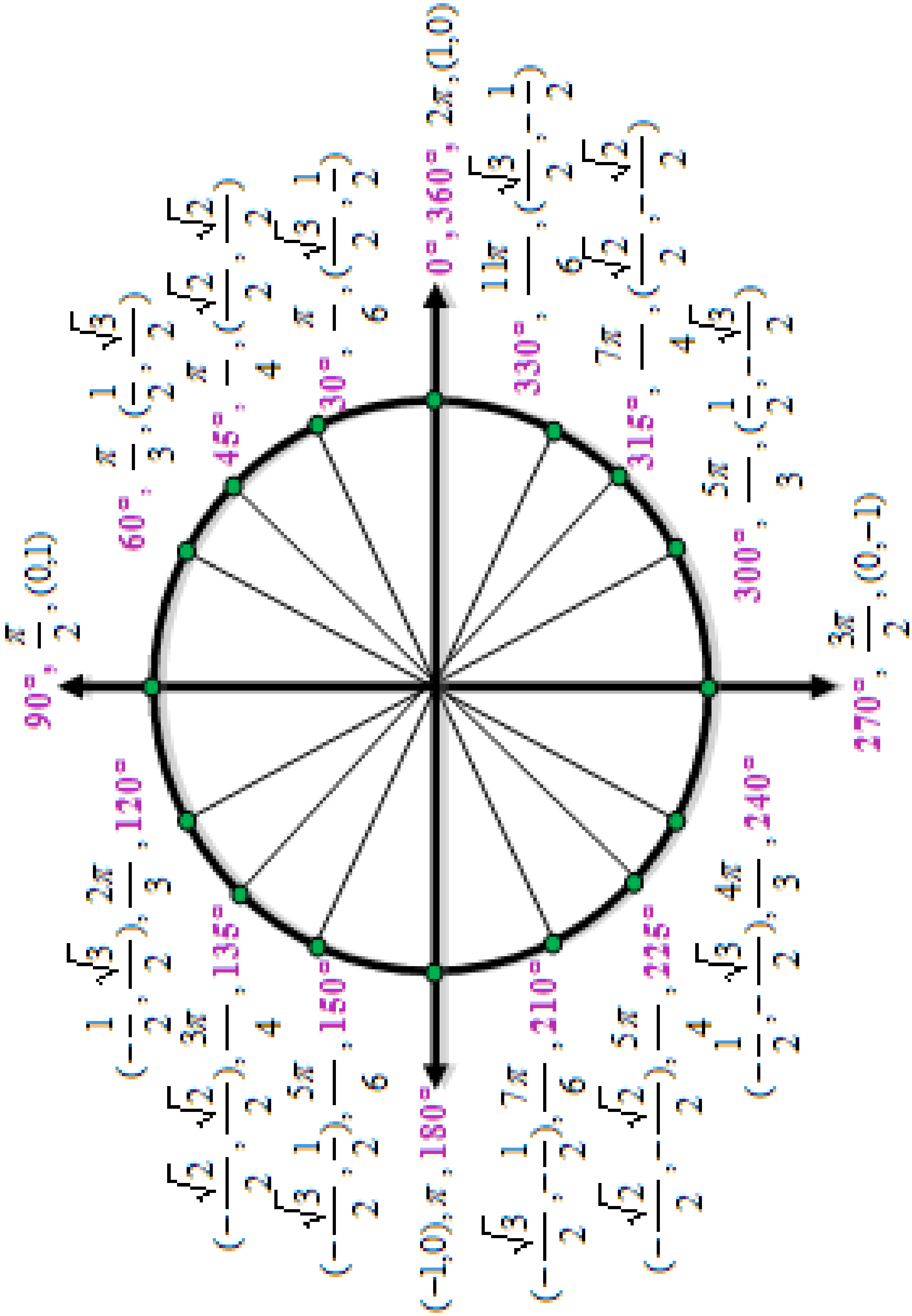
<u>Unit 3: Circle Trig</u>

<u>Objective 2.04</u> Use trigonometric (sine, cosine) functions to model and solve problems; justify results.

- Solve using tables, graphs, and algebraic properties.
- Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.

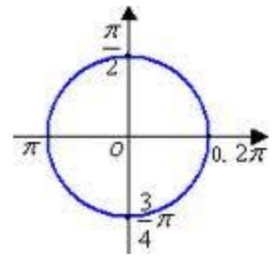
Day	Topic	Students will be able to:	Activity
1	5.1 Angle and Radian Measure	• Convert between degrees and radians. • Angles in standard position	
2	5.1 Cont'd	• Reference Angles • Reference Triangles	
3	5.1 Angle and Radian Measure	• Arc Length • Area of Sector • Use linear and angular speed to describe motion on a circular path.	
4	8.1 Unit Circle	• Intro to Unit Circle	
5	5.3/5.4Trig Functions of any Angle	• Use a unit circle to define trig functions of real numbers. • Use reference angles to evaluate trig functions.	**QUIZ 5.1 (Day 1-3)**
6	5.5 Graphs of Sine and Cosine Functions	• Understand the graph of y = sinx. • Understand the graph of y = cosx.	Discovery Lesson – Explore graphs of sine and cosine.
7	5.5 Graphs of Sine and Cosine Functions	• Graph variations of y = sinx and y = cosx. • Use vertical shifts.	
8	5.6 Sinusoidal Applications	• Sinusoidal Applications	
9		Review	**Review**
10	**TEST**	**Show mastery on circular trigonometry!** ☺	**TEST**


$0°, 360°, 2\pi, (1,0)$
$30°, \frac{\pi}{6}, (\frac{\sqrt{3}}{2}, \frac{1}{2})$
$45°, \frac{\pi}{4}, (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$60°, \frac{\pi}{3}, (\frac{1}{2}, \frac{\sqrt{3}}{2})$
$90°, \frac{\pi}{2}, (0,1)$
$(-\frac{1}{2}, \frac{\sqrt{3}}{2}), \frac{2\pi}{3}, 120°$
$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}), \frac{3\pi}{4}, 135°$
$(-\frac{\sqrt{3}}{2}, \frac{1}{2}), \frac{5\pi}{6}, 150°$
$(-1,0), \pi, 180°$
$(-\frac{\sqrt{3}}{2}, -\frac{1}{2}), \frac{7\pi}{6}, 210°$
$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}), \frac{5\pi}{4}, 225°$
$(-\frac{1}{2}, -\frac{\sqrt{3}}{2}), \frac{4\pi}{3}, 240°$
$270°, \frac{3\pi}{2}, (0,-1)$
$300°, \frac{5\pi}{3}, (\frac{1}{2}, -\frac{\sqrt{3}}{2})$
$315°, \frac{7\pi}{4}, (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
$330°, \frac{11\pi}{6}, (\frac{\sqrt{3}}{2}, -\frac{1}{2})$

Angle and Radian Measure

Measurements of Angles: Until now you have measured angles in degrees. Another unit for measuring angles is _______________.

Radians:



-The circumference of a circle with radius 1 is ___________ so a complete revolution has made __________ radians.

-A straight angle (or _________ of a circle) has measure ____________ radians.

Converting Radians and Degrees:

$$\text{Radians} = \left(\frac{\pi}{180^0}\right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^0}{\pi}\right) \times \text{radians}$$

Examples:

1. Express 60^o in radians

2. Express $\frac{\pi}{6}$ rad in degrees

Practice:

#1-8, change the given angle to radians.

