



*RTI Toolkit: A Practical Guide for Schools*

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## Effective Math Interventions

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# School-Wide Strategies for Managing... MATHEMATICS

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Mathematics instruction is a lengthy, incremental process that spans all grade levels. As children begin formal schooling in kindergarten, they develop 'number sense', an intuitive understanding of foundation number concepts and relationships among numbers. A central part of number sense is the student's ability to internalize the number line as a precursor to performing mental arithmetic. As students progress through elementary school, they must next master common math operations (addition, subtraction, multiplication, and division) and develop fluency in basic arithmetic combinations ('math facts'). In later grades, students transition to applied, or 'word', problems that relate math operations and concepts to real-world situations. Successful completion of applied problems requires that the student understand specialized math vocabulary, identify the relevant math operations needed to solve the problem while ignoring any unnecessary information also appearing in that written problem, translate the word problem from text format into a numeric equation containing digits and math symbols, and then successfully solve. It is no surprise, then, that there are a number of potential blockers to student success with applied problems, including limited reading decoding and comprehension skills, failure to acquire fluency with arithmetic combinations (math facts), and lack of proficiency with math operations. Deciding what specific math interventions might be appropriate for any student must therefore be a highly individualized process, one that is highly dependent on the student's developmental level and current math skills, the requirements of the school district's math curriculum, and the degree to which the student possesses or lacks the necessary auxiliary skills (e.g., math vocabulary, reading comprehension) for success in math. Here are some wide-ranging classroom (Tier I RTI) ideas for math interventions that extend from the primary through secondary grades.

**Applied Problems: Encourage Students to Draw to Clarify Understanding** (*Van Essen & Hamaker, 1990*). Making a drawing of an applied, or 'word', problem is one easy heuristic tool that students can use to help them to find the solution. An additional benefit of the drawing strategy is that it can reveal to the teacher any student misunderstandings about how to set up or solve the word problem. To introduce students to the drawing strategy, the teacher hands out a worksheet containing at least six word problems. The teacher explains to students that making a picture of a word problem sometimes makes that problem clearer and easier to solve. The teacher and students then independently create drawings of each of the problems on the worksheet. Next, the students show their drawings for each problem, explaining each drawing and how it relates to the word problem. The teacher also participates, explaining his or her drawings to the class or group. Then students are directed independently to make drawings as an intermediate problem-solving step when they are faced with challenging word problems. NOTE: This strategy appears to be more effective when used in later, rather than earlier, elementary grades.

**Math Computation: Boost Fluency Through Explicit Time-Drills** (*Rhymer, Skinner, Jackson, McNeill, Smith & Jackson, 2002; Skinner, Pappas & Davis, 2005; Woodward, 2006*). Explicit time-drills are a method to boost students' rate of responding on math-fact worksheets. The teacher hands out the worksheet. Students are told that they will have 3 minutes to work on problems on the sheet. The teacher starts the stop watch and tells the students to start work. At the end of the first minute in the 3-minute span, the teacher 'calls time', stops the stopwatch, and tells the students to underline the last number written and to put their pencils in the air. Then students are told to resume work and the teacher restarts the stopwatch. This process is repeated at the end of minutes 2 and 3. At the conclusion of the 3 minutes, the teacher collects the student worksheets. TIPS: Explicit time-drills work best on 'simple' math facts requiring few computation steps. They are less effective on more complex math facts. Also, a less intrusive and more flexible version of this intervention is to use time-prompts while students are working independently on math facts to speed their rate of responding. For example, at the end of every minute of seatwork, the teacher can call the time and have students draw a line under the item that they are working on when that minute expires.

