Linear Programming

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Symbolic Language

Formulas are merely shorthand for ideas. For example consider the following:

Click on the best meaning:
East and West made the Third World
Pink and Blue equal White
Couples may become families

Show me the money!

Linear Programming involves a system of linear equations and inequalities. The first step is to determine the _____ **Function**.

Since these problems involve business situations, objective functions generally involve MONEY; either revenues or costs.

FORMULAS

Objective functions take the form: Z = #X + #Y

Where: Z = the total revenues or total costs
# = coefficients
X = first variable
Y = second variable

Maximize _____ OR Minimize Costs

CONSTRAINTS

Linear Programming also involves **Constraints** which are restrictions on amounts of the factors in the objective function.

_____ functions use the same variables (X & Y) as used in the objective function.

CONSTRAINTS

Variables in constraint functions all use the same measurement units (hours, units, etc.) Constraints are almost always inequalities.

In order to be consistent, group the variables on the left side, and the numbers on the _____ side, such as: “2X + 3Y > 5”.

Steps

1. Determine objective function (follow the money)
2. Determine constraint functions (same variables)
3. Compute (X,0) and (0,Y) for constraint functions
   Sometimes it's (0,0) and (X,Y) see hints
4. Identify highest X and Y for scales (divide by 20)
5. Plot two points and connect line for each constraint
6. Identify solution area
7. Compute objective function for each corner, choose greatest revenue or least cost

For example: consider the following problem:

A financial counselor wants to develop an investment portfolio that will maximize a customer's total dollar return. Two types of independent investments are available:
Stocks, whose average return is 10 percent, and Bonds which return an average return of 6 percent.
The customer has $1,000 available and prefers to invest at least twice as much in bonds as in stocks.
As a result of other commitments, the counselor can devote not more than 90 hours to research the customer's portfolio.
Past experience indicates that stocks require 6 minutes (.1 hour) of research per dollar invested and bonds require 12 minutes (.2 hour)
Note how the problem is parsed (divided) into separate statements

$$$$

Objective Functions usually involve either earning money or _____ money.

In this case the *money* is the return to the investor.

Read the statements about returns and convert them to formulas.

DOLLARS OF RETURN

" Stocks, whose average return is 10 percent, and Bonds which return an average return of 6 percent."

The corresponding objective function is:

Stocks Bonds

Z = (.10 * X) + (.06 * Y)

Where Z is the total return

X is the dollar amount of the investment in _____

Y is the dollar amount of the investment in Bonds

Constraints

Constraints are restrictions on the objective function.

Look for phrases that involve the same factors as appear in the objective function.

For example, look at the phrase: "The customer has $1,000 available"

Remember the objective function: Z = (.10 * X) + (.06 * Y) constraints use the same variables as the _____ function.

Constraints

The constraint corresponding to "The customer has $1,000 available" is:

"The sum of stocks and bonds must be less than or _____ to $1,000."

OR

X + Y =< $1,000

Next Constraint

"The customer prefers to invest at least twice as much in bonds as in stocks."
Remember the objective function: Z = (.10 * X) + (.06 * Y)
constraints use the same variables as the _____ function

The corresponding constraint is:

Y >= 2 * X

OR

*by subtracting (2 * X) from both sides of the inequality*

Y - (2 * X) >= 0

Third Constraint

"As a result of other commitments, the counselor can devote not more than 90 hours to research of the customer's portfolio. Past experience indicates that stocks require 6 minutes of effort per dollar invested and bonds require 12 minutes"

Use consistent measurement units. Either the time for stocks and bonds must be in terms of hours to compare to the 90 hour limit; or the limit must be expressed as minutes.

Usually it is easier to convert to the larger measurement unit by dividing smaller units by number of smaller units which make up one of the larger units

In this case, there are 60 minutes in an hour, so divide by 60

Time Constraint

Remember that measurement units must be consistent
Recall the objective function: Z = (.10 * X)+(.06 * Y)
_____ use same variables as objective function
The corresponding constraint is:

(6/60 * X) + (12/60 * Y) =< 90

OR

.1X + .2Y =< 90

Positive Constraint

Normally the variables are limited to positive numbers, so the last constraint is usually: "X , Y > = 0"

This just means that both variables must be _____ numbers

Formulas describing the demo problem:

Objective Function
Z = .10X + .06Y

Constraint Functions
X + Y =< $1,000
Y - 2X >= 0
.1X + .2Y =< 90
X, Y >= 0

Graphs

First step to preparing a graph is to compute two points for each constraint _____

Treat each constraint as equality rather than inequality

Then substitute zero for X and solve for Y

Record the point (0,Y)

Next substitute zero for Y and solve for X

Again write down the result (X,0)

Single Variable

An constraint may contain only one variable (X or Y).
In that case, compute the point where the line crosses an axis.
X would be a vertical line, while Y would be a _____ line

Another difficulty arises when both answers are the same: (0,0)
which means that the line begins from the origin
In this case, just pick an X and compute Y for the 2nd point

Linear programming problems include a constraint: X, Y >= 0
Which merely means that the solution must be positive numbers,
but the solutions for points may include negative numbers.

