Influence of second-kind collisions on the electron distribution function in a He-Xe dc discharge

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The mixture of He+(2%)Xe is a mercury-free alternative for fluorescence lamps working at currents from 40 to 100 mA. A good knowledge of the electron distribution function and macroscopic properties is required to describe the plasma surrounding the cathode spot and to optimize the electrodes of the lamp. The electron velocity distribution function (evdf) in a helium-xenon low-pressure discharge is considered based on the numerical solution of the Boltzmann kinetic equation. Different models for the collision operators are analyzed, with particular attention paid to the influence of excited states and second-kind collisions on the evdf formation. The second-kind collisions were found to be important at small electric field and large densities of the excited states. The macroscopic properties are less sensitive to details of the models.

1. Introduction

Knowledge of the electron velocity distribution function (evdf) and the relevant macroscopic properties is required in many scientific and technological applications. The evdf is sensitive to approximations employed to describe the collision operators and solve the Boltzmann kinetic equation. Various approximations and resulting differences in the distribution functions and rate coefficients have been analysed in [1]. It was found, that the rate and transport coefficients calculated using the evdfs obtained by different approximations differ less pronounced than the evdfs themselves and can generally be used in discharge modelling. However, the significance of the errors depends on initial assumptions and regions of the distribution function responsible for particular macroscopic value, with an overall effect by a factor of 2.

Our specific interest in a He+(2%)Xe discharge was stipulated by broad studies of the low-pressure lamps operating in a spot mode at currents up to 100 mA. A formation of the discharge regions with very small electron mean energies has been observed. The direct ionization rate which was taken from the look-up tables as a function of the electron mean energy was too small to play a role in production of electrons. However, because very large excited atom densities were measured (10^{12}-10^{13}cm^{-3}) a corresponding pronounced influence on the evdf and plasma formation can be expected. The investigation of the second-kind collision influence becomes thus the objective of the present study.

2. The basic equations

The evdf $F(\mathbf{v},\mathbf{r})$ can be determined by solving the stationary Boltzmann equation

$$\mathbf{v}\cdot\nabla_r F(\mathbf{v},\mathbf{r}) - \frac{e\mathbf{E}}{m}\nabla_r F(\mathbf{v},\mathbf{r}) = C(F(\mathbf{v},\mathbf{r})) \qquad (1)$$

where m and $-e$ denote the electron mass and charge and $C(F)$ is the collision integral. Assuming a spatially homogeneous plasma with the electric field $\mathbf{E} = E_z\mathbf{e}_z$ and using the well-known two-term approximation for the evdf

$F(\mathbf{v}) = (2\pi)^{-1}(2/m)^{-3/2}(f_0(U)+f_1(U)v_z/v)$, $U=mv^2/2$,

we obtain the equation

$$\frac{(eE)^2}{3}\frac{d}{dU}\left[\frac{U}{N^{(He)}Q_\Sigma^{(He)} + N^{(Xe)}Q_\Sigma^{(Xe)}}\frac{df_0}{dU}\right] + 2\frac{m}{M}\frac{d}{dU}\left[U^2 N^{(He)}Q^{(He)}\left(f_0 + kT_g\frac{df_0}{dU}\right)\right] = S_I^{(He)} + S_I^{(Xe)} + S_{II}^{(He)} + S_{II}^{(Xe)} + S_{ion}^{(He)} + S_{ion}^{(Xe)}. \qquad (2)$$

Here, $N^{(He)}$ and $N^{(Xe)}$ are the helium and xenon particle densities, M is the mass of helium atom, kT_g is the gas temperature (T_g=273 K). In the term representing the energy transfer in elastic collisions only helium, the most important species has been included. The cross sections for elastic $Q^{(He,Xe)}$, inelastic $Q_{0j}^{(He,Xe)}$, and ionizing collisions $Q_{ion}^{(He,Xe)}$ are taken from [2] for He and Xe, correspondingly. Additionally, the total collision cross section is $Q_\Sigma = Q + \sum_j Q_{0j} + Q_{ion}$.