**Math Computation: Increase Accuracy and Productivity Rates Via Self-Monitoring and Performance Feedback** (*Shimabukuro, Prater, Jenkins & Edelen-Smith, 1999*). Students can bring up both their accuracy and overall productivity on math computation worksheets by independently self-monitoring, charting their daily progress, and earning rewards for improved performance. In preparation for this intervention, the teacher selects one or more computation problem types that the student needs to practice. Using that set of problem types as a guide, the teacher creates a number of standardized worksheets with similar items to be used across multiple instructional days. (A Math Worksheet Generator that will create these worksheets automatically can be accessed at <http://www.interventioncentral.org>). The teacher also prepares a progress-monitoring chart whose vertical axis extends from 0 to 100 and is labeled “Percent Accuracy/Percent Productivity”. During each day of the intervention, the student is given one of the math computation worksheets previously created by the teacher, along with an answer key. The student first consults his or her progress-monitoring chart and notes the most recent charted accuracy and productivity scores previously posted. The student is encouraged to try to exceed those scores. When the intervention starts, the student is given a pre-selected amount of time (e.g., 5 minutes) to complete as many problems on the computation worksheet as possible. The student sets a timer and works on the computation sheet until the timer rings. Then the student checks his or her work, giving credit for each correctly answered item. To compute an ACCURACY score, the student counts up the number of correct problems. Using a calculator, the student divides that number by the total number of problems attempted and then multiplies the resulting quotient by 100. This gives an accuracy score in the form of a percentage. To compute a PRODUCTIVITY score, the student counts up the number of problems completed. Using a calculator, the student divides that number by the total number of problems on the worksheet and then multiplies the resulting quotient by 100. This gives a productivity score in the form of a percentage. The student plots and labels both the accuracy and productivity scores on the progress-monitoring chart. The student receives praise, points toward a reward, or other reinforcer if he or she exceeds the most recent accuracy and productivity scores that had been previously posted.

**Math Computation: Increase Accuracy Through Interspersion of ‘Easy’ Problems** (*Hawkins, Skinner & Oliver, 2005*). Teachers can improve accuracy and positively influence the attitude of students when completing math-fact worksheets by interspersing ‘easy’ problems among the ‘challenging’ problems. The teacher first identifies the range of ‘challenging’ problem-types (number problems appropriately matched to the student’s current instructional level) that are to appear on the worksheet. Then the teacher creates a series of ‘easy’ problems that the students can complete very quickly (e.g., adding or subtracting two 1-digit numbers). The teacher next prepares a series of student math computation worksheets with ‘easy’ computation problems interspersed at a fixed rate among the ‘challenging’ problems. If the student is expected to complete the worksheet independently, ‘challenging’ and ‘easy’ problems should be interspersed at a 1:1 ratio (that is, every ‘challenging’ problem in the worksheet is preceded and/or followed by an ‘easy’ problem). If the student is to have the problems read aloud and then asked to solve the problems mentally and write down only the answer, the items should appear on the worksheet at a ratio of 3 ‘challenging’ problems for every ‘easy’ one (that is, every 3 ‘challenging’ problems are preceded and/or followed by an ‘easy’ one).

**Math Computation: Motivate With ‘Errorless Learning’ Worksheets** (*Caron, 2007*). Reluctant students can be motivated to practice math number problems to build computational fluency when given worksheets that include an answer key (number problems with correct answers) displayed at the top of the page. In this version of an ‘errorless learning’ approach, the student is directed to complete math facts as quickly as possible. If the student comes to a number problem that he or she cannot solve, the student is encouraged to locate the problem and its correct answer in the key at the top of the page and write it in. Such speed drills build computational fluency while promoting students’ ability to visualize and to use a mental number line. TIP: Consider turning this activity into a ‘speed drill’. The student is given a kitchen timer and instructed to set the timer for a predetermined span of time (e.g., 2 minutes) for each drill. The student completes as many problems as possible before the timer rings. The student then graphs the number of problems correctly computed each day on a time-series graph, attempting to better his or her previous score.

