The University of Toledo
EECS:2300 Electric Circuits I
Dr. Anthony D. Johnson

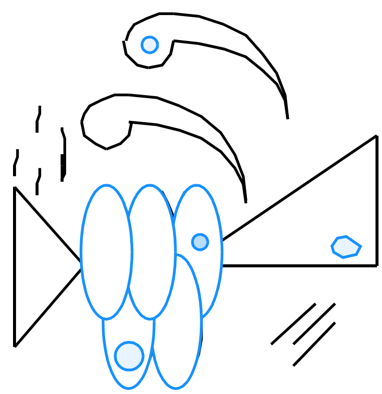
Section __________

Student Name ____

Electric Circuits I

Midterm #2

Problems	Points
1. _____	4
2. _____	5
3. _____	6
Total _____	15



Was the exam fair ?

yes ☐

no ☐

Problem 1 **4 points**

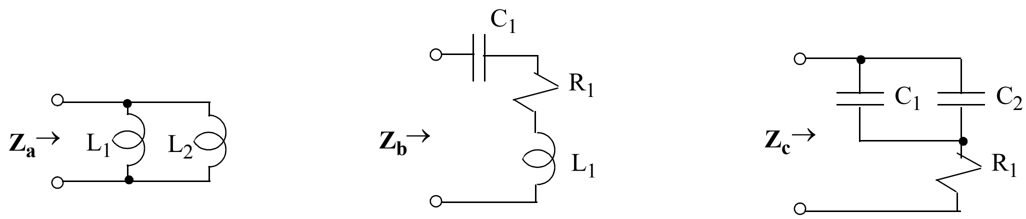
Given are three one port reactive circuits whose graphical representation is shown in Figure 1.1.

L_1=2H L_2=8H

C_1=8.2μF

C_2 = 47μF

R_1=100Ω



(a) (b) (c)

Figure 1.1 Three one-port circuits with reactive elements.

Hint #1 For full credit, give answers to all questions, prepare all required circuit diagrams, write all equations for which the space is left, and show all symbolic and numerical expressions whose evaluation produces shown numerical results.

An explicit demonstration of understanding the following solution steps is expected.

1 1.1 For the one-port circuit in Figure 1.1(a), determine the real and imaginary part representation of the impedance $\mathbf{Z_a}$ in terms of the circuit element parameters L_1 and L_2, and calculate the value of $\mathbf{Z_a}$ at the frequency of f=60Hz.

$$\mathbf{Z_a} = j\omega L_e = \frac{j\omega L_1 \cdot L_2}{L_1 + L_2} = 0 + j\frac{120\pi \cdot 2 \cdot 8}{2+8} = (0 + j603)\ \Omega$$

2 1.2 For the one-port circuit in Figure 1.1(b), determine the real and imaginary part representation of the impedance $\mathbf{Z_b}$ in terms of the circuit element parameters R_1, L_1 and C_1, and calculate the frequency f at which $\mathbf{Z_b}=R_1$.

$$\mathbf{Z_b} = R_1 + j\omega L_1 + \frac{1}{j\omega C_1} = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) = R_1 + j\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}$$

$$\mathbf{Z_b} = R_1 \text{ for } \omega^2 L_1 C_1 - 1 = 0 \quad \Rightarrow \quad f = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{2 \cdot 8.2 \cdot 10^{-6}}} = 39.3\text{Hz}$$

1 1.3 For the one-port circuit in Figure 1.1(c), determine the real and imaginary part representation of the impedance Z_c in terms of the circuit element parameters C_1, C_2, and R_1, and calculate the value of Z_c at the frequency f=120Hz.

$$\mathbf{Z_c} = R_1 + \frac{1}{j\omega C_e} = R_1 - j\frac{1}{\omega \cdot (C_1 + C_2)} = 100 - j\frac{1}{2 \cdot 120\pi \cdot (8.2 + 47) \cdot 10^{-6}} = (100 - j24)\ \Omega$$

Problem 2 **5 points**

Given is the electric circuit model with circuit element parameter values as shown in Figure 2.1(a).