First Constraint

FIRST POINT

X + Y = $1,000
If X = 0, then Y = 1,000
(0,1000)

SECOND POINT

If Y = 0, then X = 1,000
(1000,0)

Second Constraint

FIRST POINT

Y - 2X = 0
If X = 0, then 2Y = 0
(0,0)

SECOND POINT

Since the origin is the first point,
just pick a X and compute Y
If X = 300, then Y – 2*300 = 0
Y = 2 * 300
Y = 600
(300,600)

Third Constraint

FIRST POINT

.1X + .2Y = 90
If X = 0, then .2Y = 90
Y = 90/.2
(0,450)

SECOND POINT

.1X + .2Y = 90
If Y = 0, then .1X = 90
X = 90/.1
X = 900
(900,0)

Scale of the Graph

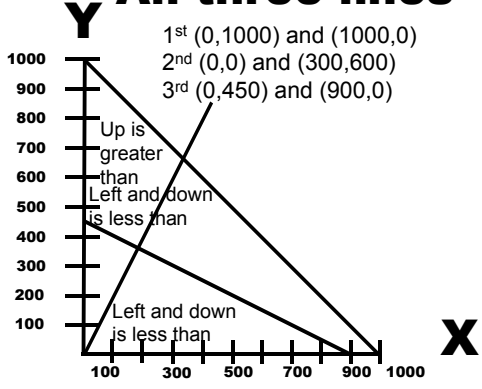
Look at all of the first coordinates for the points and the greatest X is the highest number needed for X axis

1st (0,1000) and (1000,0)
2nd (0,0) and (300,600)
3rd (0,450) and (900,0)

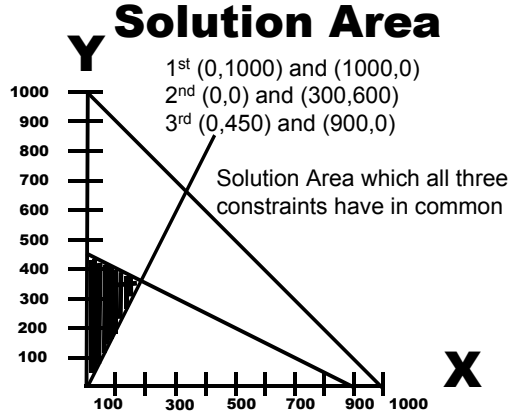
Click on the highest X

1,000 600 900

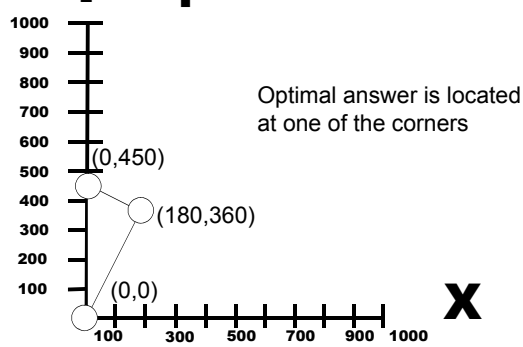
All three lines



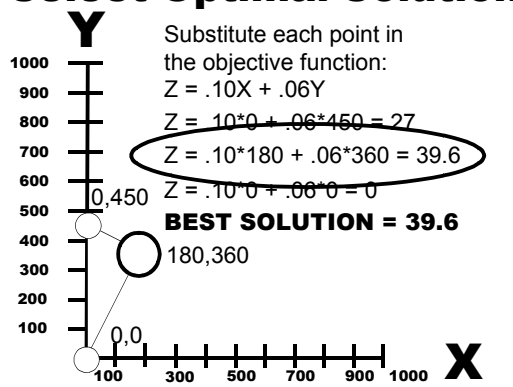
Solution Area



Optimal Solution



Select Optimal Solution



Hints

If the first point of a constraint is 0,0, then just choose a X (within the range) and compute Y

If there is only one variable in the constraint formula, the line is either vertical (X) or horizontal (Y)

Use a straightedge and graph paper
(see next slide for instructions to print graph paper)

Bigger the better for graphs

Graphical solutions are like horseshoes,
you only have to get close

Pick a point on one side of the line and plug the coordinates into the formula to see if it holds true

INSTRUCTIONS TO CREATE GRAPH PAPER
IN EXCEL (Office 2003)

Click on the blank upper-left corner square above the letter A and to the left of number 1 to highlight the entire worksheet

Click on Format, Cells, Borders – and choose “inside”

With the whole worksheet still highlighted, click on:
Format, Column, Width and change to 3

With the whole worksheet still highlighted, click on:
Format, Row, Height, and change to 15

Click on cell A1 and drag to highlight from A to Z and down to 50, and then click File, Print Area, Set Print Area

Click on File, Print; and choose: Page(s) and enter “1”