1) 315°

2) -60°

3) 212°

4) -168°

5) 12.5°

6) -310°

7) 600°

8) -720°

#9-16, change the given angle to degrees.

9) $\frac{3\pi}{4}$

10) $-\frac{9\pi}{5}$

11) $\frac{15\pi}{8}$

12) $-\frac{\pi}{10}$

13) $\frac{7\pi}{10}$

14) $-\frac{16\pi}{15}$

15) $\frac{88\pi}{9}$

16) $-\frac{29\pi}{12}$

"Why were the screams coming from the kitchen?"

Make the conversion from degrees to radians and from radians to degrees. To figure out the joke, place the letter of each problem above the answer on the line(s) below.

Convert #1-6 from degrees to radians.

E. 20° A. 120° K. -145°

H. 320° G. -245° B. 630°

Convert #7-13 from radians to degrees.

N. $\frac{\pi}{4}$ W. $\frac{2\pi}{5}$ T. $\frac{-5\pi}{60}$

S. $\frac{7\pi}{6}$ C. $\frac{13\pi}{3}$ I. $\frac{-5\pi}{4}$ O. $\frac{11\pi}{3}$

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

-15° $\frac{16\pi}{9}$ $\frac{\pi}{9}$ 780° 660° 660° $\frac{-29\pi}{36}$ 72° $\frac{2\pi}{3}$ 210°

___ ___ ___ ___ ___ ___ ___

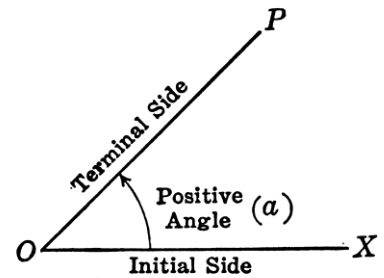
$\frac{7\pi}{2}$ $\frac{\pi}{9}$ $\frac{2\pi}{3}$ -15° -225° 45° $\frac{-49\pi}{36}$

___ ___ ___ ___ ___ ___ ___

-15° $\frac{16\pi}{9}$ $\frac{\pi}{9}$ $\frac{\pi}{9}$ $\frac{-49\pi}{36}$ $\frac{-49\pi}{36}$ 210°

Angles in Standard Position

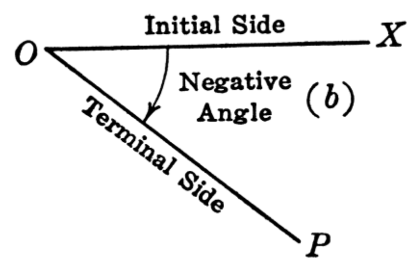
Angle:



Initial Side:

Terminal Side:

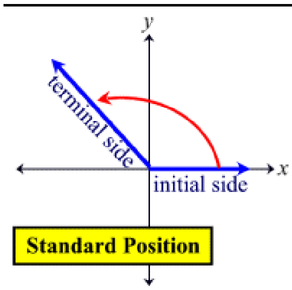
Positive Angle:



Negative Angle:

An angle is in ______________________________ if it is draw in the xy-plane with its ________________ at the

___________ and its initial side on the _________________________________.



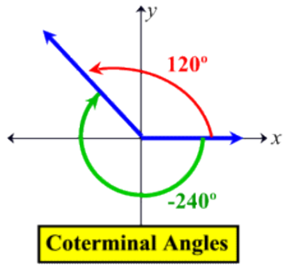
Example: Draw the given angle in standard position. State the quadrant the terminal side is in.

1. 45^0 2. 225^0 3. 270^0 4. -60^0

5. 750^0 6. -150^0 7. 180^0 8. -75^0

Coterminal Angles

Two angles in ____________________ are __________________ if their sides coincide.



To find angles that are coterminal, add any multiple of __________ for degrees or _____________ for radians.

Examples:

1. Find three angles that are coterminal with the angle $\theta = 30°$ in standard position
2. Find three angles that are coterminal with the angle $\theta = \frac{\pi}{3}$ in standard position
3. Find an angle with a measure between 0° and 360° that is coterminal with the angle of measure 1290° in standard position.

Find a coterminal angle between 0° and 360°.

13) $-330°$

14) $-435°$

15) $640°$

16) $-442°$

Find a coterminal angle between 0 and 2π for each given angle.

17) $\frac{11\pi}{3}$

18) $-\frac{35\pi}{18}$

19) $\frac{15\pi}{4}$

20) $-\frac{19\pi}{12}$

Find a positive and a negative coterminal angle for each given angle.

21) $\frac{5\pi}{4}$

22) $\frac{25\pi}{36}$

Warm Up: Day 1 Review **Name: ____________________**

Convert from degrees to radians.

1.212° 2. -85° 3. 415°

Convert from radians to degrees

4.$-\frac{3\pi}{4}$ 5.$\frac{15\pi}{19}$ 6. $\frac{6\pi}{7}$

Draw the angle in standard position and state which quadrant the terminal side is in.

7. 275° 8. -105° 9. 480° 10. 170°

11. $-\frac{3\pi}{4}$ 12. $\frac{9\pi}{2}$ 13. $\frac{13\pi}{14}$ 14. $\frac{7\pi}{4}$