**Math Computation: Two Ideas to Jump-Start Active Academic Responding** (Skinner, Pappas & Davis, 2005). Research shows that when teachers use specific techniques to motivate their classes to engage in higher rates of active and accurate academic responding, student learning rates are likely to go up. Here are two ideas to accomplish increased academic responding on math tasks. First, break longer assignments into shorter assignments with performance feedback given after each shorter 'chunk' (e.g., break a 20-minute math computation worksheet task into 3 seven-minute assignments). Breaking longer assignments into briefer segments also allows the teacher to praise struggling students more frequently for work completion and effort, providing an additional 'natural' reinforcer. Second, allow students to respond to easier practice items orally rather than in written form to speed up the rate of correct responses.

**Math Instruction: Consolidate Student Learning During Lecture Through the Peer-Guided Pause** (Hawkins, & Brady, 1994). During large-group math lectures, teachers can help students to retain more instructional content by incorporating brief Peer Guided Pause sessions into lectures. Students are trained to work in pairs. At one or more appropriate review points in a lecture period, the instructor directs students to pair up to work together for 4 minutes. During each Peer Guided Pause, students are given a worksheet that contains one or more correctly completed word or number problems illustrating the math concept(s) covered in the lecture. The sheet also contains several additional, similar problems that pairs of students work cooperatively to complete, along with an answer key. Student pairs are reminded to (a) monitor their understanding of the lesson concepts; (b) review the correctly math model problem; (c) work cooperatively on the additional problems, and (d) check their answers. The teacher can direct student pairs to write their names on the practice sheets and collect them as a convenient way to monitor student understanding.

**Math Instruction: Support Students Through a Wrap-Around Instruction Plan** (Montague, 1997; Montague, Warger & Morgan, 2000). When teachers instruct students in more complex math cognitive strategies, they must support struggling learners with a 'wrap-around' instructional plan. That plan incorporates several elements: (a) Assessment of the student's problem-solving skills. The instructor first verifies that the student has the necessary academic competencies to learn higher-level math content, including reading and writing skills, knowledge of basic math operations, and grasp of required math vocabulary. (b) Explicit instruction. The teacher presents new math content in structured, highly organized lessons. The instructor also uses teaching tools such as Guided Practice (moving students from known material to new concepts through a thoughtful series of teacher questions) and 'overlearning' (teaching and practicing a skill with the class to the point at which students develop automatic recall and control of it). (c) Process modeling. The teacher adopts a 'think aloud' approach, or process modeling, to verbally reveal his or her cognitive process to the class while using a cognitive strategy to solve a math problem. In turn, students are encouraged to think aloud when applying the same strategy—first as part of a whole-class or cooperative learning group, then independently. The teacher observes students during process modeling to verify that they are correctly applying the cognitive strategy. (d) Performance feedback. Students get regular performance feedback about their level of mastery in learning the cognitive strategy. That feedback can take many forms, including curriculum-based measurement, timely corrective feedback, specific praise and encouragement, grades, and brief teacher conferences. (e) Review of mastered skills or material. Once the student has mastered a cognitive strategy, the teacher structures future class lessons or independent work to give the student periodic opportunities to use and maintain the strategy. The teacher also provides occasional brief 'booster sessions', reteaching steps of the cognitive strategy to improve student retention.

**Math Instruction: Unlock the Thoughts of Reluctant Students Through Class Journaling** (Baxter, Woodward & Olson, 2005). Students can effectively clarify their knowledge of math concepts and problem-solving strategies through regular use of class 'math journals'. Journaling is a valuable channel of communication about math issues for students who are unsure of their skills and reluctant to contribute orally in class. At the start of the year, the teacher introduces the journaling assignment, telling students that they will be asked to write and submit responses at least weekly to teacher-posed questions. At first, the teacher presents 'safe' questions that tap into the students' opinions and

attitudes about mathematics (e.g., ‘How important do you think it is nowadays for cashiers in fast-food restaurants to be able to calculate in their head the amount of change to give a customer?’). As students become comfortable with the journaling activity, the teacher starts to pose questions about the students’ own mathematical thinking relating to specific assignments. Students are encouraged to use numerals, mathematical symbols, and diagrams in their journal entries to enhance their explanations. The teacher provides brief written comments on individual student entries, as well as periodic oral feedback and encouragement to the entire class on the general quality and content of class journal responses. Regular math journaling can prod students to move beyond simple ‘rote’ mastery of the steps for completing various math problems toward a deeper grasp of the math concepts that underlie and explain a particular problem-solving approach. Teachers will find that journal entries are a concrete method for monitoring student understanding of more abstract math concepts. To promote the quality of journal entries, the teacher might also assign them an effort grade that will be calculated into quarterly math report card grades.