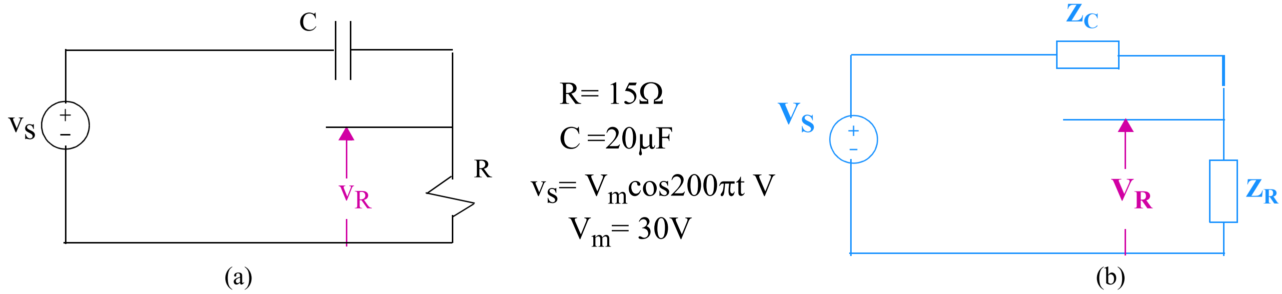


Figure 2.1 An electric circuit model.

For the time domain electric circuit model of Figure 2.1(a), demonstrate an ability to:

1. derive the corresponding phasor domain circuit model and determine its parameter values;
2. apply equivalents of resistive circuits analysis methods to the phasor domain circuit models;
3. apply specifically the voltage/current divider formula in the phasor domain analysis of electric circuits.

Problem

Prepare a partial solution of the electric circuit whose electrical model and circuit element parameter values are shown in Figure 2.1(a). For full credit, the partial solution ought to include:

(a) the phasor domain representation of the circuit as specified under 2.1 below,
(b) the phasor domain representation $\mathbf{I_R}$ of current i_R through resistor R, as specified under 2.2, 2.3 and 2.4 below,
(c) the phasor representation of the current flowing through resistor R, as specified under 2.5 below,

Hint #1 For full credit, give answers to all questions, prepare all required circuit diagrams, write all equations for which the space is reserved, and show all symbolic and numerical expressions whose evaluation produces shown numerical results.

Solution

An explicit demonstration of understanding the following solution steps is expected.

2.1 Prepare the phasor domain representation of the circuit in Figure 2.1(a), and show it in the space
1 reserved for Figure 2.1(b).

Hint#2: Denote the impedance of the resistor R **by $\mathbf{Z_R}$ and denote the impedance of the capacitor C by $\mathbf{Z_C}$.**

2.2 Applying the voltage divider formula to the circuit of Figure 2.1(b), express the real and imaginary
2 parts of the voltage $\mathbf{V_R}$ across the impedance $\mathbf{Z_R}$ in terms of circuit element parameters R and C.

Show your calculation in the space reserved for equation (2-1).

$$\mathbf{V_R} = \mathbf{V_S}\frac{\mathbf{Z_R}}{\mathbf{Z_R}+\mathbf{Z_C}} = \mathbf{V_S}\frac{\mathbf{R}}{\mathbf{R}+\frac{1}{j\omega C}} = \mathbf{V_S}\frac{j\omega CR}{1+j\omega CR} = \mathbf{V_S}\frac{j\omega CR(1-j\omega CR)}{1+(\omega CR)^2} = V_{Sm}\frac{(\omega CR)^2+j\omega CR}{1+(\omega CR)^2}$$

$$\mathrm{Re}\{\mathbf{V_R}\} = V_{Sm}\frac{(\omega CR)^2}{1+(\omega CR)^2} \qquad \mathrm{Im}\{\mathbf{V_R}\} = V_{Sm}\frac{\omega CR}{1+(\omega CR)^2} \qquad (2\text{-}1)$$