Find a coterminal angle between 0° and 360°

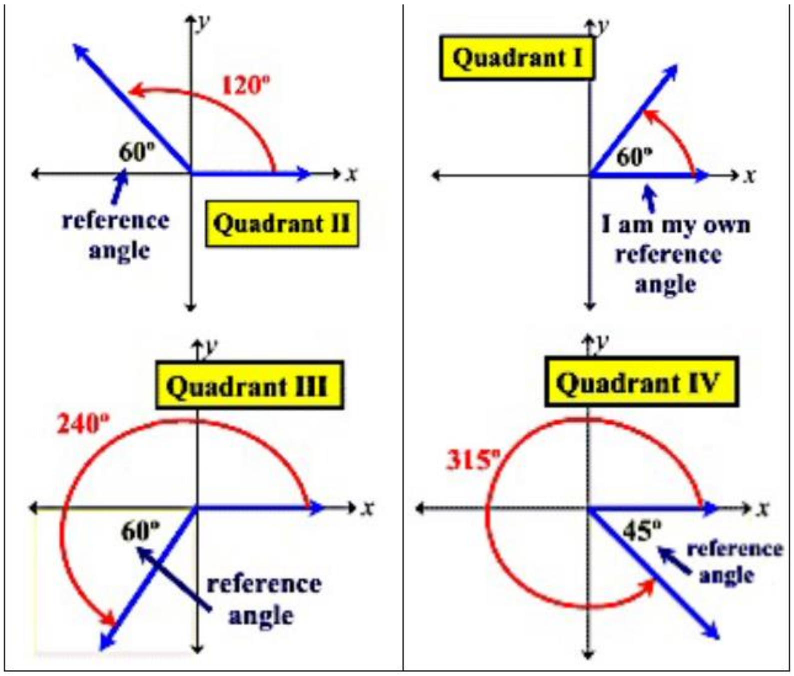
15. 1770° 16. -412°

Find a coterminal angle between 0 and 2**π**

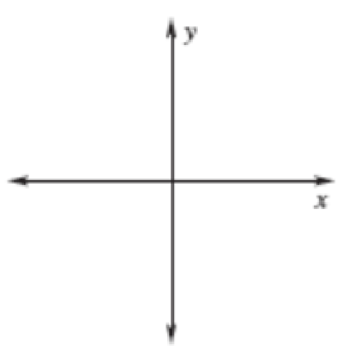
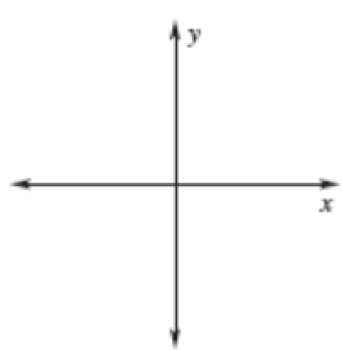
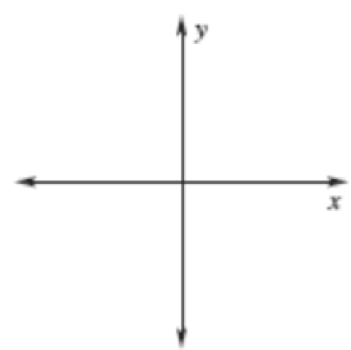
17. $-\frac{15\pi}{4}$ 18. $\frac{27\pi}{5}$

Day 2: Reference Angles

An angle drawn in standard position has a ________________________. The reference angle is an acute angle formed by the terminal side of the given angle to the ________________.

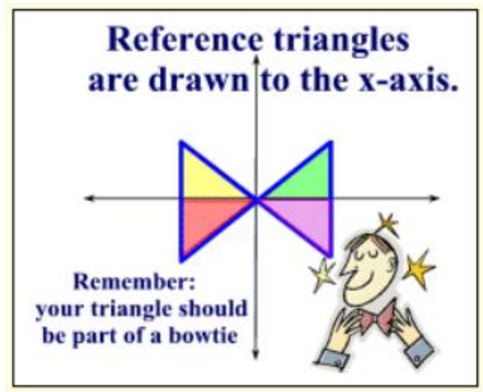


Examples: Draw the angle in standard position and then find the reference angle.

a) 150°	b) $\frac{7\pi}{6}$	c) -100°
		

Reference Triangle

- Formed by "dropping" a perpendicular from the terminal ray of a standard position angle to the x-axis.



Example 1: If ⊖ is an angle in standard position and P(-3, 4) is a point on the terminal side of ⊖, what is the value of cos⊖?

Example 2: If ⊖ is an angle in standard position and P(3, -2) is a point on the terminal side of ⊖, what is the value of csc⊖?

Practice:

1. If ⊖ is an angle in standard position and P(-4, 3) is a point on the terminal side of ⊖, what is the value of sin⊖?

2. If the terminal side of ⊖ passes through point (-8, -6), what is the value cos⊖?

Sketch the angle in standard position in the coordinate plane that passes through each given point, and find all six trigonometric ratios for that point.

3) (7, 24)

4) (-4, 0)

5) (8, 15)

6) (-2, -2)

7) (5, -12)

8) (-3, $3\sqrt{3}$)

Practice with Terminal Points

Let P be a point on the terminal side of θ. Find the 6 trig functions for the angle.

1.

a) $P(-8,6)$

b) $P(7,-24)$

c) $P(-2,-2)$

d) $P(-1,\sqrt{3})$

2.

a) $P(3,4)$

b) $P(15,8)$

c) $P(5,2)$

d) $P(1,7)$

e) $P(1,1)$

e) $P(\sqrt{2},\sqrt{7})$

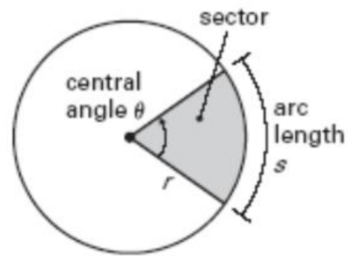
3. If ⊖ is an angle in standard position and P(-5, 13) is a point on the terminal side of ⊖, what is the value of sec⊖ and cot⊖?

4. If the terminal side of ⊖ passes through point (-5, -2), what is the value cos⊖ and sin⊖?

Day 3: Arc Length

The arc length, s, of a sector of a circle with radius r and central angle ⊖ measured in radians, is

$$\boxed{\text{Arc length: } s = r\theta}$$



Examples:

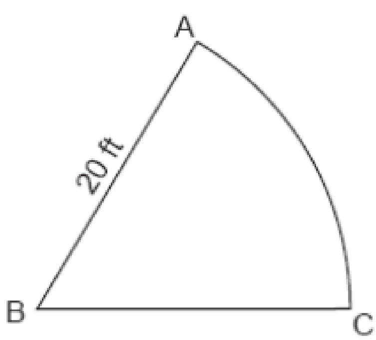
1. Find the length of an arc of a circle with radius 10 m that subtends a central angle of 30º.

2. A central angle θ in a circle of radius 4 m is subtended by an arc of length 6 m. Find the measure of θ in radians and in degrees.

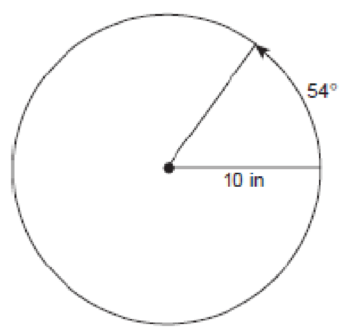
3. Memphis, TN and New Orleans, LA lie approximately on the same meridian. Memphis has latitude 35ºN and New Orleans 30ºN. Find the distance between the two cities. (Radius of earth is 3960 miles)

Practice:

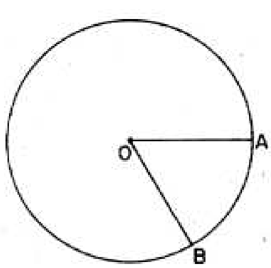
1. A sprinkler system is set up to water the sector shown in the accompanying diagram, with angle *ABC* measuring 1 radian and radius $AB = 20$ feet. What is the length of arc *AC*, in feet?



2. A ball is rolling in a circular path that has a radius of 10 inches, as shown in the accompanying diagram. What distance has the ball rolled when the subtended arc is 54°? Express your answer to the *nearest hundredth of an inch.*



3. In circle O, the length of radius $\overline{OB}$ is 5 centimeters and the length of $\overset{\frown}{AB}$ is 5 centimeters. What is the measure of $\angle AOB$?



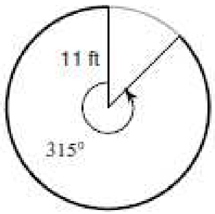
4. An arc of a circle measures 30 centimeters and the radius measures 10 centimeters. In radians, what is the measure of the central angle that subtends the arc?

Arc Length and Sector Area

Date________________ Period____

Find the length of each arc. Round your answers to the nearest tenth.

