

Prime Factoring

When a number written as the product of prime factors, it is called the prime factorization of a number.

$$48 = 3 * 2 * 2 * 2 * 2 = 3 * 2^4$$

To make finding the prime factors easy, you need to be a master of “**The Factor Facts**” and **Divisibility Rules** for 2, 3, 5, 7 and 11. This workbook will review those skills before showing you the strategies for finding prime factors.

There are 2 attack strategies for quickly and easily finding the prime factors of a given number.

1. Easy Primes
2. Ladder

Factor Facts

There are 32 numbers that are supposed to be quick and easy to recognize the factors of. Math6.org calls these numbers, “Factor Facts” and offers matching exercises and drills to help you learn them quickly and easily.

12	18	25	32	42	50	63	80
14	20	27	35	45	54	64	81
15	21	28	36	48	56	70	90
16	24	30	40	49	60	72	100

Easy Factors

Often called the tree method, easy primes involves learning to think of an easy factor and write it as the product of its prime factors. Students will think 6 and write $2 * 3$. So when they are thinking $6 * 8$, they write $3 * 2 * 2 * 2 * 2$. This process isn’t difficult, but it does take practice to make it so easy that it’s second nature.

$$4 = 2 * 2$$

$$6 = 3 * 2$$

$$8 = 2 * 2 * 2$$

$$9 = 3 * 3$$

$$10 = 5 * 2$$

Take a look at the “Factor Facts” and notice that **all of them** are made out factors that can be easily primed.

$$28 \text{ is } 7 * 4 = 7 * 2 * 2; \quad 56 \text{ is } 7 * 8 = 7 * 2 * 2 * 2; \quad 90 \text{ is } 9 * 10 = 3 * 3 * 5 * 2$$

Ladders

When you don’t know the “easy factors” of a number, you use the divisibility rules for 2, 3, 5, 7, and 11 to begin breaking a number down. Using the divisibility rules and short division, you can quickly find all of the prime factors of any number.

$$\begin{array}{r} 17 \\ 3 \overline{) 51} \\ 2 \overline{) 102} \\ 2 \overline{) 204} \end{array}$$

$$204 = 17 * 3 * 2 * 2$$

The “Factor Facts”

Lesson Box

The following numbers are numbers that you should be able to look at and instantly **know** the factors of.

12	25	42	63
14	27	45	64
15	28	48	70
16	30	49	72
18	32	50	80
20	35	54	81
21	36	56	90
24	40	60	100

If **any** of these multiplication facts are still giving you pause and stumbles, please make flash cards, study “Factor Facts” @ Math6.org, or copy them 10 times each until you are able to instantly declare the factors for these numbers.

Models

<u>4 • 3</u> 12		<u>9 • 6</u> 54
<u>7 • 2</u> 14	<u>6 • 5</u> 30	<u>8 • 7</u> 56
<u>5 • 3</u> 15	<u>8 • 4</u> 32	<u>10 • 6</u> 60
<u>4 • 4</u> 16	<u>7 • 5</u> 35	<u>9 • 7</u> 63
<u>9 • 2</u> 18	<u>6 • 6</u> 36	<u>8 • 8</u> 64
<u>5 • 4</u> 20	<u>8 • 5</u> 40	<u>10 • 7</u> 70
<u>7 • 3</u> 21	<u>7 • 6</u> 42	<u>9 • 8</u> 72
<u>6 • 4</u> 24	<u>9 • 5</u> 45	<u>10 • 8</u> 80
<u>5 • 5</u> 25	<u>8 • 6</u> 48	<u>9 • 9</u> 81
<u>9 • 3</u> 27	<u>9 • 5</u> 49	<u>10 • 9</u> 90
<u>7 • 4</u> 28	<u>10 • 5</u> 50	<u>10 • 10</u> 100

I want you to be **much faster** and make this skill **even easier**. I don’t think you need to use the multiplication sign between the factors the way that I did. As you complete the drill on this page and the drills on the next, you may drop the multiplication sign **and** remember that multiplication is commutative – **order** doesn’t matter! $120 = 12 \cdot 10$; you may write 12 10 on your paper and leave the multiplication sign out.