The first-kind collision operators $S_I^{(He)}$ and $S_I^{(Xe)}$ (excitation of normal atoms $A + e \rightarrow A_j + e$) are

$$S_I = NU\sum_j Q_{0j}(U)f_0(U) - N\sum_j (U+U_j)Q_{0j}(U+U_j)f_0(U+U_j).$$

where U_j denotes the corresponding excitation energy. The second-kind collision operators $S_{II}^{(He)}$ and $S_{II}^{(Xe)}$ (de-excitation $A_j + e \to A + e$ of excited atoms with density N_j) read

$$S_{II} = f_0(U)\sum_j N_j (g_0/g_j)(U+U_j)Q_{0j}(U+U_j)$$
$$-U\sum_j N_j (g_0/g_j)Q_{0j}(U) f_0(U-U_j).$$

where g_0 and g_j denote the statistical weights. The ionization operators $S_{ion}^{(He)}$ and $S_{ion}^{(Xe)}$ are represented according to

$$S_{ion} = NUQ_{ion} f_0(U)$$
$$-(U+U_{ion})NQ_{ion}(U+U_{ion}) f_0(U+U_{ion})$$

as particle conserving processes.

3. Results and discussions

The calculations have been performed for the (98%)He+(2%)Xe discharge at a pressure of 5 torr. The obtained evdfs are shown in Figs. 1 and 2.

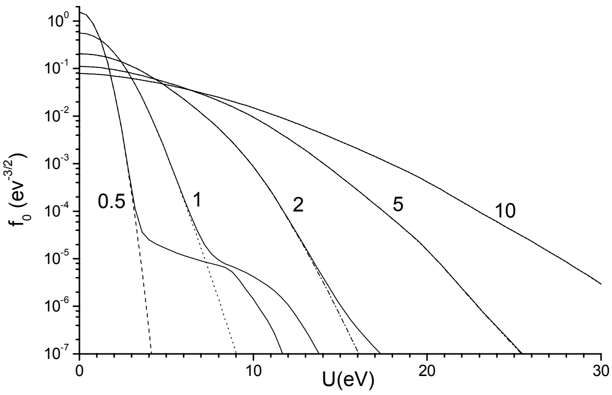

Figure 1. The evdf in dependence on electric field and the metastable atom densities. The numbers at the curves are the values of the reduced electric field *E/p* in V cm^{-1} torr^{-1}. The exited Xe atom density is $N_m^{(Xe)} = 0$ (dashed curves) and $N_m^{(Xe)} = 10^{11}$ cm^{-3} (solid curves), respectively

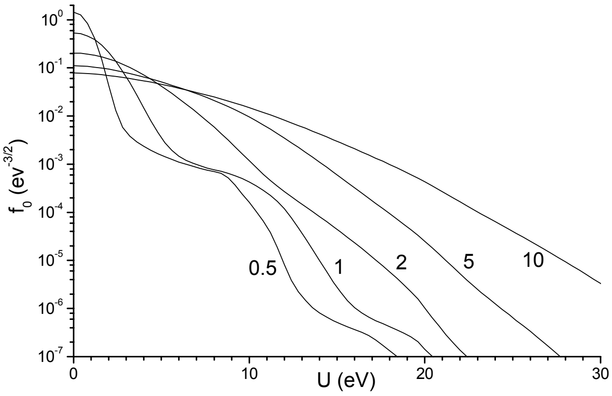

Figure 2. The evdf in dependence on electric field and the metastable atom densities. The numbers are the values of the reduced electric field *E/p* in V cm^{-1} torr^{-1}. The exited Xe atom density is $N_m^{(Xe)} = 10^{13}$ cm^{-3}.

The densities of the excited levels were assumed to have the Boltzmann distribution

$$N_j = \frac{g_j}{g_1} N_1 \exp(-(U_j - U_1)/kT_e),$$

where kT_e is the electron temperature determined from the mean energy U_e and U_1 is the first excitation energy, The density of excited He atoms is chosen $N_m^{(He)} = (g_0/g_1)N_1^{(He)} = 10^{10}$ cm^{-3} whereas that of excited Xe atoms $N_m^{(Xe)} = (g_0/g_1)N_1^{(Xe)}$ is a parameter of the calculations.

