General dispersion relation for microwave gas breakdown in the presence of static magnetic field

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Considering the interaction high-frequency circularly polarized microwave (MW) pulse field with the rarefied neutral gas, the electron distribution function (EDF) of produced plasma obtained in the presence of the static magnetic field in the non-relativistic regime. During the plasma produced process the electron generation rate is smaller than the microwave frequency and the ionization mechanism is governed by electron impact. The all elements of dielectric permittivity tensor are calculated using of the obtained EDF. Making use of the adiabatic approximation, the general dispersion relation is found for magnetized anisotropic produced plasma.

1. Introduction

Microwave breakdown of gas is the phenomenon by which the ionizing action of a high frequency electric field can turn a non-conducting gas into conducting plasma. In breakdown physics one deal with a medium ranging from an essentially neutral gas with very small densities of charge particles to weakly ionized plasma with a small fraction of charge particles together with a bulk of neutral particles. Although, the charged species only make up a small fraction of the total number of particles or particle density of the medium, their charge, and the concomitant force and conductivity, give the plasma properties completely different from those of a neutral gas.

The interaction of high-power MW field with plasma is rich in a variety of phenomena. These phenomena become particularly interesting and involved by applying a static magnetic field on the plasma. Some of the interesting phenomena that are discussed in the microwave-plasma processes include (a) Weibel instability [1] (b) low-frequency instability[2,3] (c) parametric instability [4,5] (d) relativistic effect on the Weibel instability [6] (e) effect of the magnetic field on the Weibel instability [7]..

In the following, the breakdown phenomenon in the microwave discharge is considered in the presence of a static magnetic field and the general dispersion relation is found for electron perturbation in such anisotropic plasma.

2. Plasma generation

In a microwave breakdown physics one deal with a medium ranging from an essentially neutral gas with very small densities of charge particles to weakly ionized plasma with a small fraction of charge particles together with a bulk of neutral particles. The energy needed to remove the loosest bound electron from a molecule or atom is of the order of several eV. Since the average energy of particles in a gas at room temperature is a fraction of an eV, very few particles in a tail of energy distribution of neutral atoms have the energy needed to cause ionization. Consequently, thermal ionization is negligible in gases at room temperature. For gas at room temperature, it is still possible to achieve considerable ionization by other mechanisms. Charged particles such as electrons may be accelerated to sufficiently high energies by an applied microwave field, possibly with a successive build up of energy through a series of elastic collisions. This process is called impact ionization and it is main ionization mechanisms in ordinary microwave breakdown. Since in our discussion the gas is very dilute and the collision frequency is much smaller than the MW frequency, we can ignore the collisional stochastization of the forced electron oscillation with a good accuracy. Furthermore, if plasma density $n_e(t)$ produced by the field during gas breakdown, is less than the critical density ($\omega_0^2 \rangle \omega_{Le}^2$) we can neglect the effect of the polarization field.

Moreover, the plasma density is assumed to be less than the neutral gas density n_0 so that the latter can be considered constant. Also the field was adiabatically switched on in the infinite past and the MW radiation electric field amplitude E is constant during a single field period. Under the aforementioned conditions, the kinetic equation for the plasma electrons produced in the gas breakdown by a strong pulsed field can be written as follows

$$\frac{\partial f_0}{\partial t} + v.\frac{\partial f_0}{\partial r} + e\left[E + \frac{1}{c}\left(v \times \left(B + B_0\right)\right)\right].\frac{\partial f_0}{\partial p} = n_0 \omega_{ioniz} \delta(p) \tag{1}$$

Where $E(\zeta)$ and $B(\zeta)$ are electric and magnetic fields of a wave propagating along the Z-axis and B_0 is the external static magnetic field along the wave propagating, also $\zeta = \omega_0 t - k_0 r = \omega_0 \left(t - z/c\right)$ and ω_{ioniz} is the ionization probability of the gas atoms. Here $\delta(p)$ is the delta function of electron momentum. By solving the kinetic equation (1) in the non-relativistic regime [7] the electron distribution function (EDF) is given by

$$f_0(v) = \frac{1}{2\pi V_E}\delta(v_z)\delta(V_\perp - V_E)$$
$$= \frac{1}{2\pi V_E}\delta(v_z)\delta(v_\perp - 2V_E \sin\varphi). \qquad (2)$$

Where $v_E = eE_0/m\omega_0$ is the oscillation velocity of the electron in the microwave (MW) field and E_0 is the field amplitude. Furthermore, $\Omega_e = |e|B_0/mc$ is the electron Larmour frequency. Moreover, the following definitions should be considered.

$$V_E = v_E\left(\omega_0/\Delta\omega\right),\ v_\perp^2 = v_x^2 + v_y^2,\ \Delta\omega = \omega_0 - \Omega_e$$
$$V_\perp^2 = v_\perp^2 + V_E^2 + 2V_E\left(v_x \sin\omega_0 t - v_y \cos\omega_0 t\right)$$
$$\varphi = (\omega_0 - \Omega_e)(t - t_0)/2$$