**Math Problem-Solving: Help Students Avoid Errors With the ‘Individualized Self-Correction Checklist’** (*Zrebiec Uberti, Mastropieri & Scruggs, 2004*). Students can improve their accuracy on particular types of word and number problems by using an ‘individualized self-instruction checklist’ that reminds them to pay attention to their own specific error patterns. To create such a checklist, the teacher meets with the student. Together they analyze common error patterns that the student tends to commit on a particular problem type (e.g., ‘On addition problems that require carrying, I don’t always remember to carry the number from the previously added column.’). For each type of error identified, the student and teacher together describe the appropriate step to take to prevent the error from occurring (e.g., ‘When adding each column, make sure to carry numbers when needed.’). These self-check items are compiled into a single checklist. Students are then encouraged to use their individualized self-instruction checklist whenever they work independently on their number or word problems. As older students become proficient in creating and using these individualized error checklists, they can begin to analyze their own math errors and to make their checklists independently whenever they encounter new problem types.

**Math Review: Balance Massed & Distributed Practice** (*Carmine, 1997*). Teachers can best promote students acquisition and fluency in a newly taught math skill by transitioning from massed to distributed practice. When students have just acquired a math skill but are not yet fluent in its use, they need lots of opportunities to try out the skill under teacher supervision—a technique sometimes referred to as ‘massed practice’. Once students have developed facility and independence with that new math skill, it is essential that they then be required periodically to use the skill in order to embed and retain it—a strategy also known as ‘distributed practice’. Teachers can program distributed practice of a math skill such as reducing fractions to least common denominators into instruction either by (a) regularly requiring the student to complete short assignments in which they practice that skill in isolation (e.g., completing drill sheets with fractions to be reduced), or (b) teaching a more advanced algorithm or problem-solving approach that incorporates—and therefore requires repeated use of—the previously learned math skill (e.g., requiring students to reduce fractions to least-common denominators as a necessary first step to adding the fractions together and converting the resulting improper fraction to a mixed number).

**Math Review: Promote Success Through Incremental Rehearsal** (*Burns, 2005*). Incremental rehearsal builds student fluency in basic math facts by pairing unknown computation items with a steadily increasing collection of known items. The tutor first writes down each math fact that a student should learn on an index card in ink—but without the answer. Then the tutor does a preliminary review with the student of the total collection of math facts on the index cards. Any of the math facts that the student can answer correctly within two seconds are considered to be ‘known’ problems and placed into the ‘known’ pile. Math facts that the student cannot answer correctly within two seconds are considered ‘unknown’ and placed into the ‘unknown’ pile. The tutor is now ready to follow a nine-step incremental-rehearsal sequence: First, the tutor presents the student with a single index card containing an ‘unknown’ math fact. The tutor reads the problem aloud, gives the answer, then prompts the student to read off the same unknown problem and provide the correct answer. Next the

tutor takes a math fact from the 'known' pile and pairs it with the unknown problem. When shown each of the two problems, the student is asked to read off the problem and answer it. The tutor then repeats the sequence--adding yet another known problem to the growing deck of index cards being reviewed and each time prompting the student to read off and answer the whole series of math facts--until the review deck has expanded to contain a total of one 'unknown' math fact and nine 'known' math facts (a ratio of 90 percent 'known' material to 10 percent 'unknown' material ) At this point, the last 'known' math fact that had been added to the student's review deck is discarded (placed back into the original pile of 'known' problems) and the previously 'unknown' math fact is now treated as the first 'known' math fact in new student review deck for future drills. The student is then presented with a new 'unknown' math fact to answer--and the review sequence is once again repeated each time until the 'unknown' math fact is grouped with nine 'known' math facts--and on and on. Daily review sessions are discontinued either when time runs out or when the student answers an 'unknown' math fact incorrectly three times.