Figs. 1 and 2 demonstrate that the evdfs become more and more populated at higher energies with increasing excited atom density. A staircase electron distribution is formed with a width of the step approximately equal to the first excitation potential of xenon.

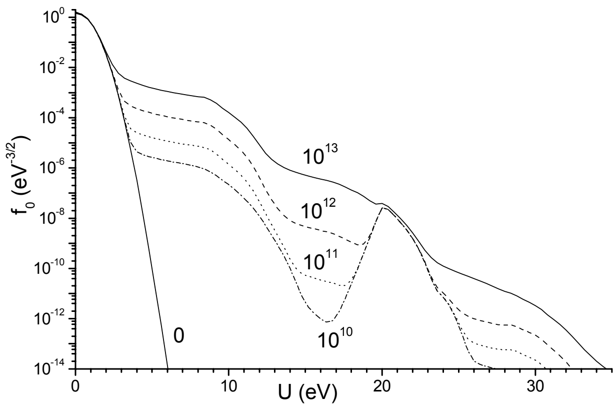

Figure 3. The evdf in dependence on the excited atom densities at $E/p = 0.5$ V cm^{-1} torr^{-1}. The curve numbers are the xenon excited atom density $N_m^{(Xe)}$ in cm^{-3}.

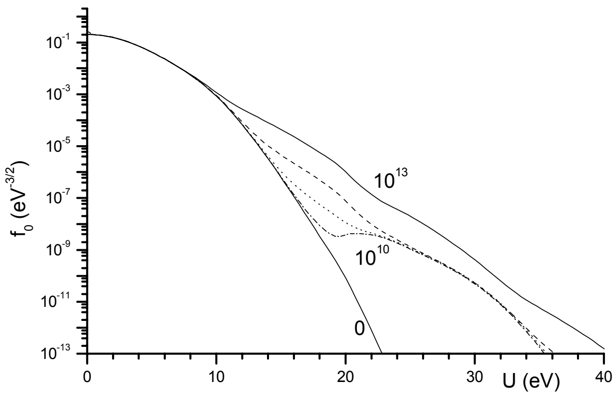

Figure 4. The evdf in dependence on the excited atom densities at $E/p = 2$ V cm^{-1} torr^{-1}. The curve numbers are the xenon excited atom density $N_m^{(Xe)}$ in cm^{-3}.

Figures 3 and 4 demonstrate the evdf behaviour at small and intermediate electric fields in dependence on the excited atom density. Two peculiarities are to be seen. Firstly, in Fig. 3 there is a plateau at energies 3-10 eV that is connected with the density of $N_m^{(Xe)}$, and, secondly a maximum placed at the excitation energy of helium. This maxiamum is formed by second kind collisions with helium atoms and its height depends on the density $N_m^{(He)}$. A similar maximum is also present at the larger electric field strength of 2 V cm^{-1} torr^{-1} (cf. Fig. 4) where the evdf is populated at intermediate energies by the field action.

It is seen from Figs. 1 and 4 that the second-kind collisions modify the distribution function at low electric fields only. At electric fields E/p>5 V cm^{-1} torr^{-1} the second-kind collisions can be neglected. This is connected with electron temperature kT_e, the evdf scale: $f_0(U+U_j)/f_0(U) \approx \exp(-U_j/kT_e)$, and the ratio of the excited to ground state atom densities $N_m^{(Xe,He)}/N^{(Xe,He)}$. Obviously, under condition

$$\left[N_m^{(Xe,He)} f_0(U)\right] / \left[N^{(Xe,He)} f_0(U+U_1^{(Xe,He)})\right] << 1$$

the second-kind collisions become negligible.

4. Macroscopic properties by different models

Different approaches to the Boltzmann kinetic equation solution exist. We shall compare the following three of them.