1)



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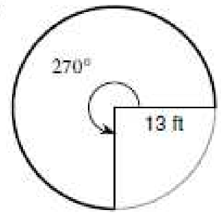
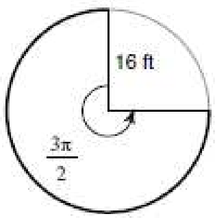
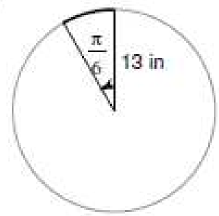


3)



4)



5) $r = 18$ cm, $\theta = 60°$

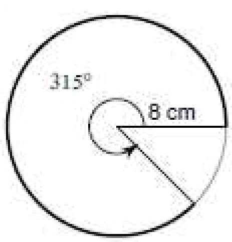
6) $r = 16$ m, $\theta = 75°$

7) $r = 9$ ft, $\theta = \dfrac{7\pi}{4}$

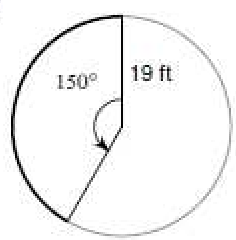
8) $r = 14$ ft, $\theta = \dfrac{19\pi}{12}$

Find the length of each arc. Do not round.

9)



10)



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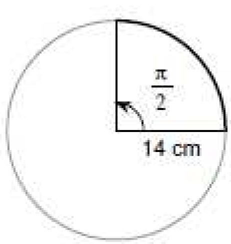

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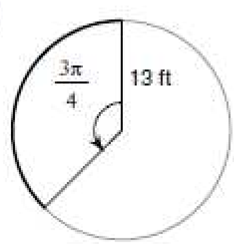


Area of a Circular Sector

Do you recall the area of a circle? A sector of this circle with central angle θ (in radians) has an area that is a fraction of the area of the entire circle. Again, if angle is given in degrees, you must convert to radians.

$$A = \frac{1}{2} r^2 \theta$$

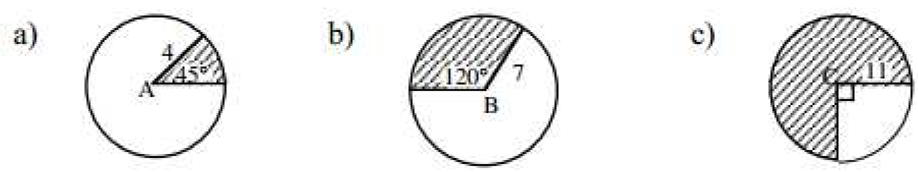
Examples:

1. Find the area of a sector of a circle with a central angle of 45º if the radius is 2 m.

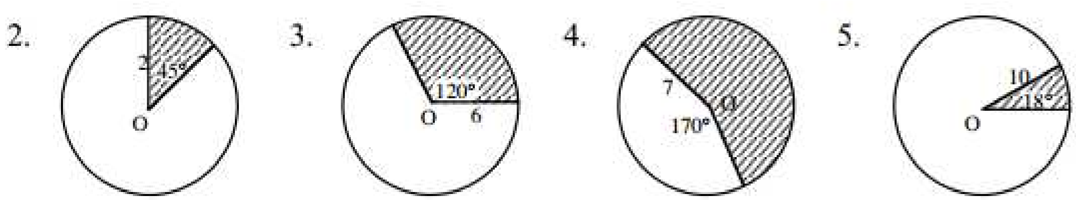
2. Find the radius of the circle if the area of a sector of a circle with a central angle of 4 radians is 2 m

Practice:

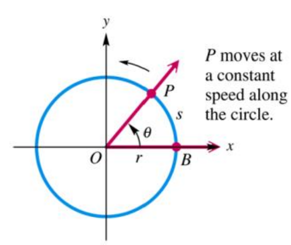
1. Find the area of the shaded sector in each circle below. Points A, B and C are the centers.



Calculate the area of the following shaded sectors. Point O is the center of each circle.



Angular and Linear Speed (Velocity)



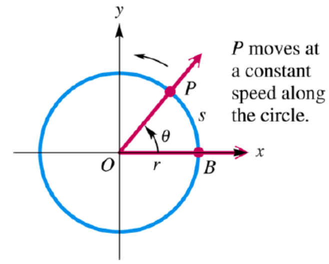
Sometimes it is important to know how fast a point is moving (__________________________) or how fast a central angle is changing (___________________________)

Linear Speed (Velocity): If P is a point on a circle of radius r, and P moves a distance s on the circumference of the circle in an amount of time t, then the ***linear velocity***, v, of P is given by the formula

$$speed = \frac{distance}{time} \qquad or \qquad v = \frac{s}{t}$$

Example: A runner of a 4.2 mile race finished in 28 minutes and 4 seconds. What was the runners average velocity in miles per hour?

Angular Speed (Velocity): The measure of how fast an angle is changing, *angular velocity*, ω(omega)



$$\omega = \frac{\theta}{t} \qquad where\ \theta\ is\ measured\ of\ angle\ in\ radians\ at\ time, t$$

Example: A mechanical arm rotates 1/3 of a rotation in 0.25 seconds. Determine the angular speed in radians per second.
(One rotation is 2pi radians—change to radians)

Linear Speed Around a Circle

Example: A tire with radius of 9 inches is spinning at 80 revolutions per minute.

a) Find the angular speed of the tire in radians per second

$$\omega = \frac{\theta}{t}$$

b) Find the linear speed in miles per minute

$$v = \frac{r\theta}{t}$$

Practice:
1. A record is spinning at the rate of 25 rpm. If a ladybug is sitting 10 cm from the center of the record:

a) What is the angular velocity of the ladybug? (in radians/sec)
b) What is the speed of the ladybug? (in cm/sec)
c) After 20 seconds, how far has the ladybug traveled? (in cm)
d) After 20 seconds, what angle has the ladybug turned through? (in radians)

2. An ant sits on a cd at a distance of 17 cm from the center. If it sits there for 42 seconds, it travels a total distance of 913 cm.

a) What angle has the ant turned through? (in radians)
b) What speed has the ant been traveling at? (in cm/sec)
c) What angular velocity has the ant been spinning at? (in radians/sec)
d) What rpm is the cd turning at?

Day 3 Angles and Radian Measure Applications

1. Find the distance s covered by a point moving with linear velocity $v = 55$ mi/hr and $t = 0.5$ hr.

2. A bicycle traveled a distance of 100 meters. The diameter of the wheel of this bicycle is 40 cm. Find the number of rotations of the wheel.

3. The wheel of a car made 100 rotations. What distance has the car traveled if the diameter of the wheel is 60 cm?

4. The wheel of a machine rotates at the rate of 300 rpm (rotation per minute). If the diameter of the wheel is 80 cm, what are the angular (in radian per second) and linear speed (in cm per second) of a point on the wheel?

5. The Earth rotates about its axis once every 24 hours (approximately). The radius R of the equator is approximately 4000 miles. Find the angular (radians / second) and linear (feet / second) speed of a point on the equator.

6. The diameter of the Ferris wheel is 250 *ft*, the distance from the ground to the bottom of the wheel is 14 *ft*, and one complete revolution takes 20 minutes, find
   a. The linear velocity, in miles per hour, of a person riding on the wheel.

b. The height of the rider in terms of the time *t*, where *t* is measured in minutes.

7. A man rides a bicycle for 2 hr and travels 15 km. find the angular velocity of the wheel if the radius is 30 cm.

8. A lawnmower blade of 1.5 feet rotates 55 rpm has a linear velocity of what?

9. A 78 rpm Bing Crosby record has a radius of 6 inches. What is the linear velocity?

10. A boy is twirling a model airplane on a string 5 ft long. If he twirls the plane at 0.6rpm, how far does the plane travel in 2 min?