2. General dispersion relation

The (EDF) founded in the previous section shows the produced plasma is highly anisotropic and it is expected to have the various instability in such system. Therefore, we turn to the adiabatic approximation [8], assuming that the instability grows faster than the plasma density. In this approximation, we can use the following unperturbed dispersion relation for small electron perturbation

$$\left| k^2\delta_{ij} - k_i k_j - \frac{\omega^2}{c^2}\varepsilon_{ij}\left(\omega, \Omega_\beta, \vec{k}\right)\right| = 0 \qquad (3)$$

Here $\varepsilon_{ij}\left(\omega, \Omega_\beta, \vec{k}\right) = 1 + \delta\varepsilon_{ij}^e + \delta\varepsilon_{ij}^I$ is the dielectric tensor of the magnetized plasma, in which, $\delta\varepsilon_{ij}^e$ and $\delta\varepsilon_{ij}^I$ denote the ion and electron contribution in the plasma dielectric permittivity tensor. It should be noted that in this system the ions are could and the contribution of the electron dielectric permittivity are given by [7]

$$\delta\varepsilon_{11}^e = -\frac{\omega_{Le}^2}{2\omega^2}\left\{\frac{2\omega^2}{\left(\omega^2 - \Omega_e^2\right)}\left(1 - \frac{4}{3}\alpha^2 + \frac{64}{241}\alpha^4\right) + \frac{8\omega^2}{3\left(\omega^2 - 4\Omega_e^2\right)}\left(\alpha^2 - \frac{4}{5}\alpha^4\right) + \frac{36\omega^2}{45\left(\omega^2 - 9\Omega_e^2\right)}\alpha^4\right\}$$
$$-\frac{2\omega_{Le}^2 k_z^2 V_E^2}{3\omega^2}\left\{\frac{2\left(\omega^2 + \Omega_e^2\right)}{\left(\omega^2 - \Omega_e^2\right)^2}\left(1 - \frac{4}{5}\alpha^2 + \frac{4}{7}\alpha^4\right) + \frac{16\left(\omega^2 + 4\Omega_e^2\right)}{15\left(\omega^2 - 4\Omega_e^2\right)^2}\left(\alpha^2 - \frac{4}{7}\alpha^4\right) + \frac{144\left(\omega^2 + 9\Omega_e^2\right)}{315\left(\omega^2 - 9\Omega_e^2\right)^2}\alpha^4\right\}$$

$$\delta\varepsilon_{12}^e = -\delta\varepsilon_{21}^e = -\frac{i\omega_{Le}^2}{\omega^2}\left\{\frac{2\omega\Omega_e}{\left(\omega^2 - \Omega_e^2\right)}\left(1 - \frac{8}{3}\alpha^2 + 2\alpha^4\right) + \frac{16\omega\Omega_e}{\left(\omega^2 - 4\Omega_e^2\right)}\left(\frac{1}{3}\alpha^2 - \frac{2}{5}\alpha^4\right) + \frac{36\omega\Omega_e}{15\left(\omega^2 - 4\Omega_e^2\right)}\alpha^4\right\}$$
$$-\frac{2i\omega_{Le}^2 k_z^2 v_z^2}{\omega^2}\left\{\frac{\omega\Omega_e}{\left(\omega^2 - \Omega_e^2\right)^2}\left(-\frac{32}{15}\alpha^2 + \frac{8}{7}\alpha^4\right) + \frac{8\omega\Omega_e}{\left(\omega^2 - 4\Omega_e^2\right)^2}\left(\frac{8}{15}\alpha^2 - \frac{16}{35}\alpha^4\right) + \frac{144\omega\Omega_e}{\left(\omega^2 - 9\Omega_e^2\right)^2}\alpha^4\right\}$$
$$\delta\varepsilon_{13}^e = -\delta\varepsilon_{31}^e = -\frac{4\omega_{Le}^2 k_z \Omega_e}{3\omega^2 k_\perp}\left\{\frac{\Omega_e}{\left(\omega^2 - \Omega_e^2\right)}\left(\alpha^2 - \frac{4}{5}\alpha^4\right) + \frac{4\Omega_e}{5\left(\omega^2 - 4\Omega_e^2\right)}\alpha^4\right\} \qquad (4)$$

$$\delta\varepsilon_{22}^{e}=-\frac{\omega_{Le}^{2}}{2\omega^{2}}\left\{\left(\frac{16}{3}\alpha^{2}-\frac{32}{5}\alpha^{4}\right)+\frac{2\omega^{2}}{(\omega^{2}-\Omega_{e}^{2})}\left(1-4\alpha^{2}+\frac{74}{15}\alpha^{4}\right)+\frac{8\omega^{2}}{3(\omega^{2}-4\Omega_{e}^{2})}\left(\alpha^{2}-\frac{8}{5}\alpha^{4}\right)+\frac{4\omega^{2}}{5(\omega^{2}-9\Omega_{e}^{2})}\alpha^{4}\right\}$$
$$-\frac{\omega_{Le}^{2}k_{z}^{2}V_{E}^{2}}{\omega^{2}}\left\{\frac{1}{\omega^{2}}\left(\frac{32}{15}\alpha^{2}-\frac{64}{35}\alpha^{4}\right)+\frac{(\omega^{2}+\Omega_{e}^{2})}{(\omega^{2}-\Omega_{e}^{2})^{2}}\left(\frac{4}{3}-\frac{16}{5}\alpha^{2}+\frac{216}{105}\alpha^{4}\right)+\frac{(\omega^{2}+4\Omega_{e}^{2})}{(\omega^{2}-4\Omega_{e}^{2})^{2}}\left(\frac{16}{15}\alpha^{2}-\frac{128}{105}\alpha^{4}\right)+\frac{8(\omega^{2}+9\Omega_{e}^{2})}{35(\omega^{2}-9\Omega_{e}^{2})^{2}}\alpha^{4}\right\}$$