One of the strictest approaches is that performed in [1] where the multi-term expansion of the distribution function was used and ionization was described as a non-conservative process. This model we shall call model 1. Its disadvantages are connected with the absence of the second-kind collisions and the use of one effective excitation level. The model described in sections 2 and 3 will be called model 2.

Often, the kinetic equation is solved in the most simplified way in two-term approximation with one effective excitation level, no second-kind collisions and ionization as conservative process. This model will be called model 3.

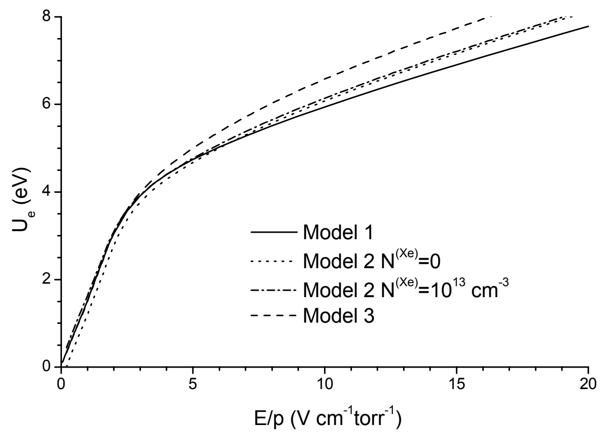

Figure 5. Mean electron energy in dependence on the electric field calculated by different models.

The electron mean energy and direct ionization rate coefficient for Xe are represented in Figs. 5 and 6 for different models. It is seen that the mean energy has a weak dependence on details of the model and a negligible dependence on the excited state density.

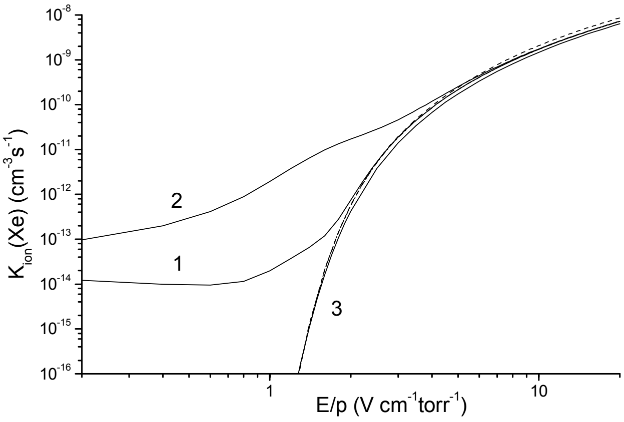

Figure 6. Rate coefficient for direct ionization of xenon in dependence on electric field calculated by different models. Curve 1: model 2 at $N_m^{(Xe)}$=10^{11} cm^{-3}, curve 2: model 2 at $N_m^{(Xe)}$=10^{13} cm^{-3} and curve 3: models 1,3 and model 2 at $N_m^{(Xe)}$=0 .

Because the second-kind collisions modify the high-energy part of the electron distribution their impact is most evident with respect to rate coefficients of inelastic collision processes. As an example, Fig. 6 illustrates that a relatively large ionization rate coefficient can be reached at very low electric fields or mean energies if second kind collisions are taken into account.

5. Conclusion

The influence of the second-kind collisions on the electron distribution function has been considered based on different approaches to represent the collision processes in the Boltzmann kinetic equation. The second-kind collisions were found to be important at small electric field and large densities of the excited states. Those macroscopic properties which depend on the low energy part of the evdf are less sensitive to details of the models. However a large impact has been found with respect to collision rate coefficients which are strongly increased at lower field strength.

6. References

[1] N. R. Pinhao, Z. Donko, D. Loffhagen, M. J. Pinheiro and E. A. Richley, Plasma Sources Sci. Technol. **13** (2004) 719.

[2] http://www.siglo-kinema.com/BOLSIG. Boltzmann solver for the SIGLO-series. CPA Tolouse&Kinema Software.