2.3 Using the derived expression (2-1), calculate the numerical values of the real and imaginary parts of the phasor domain voltage $\mathbf{V_R}$. Show your calculation in the space reserved for equation (2-2)

2

$$\omega CR = 200\pi \cdot 20 \cdot 10^{-6} \cdot 15 = 189 \cdot 10^{-3} \qquad (\omega CR)^2 = (189 \cdot 10^{-3})^2 = 0.0355$$

$$\mathrm{Re}\{\mathbf{V_R}\} = V_{Sm}\frac{(\omega CR)^2}{1+(\omega CR)^2} = \frac{30 \cdot 0.0355}{1+0.0355} = 1.06\ \mathrm{V} \qquad (2\text{-}2)$$

$$\mathrm{Im}\{\mathbf{V_R}\} = V_{Sm}\frac{\omega CR}{1+(\omega CR)^2} = \frac{30 \cdot 18.8 \cdot 10^{-3}}{1+0.0355} = 5.67\ \mathrm{V}$$

2.4 Calculate the values of the modul and argument of the phasor domain voltage $\mathbf{V_R}$ in the circuit of Figure 2.1(b). Use the space reserved for equation (2-3) to show your calculation, and the numerical values of the polar representation of the of the phasor $\mathbf{V_R}$.

2

$$\mathrm{mod}\mathbf{V_R} = |\mathbf{V_R}| = 10^{-3}\sqrt{1.06^2+5.67^2} = 5.77\ \mathrm{V}$$

$$\arg\mathbf{V_{R2}} = \mathrm{arctg}\frac{5.67}{1.06} = \mathrm{arctg}(5.35) = 79.4^{o} = 1.39\ \mathrm{rad} \qquad (2\text{-}3)$$

$$\mathbf{V_R} = 5.77\angle 79.4^{o}\ \mathrm{V}$$

2.5 Determine the expression for the phasor representation of the current through resistor R. Show your calculation in the space reserved for equation (2-4).

1

$$G_R = \frac{1}{R} = \frac{1}{15} = 0.067\ \mathrm{S}$$

$$\mathbf{I_R} = \mathbf{Y_R V_R} = \mathbf{G_R V_R} = 0.067 \cdot 5.77\angle 79.4^{o} = 0.39\angle 79.4^{o}\ \mathrm{A} \qquad (2\text{-}4)$$

Problem 3 6 points

Given is the electric circuit model whose graphical representation and circuit element parameter values are shown in Figure 3.1(a).

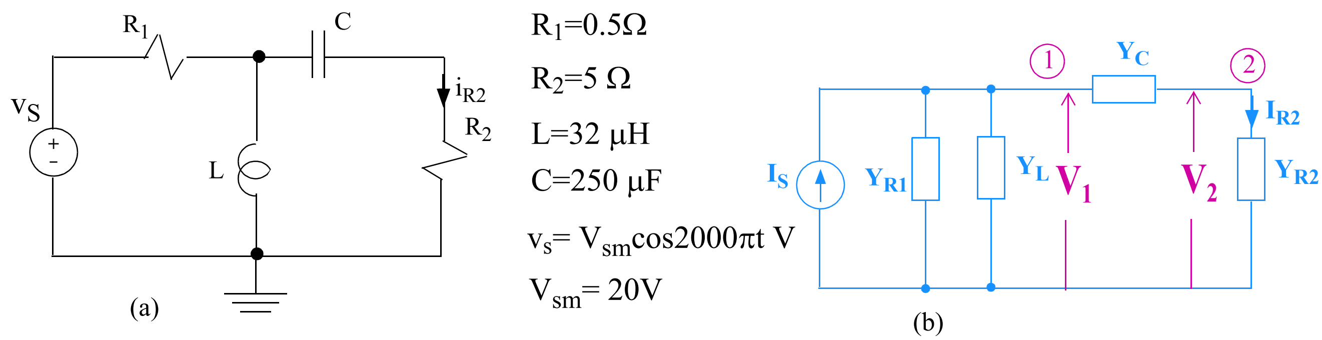


Figure 3.1 An electric circuit description. (a)Time domain representation of the circuit. (b)Phasor representation of the circuit, with mesh currents selected for minimum number of unknown variables in the system of mesh current equations.