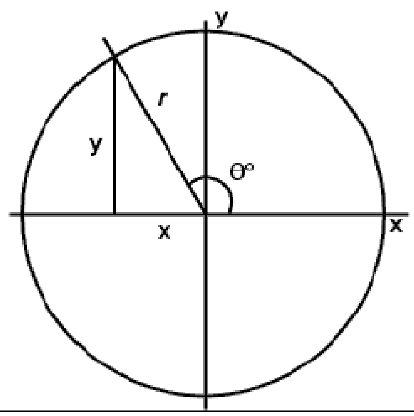
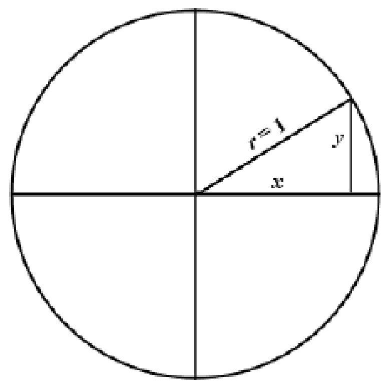
11. Find the angular velocity associated with the given rpm of 16 and 2/3 rpm

12. How far does the tip of a 11 cm minute hand on a clock travel in 1 day?

Day 4 Notes : The Unit Circle

The ________________________ is the circle of radius 1 centered at the origin in the xy-plane.

The equation of the unit circle is __________________________

If the point (x, y) is on a circle with radius r, then:	In a unit circle, the radius is one, so:
 $\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$	 $\cos\theta = \frac{x}{1} = x$ $\sin\theta = \frac{y}{1} = y$

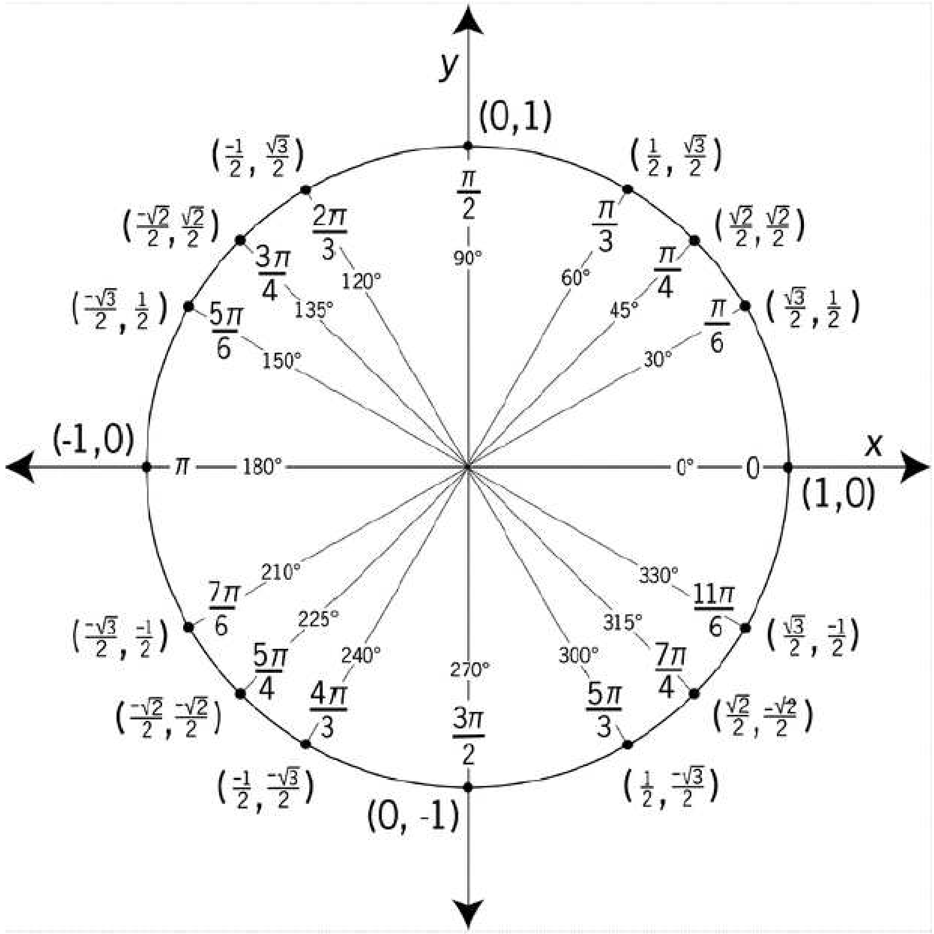
Using the unit circle makes finding sine and cosine (and other trig functions) simpler since cos ⊖ = x and sin ⊖ = y

There are important values you need to know exact values of sine and cosine and be able to use those to find the other trig values.

θ (in degrees)	0°	30°	45°	60°	90°	180°	270°
θ (in radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$ (0.5)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{\sqrt{3}}{2}$ (0.866)	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$ (0.866)	$\frac{\sqrt{2}}{2}$ (0.707)	$\frac{1}{2}$ (0.5)	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (0.577)	1	$\sqrt{3}$ (1.732)	undefined	0	undefined

It may be helpful for you to look at the unit circle. You should memorize the first quadrant and then be able to figure out the other quadrants.

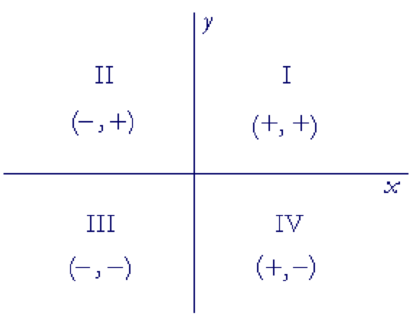
Unit Circle:



Things to remember about the unit circle:

- **Cos** = “x” values, **Sin** = “y” values, and **Tan** = $\frac{y}{x}$ values.
- <u>A</u>ll <u>S</u>tudents <u>T</u>ake <u>C</u>lasses 1st Quadrant (____________), 2nd quadrant (______________),

  3rd Quadrant (____________), 4th quadrant (______________).



- The main points you need to learn are in the first quadrant because everything is derived from the 1st quadrant.

Unit Circle Practice
Example: Find the exact trig value for the following:

1. a) $\sin 0$ b) $\cos 0$ 2. a) $\sin(-\pi)$ b) $\cos(-\pi)$

Practice: Find the exact value of the trigonometric function at the given real number. Do NOT use your calculator!

1. a) $\sin(\pi/2)$ b) $\sin(3\pi/2)$ 2. a) $\cos(7\pi/3)$ b) $\sec(7\pi/3)$

3. a) $\sec(11\pi/3)$ b) $\csc(11\pi/3)$ 4. a) $\tan(-\pi/4)$ b) $\cot(-\pi/4)$

Example: From the information given, determine the quadrant in which the point lies.
Cos $t > 0$ and Tan $t < 0$

Practice:

1. From the information given, find the quadrant in which the terminal point determined by t lies.

(a) $\sin t > 0$ and $\cos t < 0$

(b) $\tan t > 0$ and $\sin t < 0$

2. In what quadrant is…

a) $\sin\theta > 0$ and $\cos\theta < 0$

b) $\sec\theta < 0$ and $\cot\theta < 0$

c) $\csc\theta < 0$ and $\cos\theta > 0$

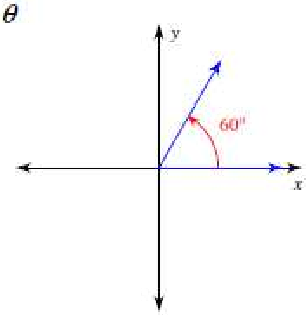
d) all trig functions are negative?

