Homework 1

Date Assigned: 8/23/06
Date Due: 8/30/06 for conceptual, 9/6/06 for programming

Conceptual - Think about your research or work or a previous class that you have taken. Formulate a realistic optimization problem, including:

1. An objective function that should be minimized or maximized
2. Inequality and/or equality constraints that correspond to practical, manufacturing, aesthetic, etc. constraints.
3. Optimization variables - these may represent size, shape, topology, number, material parameters - think of whatever you want as long as you can formulate it mathematically based on the equations you are using for your system.
4. Simple bounds on the optimization variables.

In addition to mathematically defining the problem, the use of picture can be helpful in defining the problem and variables.

Also, try to guess what the optimal solution might look like, including which constraints you think will be active. Understanding the problem goes a long way towards using optimization tools effectively, especially in the formulation of the problem.

Programming - Program the following three line search methods in order to solve unconstrained optimization problems:

1. Equal interval method
2. Golden section method
3. Quadratic polynomial fit method

For each of these methods, solve both the following two optimization problems, which have one and two variables, respectively:

$$min_x\, f(x) = 4x^2 - \frac{1}{2}x^3 + 0.015x^4 \quad (1)$$

$$min_{\mathbf{x}}\, f(\mathbf{x}) = 3x_1^2 + 2x_1x_2 + 2x_2^2 + 7 \quad (2)$$

For both of these problems, use $\mathbf{d}_k = -\nabla f(\mathbf{x}_k)$ (the steepest descent direction).

For the one variable problem, an accurate line search should converge to the optimum solution in one iteration, whereas more than one iteration will be required for the two variable problem. However, the least number of overall iterations is not necessarily the most efficient algorithm. When running the programs, keep track of the number of times $f(\mathbf{x})$ is evaluated. This is generally what determines the overall efficiency of your algorithm.

Along with the write-up discussing you optimization formulation, make some comments on the line-search algorithms that you have implemented, especially regarding the efficiency and convergence of the algorithms.

Please email me all computer files in a zip file, along with a PDF write-up. Make sure you comment your computer files well! If you think that I will need special instructions in order to run your programs, make sure to include them in the write-up.