_____ 42	_____ 40	_____ 18	_____ 70
_____ 60	_____ 24	_____ 28	_____ 35
_____ 54	_____ 64	_____ 100	_____ 42
_____ 56	_____ 20	_____ 63	_____ 64
_____ 30	_____ 25	_____ 50	_____ 48
_____ 21	_____ 45	_____ 12	_____ 15
_____ 70	_____ 35	_____ 16	_____ 60
_____ 27	_____ 80	_____ 72	_____ 72
_____ 49	_____ 81	_____ 14	_____ 56
_____ 48	_____ 90	_____ 36	_____ 28
_____ 32	_____ 15	_____ 90	_____ 81

[use **Key 1** to check your answers]

Factor Facts Drills

Use one drill each day as a warm-up before completing the assignments for the next 3 sections of the workbook.

[Key 3]	[Key 4]	[Key 5]	[Key 6]
_____ 70	_____ 64	_____ 90	_____ 14
_____ 72	_____ 70	_____ 60	_____ 32
_____ 48	_____ 72	_____ 50	_____ 25
_____ 45	_____ 12	_____ 54	_____ 56
_____ 25	_____ 40	_____ 49	_____ 90
_____ 14	_____ 15	_____ 20	_____ 21
_____ 54	_____ 45	_____ 12	_____ 45
_____ 15	_____ 48	_____ 80	_____ 16
_____ 27	_____ 42	_____ 16	_____ 20
_____ 60	_____ 24	_____ 63	_____ 28
_____ 81	_____ 56	_____ 70	_____ 24
_____ 100	_____ 60	_____ 40	_____ 60
_____ 64	_____ 32	_____ 35	_____ 54
_____ 40	_____ 18	_____ 15	_____ 72
_____ 24	_____ 30	_____ 72	_____ 48
_____ 56	_____ 49	_____ 27	_____ 70
_____ 16	_____ 25	_____ 28	_____ 35
_____ 32	_____ 27	_____ 36	_____ 18
_____ 20	_____ 80	_____ 32	_____ 40
_____ 36	_____ 90	_____ 14	_____ 36
_____ 63	_____ 14	_____ 45	_____ 64
_____ 42	_____ 28	_____ 48	_____ 42
_____ 12	_____ 16	_____ 24	_____ 15
_____ 18	_____ 50	_____ 18	_____ 63
_____ 49	_____ 20	_____ 64	_____ 81

The “Easy Primes”

Lesson Box

We need to learn to think of a few numbers as the product of their primes. The easy primes are 4, 6, 8, 9, and 10!

$$4 = 2 \cdot 2$$

$$6 = 3 \cdot 2$$

$$8 = 2 \cdot 2 \cdot 2$$

$$9 = 3 \cdot 3$$

$$10 = 5 \cdot 2$$

When you get good at thinking “10” and writing $5 \cdot 2$, the rest of the prime factoring skill will be much easier. Use the drills on this page and/or practice “[Easy Primes](http://Math6.org)” @ Math6.org!

Models

$$\underline{\quad 2 \quad} = 2$$

$$\underline{\quad 3 \quad} = 3$$

$$\underline{\quad 2 \cdot 2 \quad} = 4$$

$$\underline{\quad 5 \quad} = 5$$

$$\underline{\quad 3 \cdot 2 \quad} = 6$$

$$\underline{\quad 7 \quad} = 7$$

$$\underline{\quad 2 \cdot 2 \cdot 2 \quad} = 8$$

$$\underline{\quad 3 \cdot 3 \quad} = 9$$

$$\underline{\quad 5 \cdot 2 \quad} = 10$$

I want you to be **much faster** and make this skill **even easier**. I don’t think you need to use the multiplication sign between the factors the way that I did. As you complete the drill on this page, you may drop the multiplication sign **and** remember that multiplication is commutative – **order** doesn’t matter! $6 = 3 \cdot 2$; you may write 3 2 on your paper and leave the multiplication sign out.