## References

- Baxter, J. A., Woodward, J., & Olson, D. (2005). Writing in mathematics: An alternative form of communication for academically low-achieving students. *Learning Disabilities Research & Practice, 20*(2), 119-135.
- Burns, M. K. (2005). Using incremental rehearsal to increase fluency of single-digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children, 28*, 237-249.
- Carnine, D. (1997). Instructional design in mathematics for students with learning disabilities. *Journal of Learning Disabilities, 30*, 130-141.
- Caron, T. A. (2007). Learning multiplication the easy way. *The Clearing House, 80*, 278-282.
- Hawkins, J., & Brady, M. P. (1994). The effects of independent and peer guided practice during instructional pauses on the academic performance of students with mild handicaps. *Education & Treatment of Children, 17* (1), 1-28.
- Hawkins, J., Skinner, C. H., & Oliver, R. (2005). The effects of task demands and additive interspersal ratios on fifth-grade students' mathematics accuracy. *School Psychology Review, 34*, 543-555.
- Montague, M. (1997). Cognitive strategy instruction in mathematics for students with learning disabilities. *Journal of Learning Disabilities, 30*, 164-177.
- Montague, M., Warger, C., & Morgan, T. H. (2000). Solve it! Strategy instruction to improve mathematical problem solving.. *Learning Disabilities Research & Practice, 15*, 110-116.
- Rhymer, K. N., Skinner, C. H., Jackson, S., McNeill, S., Smith, T., & Jackson, B. (2002). The 1-minute explicit timing intervention: The influence of mathematics problem difficulty. *Journal of Instructional Psychology, 29*(4), 305-311.
- Shimabukuro, S. M., Prater, M. A., Jenkins, A., & Edelen-Smith, P. (1999). The effects of self-monitoring of academic performance on students with learning disabilities and ADD/ADHD. *Education and Treatment of Children, 22*, 397-414.
- Skinner, C. H., Pappas, D. N., & Davis, K. A. (2005). Enhancing academic engagement: Providing opportunities for responding and influencing students to choose to respond. *Psychology in the Schools, 42*, 389-403.
- Van Essen, G., & Hamaker, C. (1990). Using self-generated drawings to solve arithmetic word problems. *Journal of Educational Research, 83*, 301-312.
- Woodward, J. (2006). Developing automaticity in multiplication facts integrating strategy instruction with timed practice drills. *Learning Disability Quarterly, 29*, 269-289.
- Zrebiec Uberti, H., Mastropieri, M. A., & Scruggs, T. E. (2004). Check it off: Individualizing a math algorithm for students with disabilities via self-monitoring checklists. *Intervention in School and Clinic, 39*, 269-275.

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# Schoolwork Motivation Assessment

(adapted from Witt & Beck, 1999; Witt, VanDerHeyden & Gilbertson, 2004)

Student: \_\_\_\_\_ Teacher/Classroom: \_\_\_\_\_

Date of Assessment: \_\_\_/\_\_\_/\_\_\_ Person Completing Assessment: \_\_\_\_\_

<p><b>Step 1: Assemble an incentive menu.</b> Create a 4-5 item menu of modest incentives or rewards that students in the class are most likely to find motivating. Examples of popular incentives include:</p> <ul style="list-style-type: none"> <li>• small prizes such as pencils or stickers,</li> <li>• 5 minutes of extra free time,</li> <li>• an opportunity to play a computer game,</li> <li>• praise note or positive phone call to parent</li> </ul>	<p style="text-align: center;">Incentive / Reward Menu</p> <p>Idea 1: _____</p> <p>Idea 2: _____</p> <p>Idea 3: _____</p> <p>Idea 4: _____</p> <p>Idea 5: _____</p>
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**Step 2: Create two versions of a CBM probe or timed worksheet.** Make up two versions of a structured, timed worksheet with items of the type that the student appears to find challenging. Use one of the options below:

