34.43 m = 56.14 m
The student-engineer is not correct. The base of the dike needs to be 56.1 m wide, not 54.8 m.

12. Sketch a simplified diagram to illustrate the problem, converting all distances to metres.




**a)** Find the distance $d$, in metres, from surveyor #1 to a point directly under the peak:

$$\sin 78° = \frac{d}{3750 \text{ m}}$$
$$(3750 \text{ m}) \times \sin 78° = d$$
$$3668 \text{ m} = d$$

Now, use this distance in the other triangle to find $h$:

$$\tan 43° = \frac{h}{3668 \text{ m}}$$
$$(3668 \text{ m}) \times \tan 43° = h$$
$$3420 \text{ m} = h$$

The mountain is 3420 m high.

**b)** Find the hypotenuse, $y$, of the second triangle:

$$\cos 43° = \frac{3668 \text{ m}}{y}$$
$$y = \frac{3668 \text{ m}}{\cos 43°}$$
$$y = 5015 \text{ m}$$

The cable needs to be 5015 m long.

**c)** Use trigonometric ratios as in part a) to determine the distance, in metres, of each of $d$ and $h$. Then, use the Pythagorean theorem to determine the distance $y$, in metres:

$y^2 = d^2 + h^2$

$y^2 = (3668)^2 + (3420)^2$

$y^2 = 13\,454\,224 + 11\,696\,400$

$y^2 = 25\,150\,624$

$y = \sqrt{25\,150\,624}$

$y = 5015.03\ldots$, or 5015 m, to the nearest metre