<u> </u> = 5	<u> </u> = 10	<u> </u> = 2	<u> </u> = 3
<u> </u> = 8	<u> </u> = 4	<u> </u> = 4	<u> </u> = 9
<u> </u> = 10	<u> </u> = 3	<u> </u> = 9	<u> </u> = 4
<u> </u> = 4	<u> </u> = 2	<u> </u> = 3	<u> </u> = 5
<u> </u> = 9	<u> </u> = 8	<u> </u> = 7	<u> </u> = 6
<u> </u> = 2	<u> </u> = 6	<u> </u> = 6	<u> </u> = 8
<u> </u> = 3	<u> </u> = 5	<u> </u> = 5	<u> </u> = 4
<u> </u> = 7	<u> </u> = 10	<u> </u> = 2	<u> </u> = 2
<u> </u> = 6	<u> </u> = 5	<u> </u> = 8	<u> </u> = 7
<u> </u> = 7	<u> </u> = 8	<u> </u> = 10	<u> </u> = 10
<u> </u> = 9	<u> </u> = 6	<u> </u> = 7	<u> </u> = 9

[use **Key 7** to check your answers]

The “Easy Factors”

Lesson Box

Now that we know the “Easy Primes”, we put it together with the “Factor Facts” and easily factor these numbers! When you see a “Factor Fact” you will know the prime factorization!

$$40 = 8 \cdot 5 = 2 \cdot 2 \cdot 2 \cdot 5$$

$$42 = 7 \cdot 6 = 7 \cdot 3 \cdot 2$$

$$63 = 9 \cdot 7 = 3 \cdot 3 \cdot 7$$

$$72 = 9 \cdot 8 = 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$$

Think of the factors and write those factors as easy primes. You will quickly and easily be able to factor most of the fractions that you will soon face! Use the drills on this page and/or practice “Easy Factors” @ Math6.org!

Models

$$\underline{\quad 7 \cdot 5 \quad} = 35$$

$$\underline{\quad 3 \cdot 3 \cdot 7 \quad} = 63$$

$$\underline{\quad 5 \cdot 3 \quad} = 15$$

$$\underline{\quad 5 \cdot 2 \cdot 3 \cdot 3 \quad} = 90$$

$$\underline{\quad 2 \cdot 2 \cdot 2 \cdot 2 \quad} = 16$$

$$\underline{\quad 7 \cdot 2 \quad} = 14$$

$$\underline{\quad 3 \cdot 2 \cdot 2 \cdot 2 \quad} = 24$$

I want you to be **much faster** and make this skill **even easier**. I don’t think you need to use the multiplication sign between the factors the way that I did. As you complete the drills on this page and the next, you may drop the multiplication sign **and** remember that multiplication is commutative – **order** doesn’t matter! $6 = 3 \cdot 2$; you may write 3 2 on your paper and leave the multiplication sign out.

$$\underline{\quad \quad} = 30$$

$$\underline{\quad \quad} = 36$$

$$\underline{\quad \quad} = 18$$

$$\underline{\quad \quad} = 56$$

$$\underline{\quad \quad} = 21$$

$$\underline{\quad \quad} = 35$$

$$\underline{\quad \quad} = 48$$

$$\underline{\quad \quad} = 25$$

$$\underline{\quad \quad} = 40$$

$$\underline{\quad \quad} = 24$$

$$\underline{\quad \quad} = 63$$

$$\underline{\quad \quad} = 42$$

$$\underline{\quad \quad} = 16$$

$$\underline{\quad \quad} = 81$$

$$\underline{\quad \quad} = 54$$

$$\underline{\quad \quad} = 45$$

$$\underline{\quad \quad} = 70$$

$$\underline{\quad \quad} = 80$$

$$\underline{\quad \quad} = 15$$

$$\underline{\quad \quad} = 32$$

$$\underline{\quad \quad} = 12$$

$$\underline{\quad \quad} = 14$$

$$\underline{\quad \quad} = 50$$

$$\underline{\quad \quad} = 64$$

$$\underline{\quad \quad} = 60$$

$$\underline{\quad \quad} = 90$$

$$\underline{\quad \quad} = 100$$

$$\underline{\quad \quad} = 72$$

$$\underline{\quad \quad} = 20$$

$$\underline{\quad \quad} = 49$$

$$\underline{\quad \quad} = 28$$

$$\underline{\quad \quad} = 27$$

$$\underline{\quad \quad} = 24$$

[use **Key 8** to check your answers]

Easy Factor Drills

Use one drill each day as a warm-up before completing the assignments for the next 3 sections of the workbook.