Option 1: Create Curriculum-Based Measurement probes. The probes should be at the same level of difficulty, but each probe should have different items or content to avoid a practice effect. NOTE: CBM probes in oral reading fluency, math computation, writing, and spelling can all be used.

Option 2: Make up two versions of custom student worksheets. The worksheets should be at the same level of difficulty, but each worksheet should have different items or content to avoid a practice effect. NOTE: If possible, the worksheets should contain standardized short-answer items (e.g., matching vocabulary words to their definitions) to allow you to calculate the student's rate of work completion.

**Step 3: Administer the first CBM probe or timed worksheet to the student WITHOUT incentives.** In a quiet, non-distracting location, administer the first worksheet or CBM probe under timed, standardized conditions. Collect the probe or worksheet and score.

<p><b>Step 4: Compute an improvement goal.</b> After you have scored the first CBM probe or worksheet, compute a '20 percent improvement goal'. Multiply the student's score on the worksheet by 1.2. This product represents the student's minimum goal for improvement.</p>	<p>Student Score on First CBM Probe or Worksheet _____</p> <p style="text-align: right;">Multiplied by: <b>1.2</b></p> <p>Yields an improvement goal of: _____</p>
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**Step 5: Have the student select an incentive for improved performance.** Tell the student that if he or she can attain a score on the second worksheet that meets or exceeds your goal for improvement (Step 3), the student can earn an incentive. Show the student the reward menu. Ask the student to select the incentive that he or she will earn if the student makes or exceeds the goal.

**Step 6: Administer the second timed worksheet to the student WITH incentives.** Give the student the second CBM probe. Collect and score. If the student meets or exceeds the pre-set improvement goal, award the student the incentive.

Student Score on Second  
CBM Probe or Worksheet \_\_\_\_\_

Compared to:

Improvement goal of: \_\_\_\_\_

**Step 7: Interpret the results of the academic motivation assessment to select appropriate interventions.** Use the decision-rules below to determine recommended type(s) of intervention:

- ❑ **ACADEMIC INTERVENTIONS ONLY.** If the student fails to meet or exceed the improvement goal, an academic intervention should be selected to teach the appropriate skills or to provide the student with drill and practice opportunities to build fluency in the targeted academic area(s).
- ❑ **COMBINED ACADEMIC AND PERFORMANCE INTERVENTIONS.** If the student meets or exceeds the improvement goal but continues to function significantly below the level of classmates, an intervention should be tailored that includes strategies to both improve academic performance and to increase the student's work motivation. The academic portion of the intervention should teach the appropriate skills or to provide the student with drill and practice opportunities to build fluency in the targeted academic area(s). Ideas for performance interventions include (a) providing the student with incentives or 'pay-offs' for participation and/or (b) structuring academic lessons around topics or functional outcomes valued by the student.
- ❑ **PERFORMANCE INTERVENTIONS ONLY.** If the student meets or exceeds the improvement goal with an incentive and shows academic skills that fall within the range of 'typical' classmates, the intervention should target only student work performance or motivation. Ideas for performance interventions include (a) providing the student with incentives or 'pay-offs' for participation and/or (b) structuring academic lessons around topics or functional outcomes valued by the student.

References:

Witt, J., & Beck, R. (1999). *One minute academic functional assessment and interventions: "Can't" do it...or "won't" do it?* Longmont, CO: Sopris West.

Witt, J. C., VanDerHeyden, A. M., Gilbertson, D. (2004). Troubleshooting behavioral interventions: A systematic process for finding and eliminating problems. *School Psychology Review*, 33, 363-381.