Exact Trig Values of Special Angles

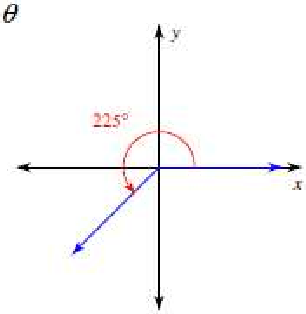
Date_______________ Period____

Find the exact value of each trigonometric function.

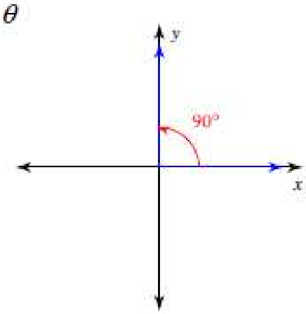
1) $\tan \theta$



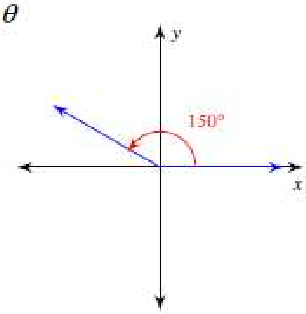
2) $\sin \theta$



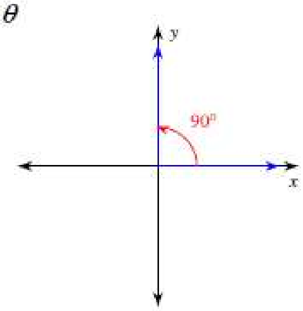
3) $\sin \theta$



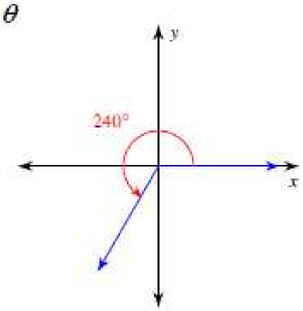
4) $\cos \theta$



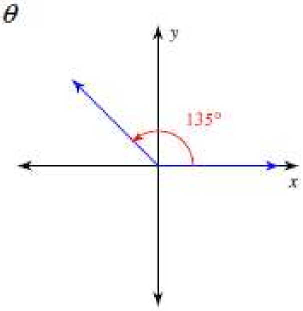
5) $\cos \theta$



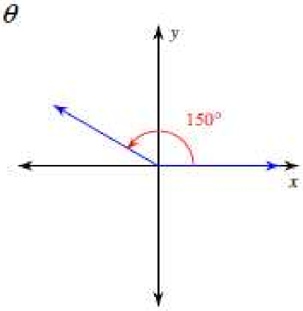
6) $\tan \theta$



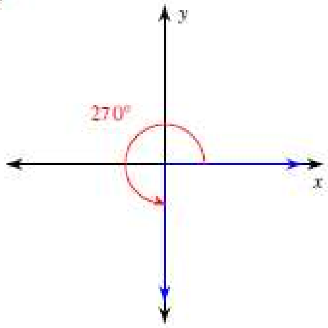
7) $\cos \theta$



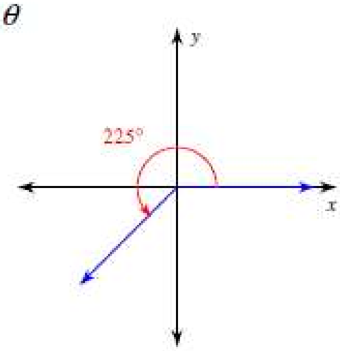
8) $\tan \theta$



9) $\cos \theta$



10) $\tan \theta$



11) $\cos 270°$

12) $\sin 0$

13) $\cot \frac{7\pi}{4}$

14) $\csc \frac{2\pi}{3}$

15) $\csc 225°$

16) $\sin 300°$

17) $\csc 90°$

18) $\tan 240°$

19) $\sin \frac{\pi}{4}$

20) $\tan 120°$

21) $\tan -\frac{13\pi}{6}$

22) $\cos -630°$

23) $\cos 990°$

24) $\csc -\frac{31\pi}{6}$

25) $\csc -\frac{5\pi}{6}$

26) $\cos -\frac{17\pi}{3}$

27) $\sin \frac{29\pi}{6}$

28) $\sec 945°$

29) $\cos -\frac{11\pi}{2}$

30) $\sin -2\pi$

Graphs of Sine and Cosine

From your discovery activity yesterday, you should have discovered that sine and cosine values repeat themselves. Thus, the sine and cosine functions are ____________________.

A function is ____________________ if there is a positive number p such that f(t + p) = f(t) for every t. The least such positive number is called the __________.

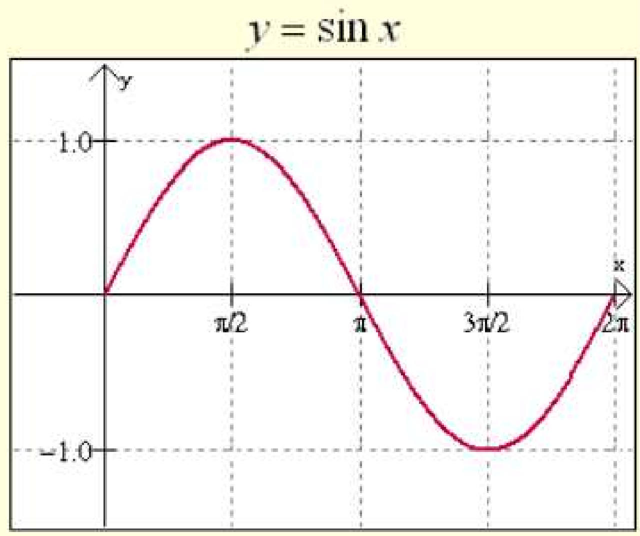
Points you should know from the unit circle

x	Angle in Radians	0	π/2	π	3π/2	2π
y	sin x					

Sine Function: $y = \sin x$

- called a "wave" because of its rolling wave-like appearance (also referred to as oscillating)
- amplitude: 1 (height)
- period: 2π (length of one cycle)
- frequency: 1 cycle in 2π radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \le y \le 1\}$

At $x = 0$, the sine wave is on the shoreline! (meaning the y-value is equal to zero)