$$\delta\varepsilon_{23}^{e}=-\delta\varepsilon_{32}^{e}=-\frac{4i\omega_{Le}^{2}k_{z}V_{E}}{\omega^{2}}\left\{\frac{1}{\omega}\left(-\frac{1}{3}\alpha+\frac{2}{5}\alpha^{3}\right)+\frac{2\omega}{(\omega^{2}-\Omega_{e}^{2})}\left(\frac{1}{3}\alpha-\frac{8}{15}\alpha^{3}\right)+\frac{2\omega}{15(\omega^{2}-4\Omega_{e}^{2})}\alpha^{3}\right\}$$

$$\delta\varepsilon_{33}^{e}=-\frac{\omega_{Le}^{2}}{\omega^{2}}$$

Where, $\alpha=k_{\perp}V_{E}/\Omega_{e}\langle 1$. To obtain the dielectric elements we assume in the strong magnetic fields the electrons are strongly magnetized ($\Omega_{e}^{2}\rangle\rangle\omega_{le}^{2}$) and the ions, on the contrary, are not magnetized ($\Omega_{i}^{2}\langle\langle\omega_{Li}^{2}$). Furthermore, in the weak magnetic fields the electrons may be weakly magnetized ($\Omega_{e}^{2}\langle\omega_{Le}^{2}$).

Here $\omega_{Le}=\left(4\pi n_{e}e^{2}/m\right)^{1/2}$ and $\omega_{Li}=\left(4\pi n_{i}e^{2}/M\right)^{1/2}$ are electron and ion plasma frequency and strength of the magnetic field restricted by condition, $c\sqrt{4\pi n_{e}m}\langle\langle B_{0}\langle\langle c\sqrt{4\pi n_{i}M}$. By substituting the obtained electron dielectric permittivity in dispersion equation (3) we get the following dispersion relation for electron perturbation as follows:

$$\left(k^{2}-k_{x}^{2}-\frac{\omega^{2}}{c^{2}}(1+\delta\varepsilon_{11}^{e})\right)\times\left\{\left(k^{2}-k_{x}^{2}-\frac{\omega^{2}}{c^{2}}(1+\delta\varepsilon_{22}^{e})\right)\times\left(k_{\perp}^{2}-\frac{\omega^{2}}{c^{2}}(1+\delta\varepsilon_{33}^{e})\right)-\left(k_{y}k_{z}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{32}^{e}\right)\times\left(k_{y}k_{z}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{32}^{e}\right)\right\}$$
$$-\left(k_{x}k_{y}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{12}^{e}\right)\times\left\{\left(k_{x}k_{y}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{21}^{e}\right)\times\left(k_{\perp}^{2}-\frac{\omega^{2}}{c^{2}}(1+\delta\varepsilon_{33}^{e})\right)-\left(k_{y}k_{z}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{31}^{e}\right)\times\left(k_{y}k_{z}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{23}^{e}\right)\right\}$$
$$+\left(k_{y}k_{z}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{13}^{e}\right)\times\left\{\left(k_{x}k_{y}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{21}^{e}\right)\times\left(k_{y}k_{z}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{32}^{e}\right)-\left(k_{y}k_{z}-\frac{\omega^{2}}{c^{2}}\delta\varepsilon_{31}^{e}\right)\times\left(k^{2}-k_{y}^{2}-\frac{\omega^{2}}{c^{2}}(1+\delta\varepsilon_{22}^{e})\right)\right\}=0 \qquad (5)$$

Where

$$k=(k\sin\theta\cos\psi,k\sin\theta\sin\psi,k\cos\theta)$$
$$k_{\perp}=\sqrt{k_{x}^{2}+k_{y}^{2}}=k\sin\theta$$

Here θ and ψ are polar and azimuthally angels of wave vector $\vec{k}$ in the spherical coordinates respectively, in which, θ is the angel between wave vector $\vec{k}$ and the external static magnetic field. We have found as well known Weibel instability for perturbation along the magnetic field ($\theta=0$) in the recent paper and it is shown that the magnetic field decreases the Weibel instability growth rate [7]. Therefore, we can use of the magnetic field for stabilizing purpose and reduced the threshold of microwave breakdown. The dispersion relation (5) is reported in the present paper for the first time and we wish to study the stability of the produced plasma generally in the next works.

3. References

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