[Key 9]

_____ = 15
_____ = 35
_____ = 54
_____ = 16
_____ = 36
_____ = 49
_____ = 12
_____ = 24
_____ = 81
_____ = 32
_____ = 72
_____ = 45
_____ = 100
_____ = 27
_____ = 20
_____ = 60
_____ = 42
_____ = 30
_____ = 28
_____ = 70
_____ = 14
_____ = 50
_____ = 48
_____ = 56
_____ = 21

[Key 10]

_____ = 40
_____ = 80
_____ = 24
_____ = 14
_____ = 81
_____ = 12
_____ = 30
_____ = 28
_____ = 42
_____ = 60
_____ = 27
_____ = 64
_____ = 100
_____ = 25
_____ = 15
_____ = 21
_____ = 90
_____ = 50
_____ = 16
_____ = 20
_____ = 70
_____ = 72
_____ = 45
_____ = 36
_____ = 48

[Key 11]

_____ = 100
_____ = 25
_____ = 35
_____ = 36
_____ = 81
_____ = 80
_____ = 21
_____ = 56
_____ = 70
_____ = 64
_____ = 24
_____ = 27
_____ = 30
_____ = 50
_____ = 16
_____ = 60
_____ = 28
_____ = 49
_____ = 14
_____ = 90
_____ = 15
_____ = 45
_____ = 20
_____ = 12
_____ = 42

Factor Ladders

Lesson Box

When a the prime factorization of a number is not easily known using “Factor Facts”, you apply the divisibility rules for 2, 3, 5, 7, and 11 to find a divisor. Use short division to build a “Factor Ladder”.

1. Check for divisibility by 2, 3, 5, 7, and/or 11.
2. Use short division to find the quotient.
3. Repeat steps 1 and 2 on the quotient.

Continue until the quotient is a prime number.

Models

$$\begin{array}{r} 113 \\ 2 \overline{) 226} \end{array}$$

$$226 = 113 \cdot 2$$

$$\begin{array}{r} 53 \\ 3 \overline{) 159} \end{array}$$

$$159 = 53 \cdot 3$$

$$\begin{array}{r} 23 \\ 2 \overline{) 46} \\ 5 \overline{) 230} \end{array}$$

$$230 = 23 \cdot 5 \cdot 2$$

$$\begin{array}{r} 23 \\ 7 \overline{) 161} \\ 2 \overline{) 322} \end{array}$$

$$322 = 23 \cdot 7 \cdot 2$$

Use factor ladders to find the prime factorization of each of these numbers. Write the prime factorization on the line and check your work. Remember that multiplication is commutative so the **order** of your answers will not matter.

_____ = 129

_____ = 143

_____ = 147

_____ = 86

_____ = 58

_____ = 164

_____ = 102

_____ = 69

_____ = 144

_____ = 148

_____ = 26

_____ = 170

_____ = 74

_____ = 133

_____ = 96

_____ = 171

_____ = 51

_____ = 156

_____ = 126

_____ = 152

_____ = 180

_____ = 159

_____ = 177

_____ = 93

_____ = 127

_____ = 84

_____ = 118

_____ = 166

_____ = 122

_____ = 56

_____ = 34

_____ = 132

_____ = 38

_____ = 168

_____ = 172

_____ = 46

[use **Key 12** to check your answers]

Find the Prime Factorization of each of the numbers below. You should use exponents to make writing and checking your work easier.

Drill A

_____ = 24

_____ = 25

_____ = 32

_____ = 40

_____ = 42

_____ = 51

_____ = 78

_____ = 80

_____ = 81

_____ = 82

_____ = 84

_____ = 90

_____ = 91

_____ = 94

_____ = 210

_____ = 500

Drill B

_____ = 17

_____ = 18

_____ = 20

_____ = 21

_____ = 44

_____ = 49

_____ = 50

_____ = 58

_____ = 63

_____ = 72

_____ = 87

_____ = 95

_____ = 98

_____ = 220

_____ = 225

_____ = 400

Drill C

_____ = 8

_____ = 12

_____ = 16

_____ = 22

_____ = 30

_____ = 34

_____ = 45

_____ = 48

_____ = 64

_____ = 66

_____ = 76

_____ = 88

_____ = 100

_____ = 102

_____ = 350

_____ = 474