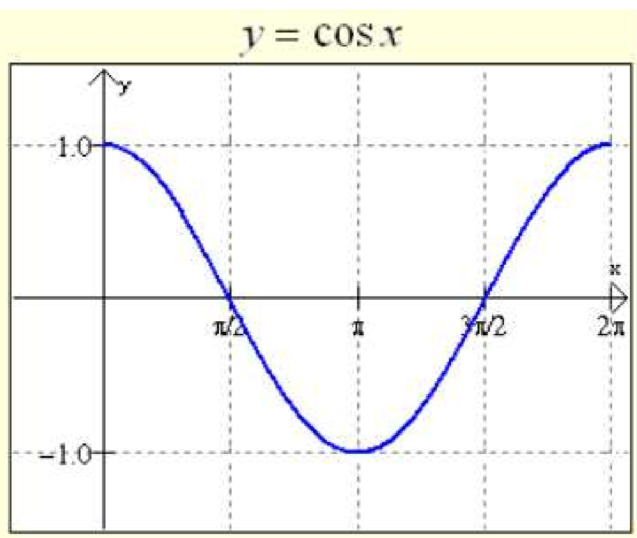


x	Angle in Radians	0	π/2	π	3π/2	2π
y	cos x					

Cosine Function: $y = \cos x$

- called a "wave" because of its rolling wave-like appearance
- amplitude: 1
- period: 2π
- frequency: 1 cycle in 2π radians [or $1/(2\pi)$]
- domain: $\{x \mid x \in \mathbb{R}\}$
- range: $\{y \mid -1 \le y \le 1\}$

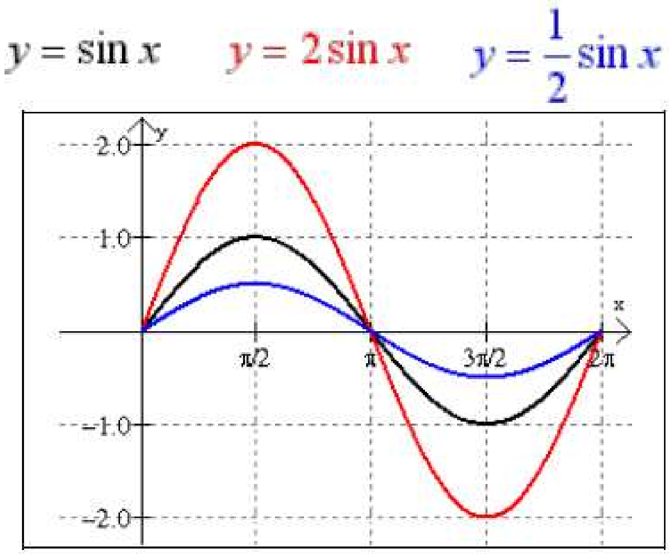
At $x = 0$, the cosine wave breaks off the cliff! (meaning the y-value is equal to one)

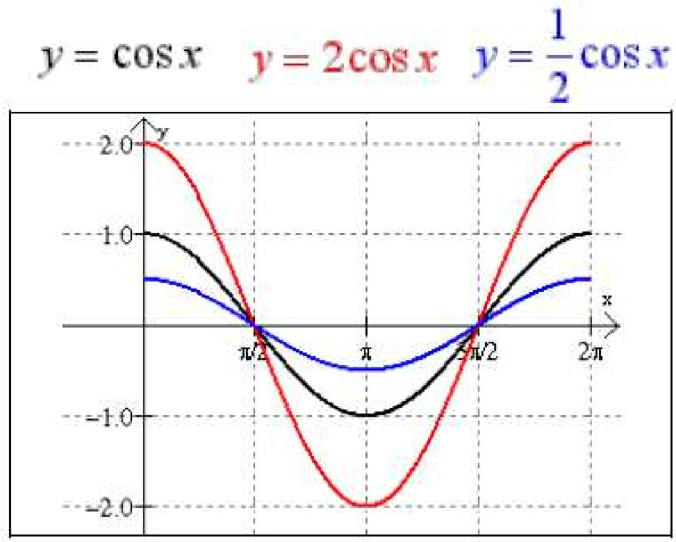


The sine and cosine curves

$$y = A\sin(B(x - C)) + D \ \ and \ y = A\cos(B(x - C)) + D$$

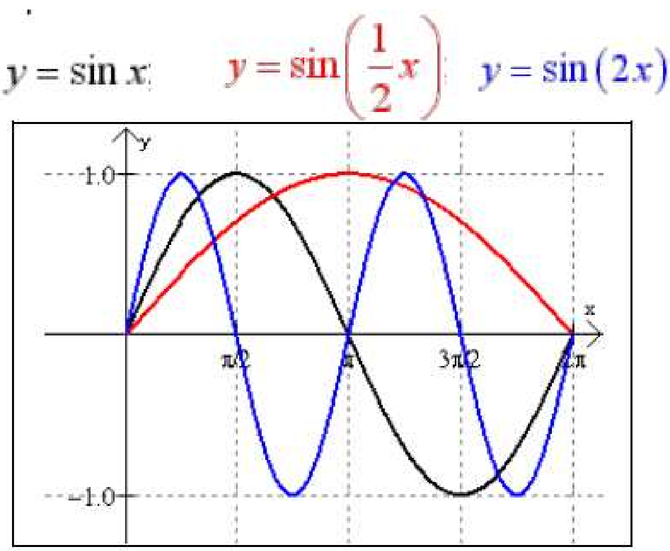
The value **A** affects the amplitude. The amplitude (half the distance from the max to the min) will be |A| because distance is always ___________________. Increasing or decreasing an A value with vertically _____________ or _________________ a graph.

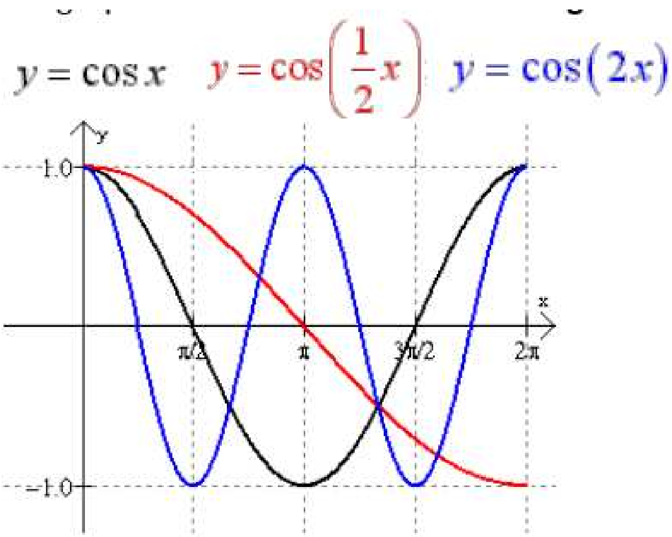




The change in amplitude changes the "height" but not the width. This graph still reaches from 0 to 2π.

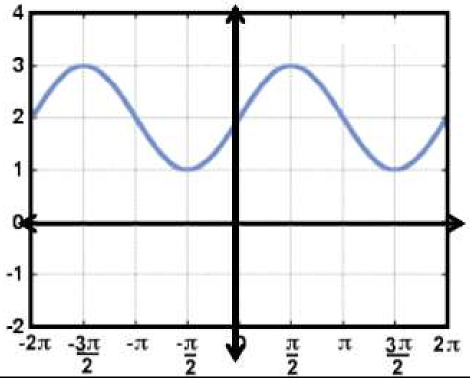
The **B** value is the number of cycles it completes in an interval of _______________ or ________. The **B** value affects the period. The period of sine and cosine is $|\frac{2\pi}{b}|$. When 0<B<1 the period of the function is ______________ than 2π and the graph will have a ___________________ ___________________. When **B**>1, the period is _______________ than 2π and the graph will have a horizontal ____________________.