For the time domain electric circuit model of Figure 3.1(a), demonstrate an ability to:

1. derive the corresponding phasor domain circuit model and determine its parameter values;
2. apply equivalents of resistive circuits analysis methods to the phasor domain circuit models;
3. apply specifically the nodal voltage method to the analysis of electric circuits in phasor domain.

Problem

1. prepare the phasor domain representation of the circuit as specified under 3.1 below,
2. prepare the admittances of the passive circuit elements, as specified under 3.2 below,
3. **u**se the **nodal voltage method** of analysis to determine:
   (a) phasor domain representation $\mathbf{V_L}$ of the voltage across inductor L, as specified under 3.3 through 3.5 below,
   (b) phasor domain representation $\mathbf{I_{R2}}$ of the current through resistor R_2, as specified under 3.6 below,
   (c) the complex powers $\mathbf{S_L}$ and $\mathbf{S_{R2}}$ delivered to inductor L and resistor R respectively, as specified under 3.5 and 3.6 below.

Hint #1 For full credit, give answers to all questions, prepare all required circuit diagrams, write all equations for which the space is reserved, and show all symbolic and numerical expressions whose evaluation produces shown numerical results.

An explicit demonstration of understanding the following solution steps is expected.

1 **3.1** Showing the admittances of the passive circuit elements, prepare a phasor representation of the circuit in Figure 3.1(a) and show it in the space reserved for Figure 3.1(b); also show in the circuit of Figure 3.1(b) the positive reference directions for the nodal voltages.

Hint #2 As the preparation for writing the nodal voltage equations, replace in the phasor representation of Figure 3.1(b) the series connection of the voltage source and the resistor R_1 by its Norton's equivalent circuit.

1 3.2 For the four admittances in the circuit of Figure 3.1(b), prepare the symbolic expressions of their real and imaginary parts in terms of the circuit parameters, and calculate their values. Show the work

in the space reserved for equations (3-1).

$$\mathbf{Y_{R1}} = \frac{1}{R_1} = \frac{1}{0.5} = 2\angle 0\ \text{S} \qquad \mathbf{Y_{R2}} = \frac{1}{R_2} = \frac{1}{5} = 0.2\angle 0\ \text{S}$$

(3-1)

$$\mathbf{Y_C} = j\omega C = j2000\pi \cdot 250 \cdot 10^{-6} = 0 + j1.57\ \text{S} \qquad \mathbf{Y_L} = \frac{1}{j\omega L} = \frac{-j}{2000\pi \cdot 32 \cdot 10^{-6}} = (0 - j5)\ \text{S}$$

$$\mathbf{V_S} = 20\angle 0\ \text{V} \qquad \mathbf{I_S} = \mathbf{Y_{R1}V_S} = 2\angle 0 \cdot 20\angle 0 = 40\angle 0\ \text{A}$$

1 **3.3** Prepare the the canonical form of nodal voltage equations for the circuit in Figure 3.1(b) and show it in the space reserved for equations (3-2); also show in the same space the calculation of the numerical values of the self and mutual admittances using the values of circuit element admittances calculated in Section 3.2.

$$\mathbf{Y_{11}V_1} - \mathbf{Y_{12}V_2} = \mathbf{I_S}$$

$$-\mathbf{Y_{21}V_1} + \mathbf{Y_{22}V_2} = 0$$

where,

$$\mathbf{Y_{11}} = \mathbf{Y_{R1}} + \mathbf{Y_L} + \mathbf{Y_C} = 2 - j5 + j1.57 = 2 - j3.43 = 4\angle -59.8^o\ \text{S}$$

(3-2)

$$\mathbf{Y_{22}} = \mathbf{Y_{R2}} + \mathbf{Y_C} = 0.2 + j1.57 = 1.58\angle 82.7^o\ \text{S}$$

$$\mathbf{Y_{12}} = \mathbf{Y_{21}} = \mathbf{Y_C} = j1.57 = 1.57\angle 90^o\ \text{S}$$

3.4 Solve the Equations (3-2) and show the solution process in the space reserved for equations (3-3)

1 The three determinant needed for the solution of equations (3-2) are,

$$\Delta = \begin{vmatrix} \mathbf{Y_{11}} & -\mathbf{Y_{12}} \\ -\mathbf{Y_{21}} & \mathbf{Y_{22}} \end{vmatrix} = \mathbf{Y_{11}Y_{22}} - \mathbf{Y_{12}Y_{21}} = 4\angle -60^o \cdot 1.58\angle 82.7^o - (1.57\angle 90^o)^2 =$$

$$= 6.3\angle 22.7^o - 2.46\angle 180^o = 5.81 + j2.43 + 2.46 = 8.24 + j2.43 = 8.6\angle 73.5^o\ \text{S}^2$$

$$\Delta_1 = \begin{vmatrix} \mathbf{I_S} & -\mathbf{Y_{12}} \\ 0 & \mathbf{Y_{22}} \end{vmatrix} = \mathbf{I_S Y_{22}} = 40 \cdot 1.58\angle 82.7^o = 63.2\angle 82.7^o\ \text{AS}$$

(3-3)

$$\Delta_2 = \begin{vmatrix} \mathbf{Y_{11}} & \mathbf{I_S} \\ -\mathbf{Y_{21}} & 0 \end{vmatrix} = \mathbf{I_S Y_{21}} = 40 \cdot 1.57\angle 90^o = 62.8\angle 90^o\ \text{AS}$$

which then yield the nodal voltages as,

$$\mathbf{V_1} = \frac{\Delta_1}{\Delta} = \frac{63.2\angle 82.7^o}{8.6\angle 73.5^o} = (7.34\angle 9.2^o)\ \text{V}$$

$$\mathbf{V_2} = \frac{\Delta_2}{\Delta} = \frac{62.8\angle 90^o}{8.6\angle 73.5^o} = (7.30\angle 16.5^o)\ \text{V} \qquad (3\text{-}3)$$

3.5 Determine the symbolic expression, and calculate the value of the phasor representation of
1 inductor's voltage $\mathbf{V_L}$, and the complex power $\mathbf{S_L}$ delivered to the admittance $\mathbf{Y_L}$. Show your work in the space reserved for equation (3-4)

Voltage across the inductor is equal to the nodal voltage V_1,

$$\mathbf{V_L} = \mathbf{V_1} = 7.34\angle 9.2^o\ \text{V} \qquad (3\text{-}4)$$

and the power follows as

$$\mathbf{S_L} = \frac{\mathbf{V_L V_L^* Y_L^*}}{2} = \frac{|\mathbf{V_L}|^2\mathbf{Y_L^*}}{2} = \frac{|7.34|^2\,(0+j5)}{2} = 135\angle 90^o\ \text{VAR}$$

3.6 Determine the symbolic expression, and calculate the value of the phasor representation of the
1 current $\mathbf{I_{R2}}$ through admittance $\mathbf{Y_{R2}}$, and the complex power $\mathbf{S_{R2}}$ delivered to the admittance $\mathbf{Y_{R2}}$. Show your work in the space reserved for equation (3-5). Show the positive reference direction of the current $\mathbf{I_{R2}}$ jn the circuit of Figure 3.1(b).

Current through Y_{R2} can be expressed, and calculated as,

$$\mathbf{I_{R2}} = \mathbf{V_2 Y_{R2}} = 7.3\angle 16.5^o \cdot 0.2\angle 0 = 1.46\angle 16.5^o\ \text{A} \qquad (3\text{-}5)$$

and the power follows as

$$\mathbf{S_{R2}} = \frac{\mathbf{V_{R2} I_{R2}^*}}{2} = \frac{\mathbf{7.3}\angle 16.5^o\ \ 1.46\angle -16.5^o}{2} = (5.4+j0) = 5.4\ \text{W}$$