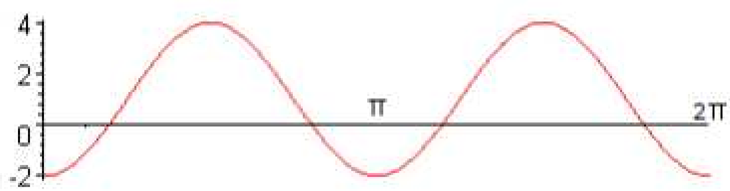
These graphs change horizontal "width" but do not change height. The two red graphs only show us half of the original graphs in their 0 to 2π windows. We would need to "stretch" the domain window to 4π to see entire cycles of those two graphs. The two blue graphs show us two complete cycles of the graphs in their 0 to 2π windows, which would allow us to "shrink" the domain window and still see complete cycles of the graphs.

Practice: Complete the table with the missing information. Graph when necessary.

<table>
<tr><td></td><td>Midline:

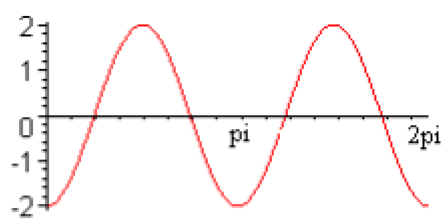
Amplitude:

Period:

Equation:</td></tr>
<tr><td></td><td>Midline:

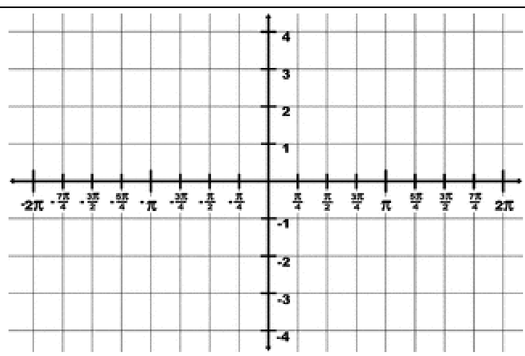
Amplitude:

Period:

Equation:</td></tr>
<tr><td></td><td>Midline:

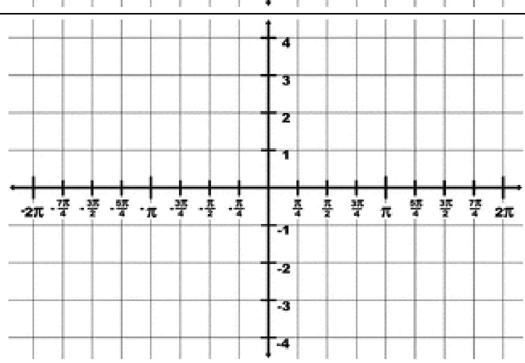
Amplitude:

Period:

Equation:</td></tr>
<tr><td></td><td>Equation:

Midline:

Amplitude:

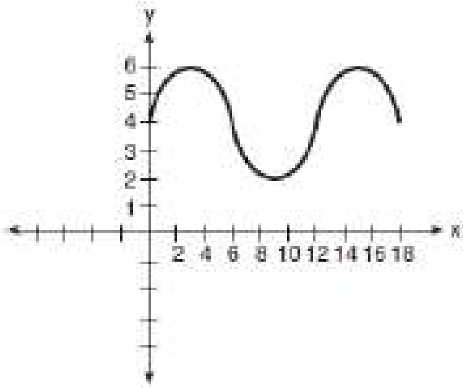
Period:</td></tr>
<tr><td></td><td>Equation:

Midline:

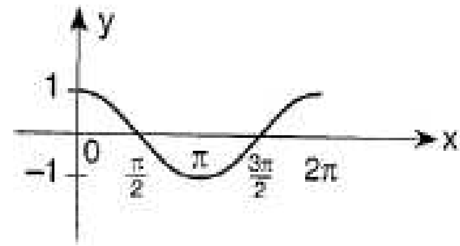
Amplitude:

Period:</td></tr>
</table>

1. What are the midline and amplitude of the sine graph below?

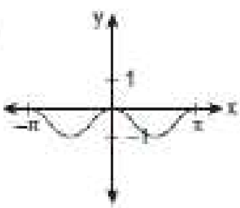


2. The graph below *incorrectly* represents the equation $y = 2\cos x$. Write a mathematical explanation of why this graph is incorrect.

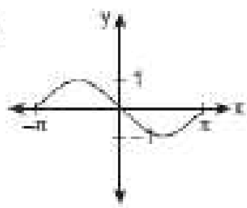


3. Which graph represents the function $f(x) = -\sin x$ in the interval $-\pi \le x \le \pi$?

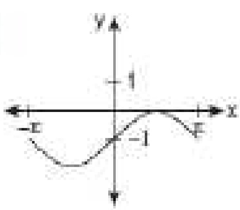
1)



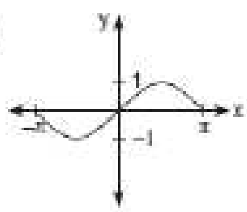
3)



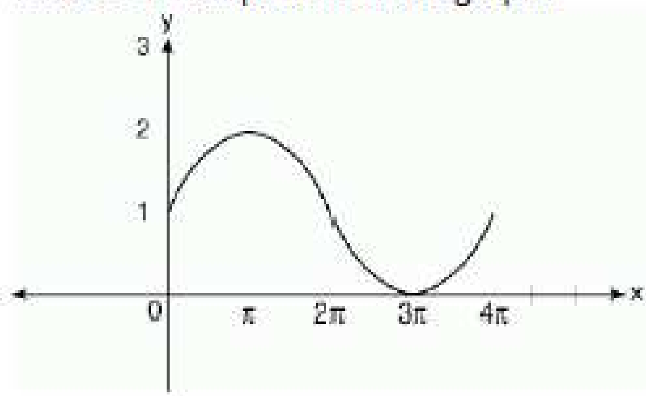
2)



4)

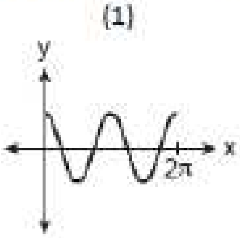


4. What is the amplitude of the graph?

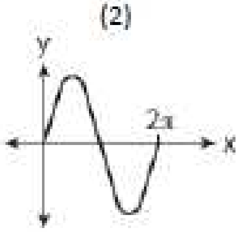


5. Which graph represents a sound wave that follows a curve whose period is π and that is in the form $y = a\sin bx$?

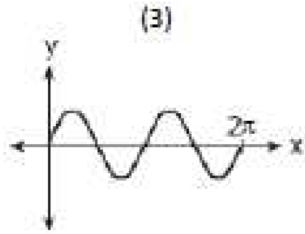
(1)



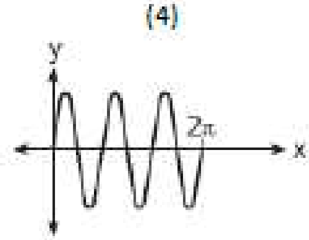
(2)



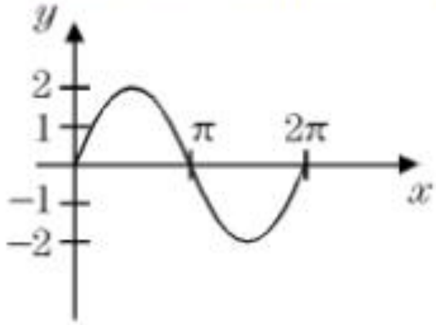
(3)



(4)



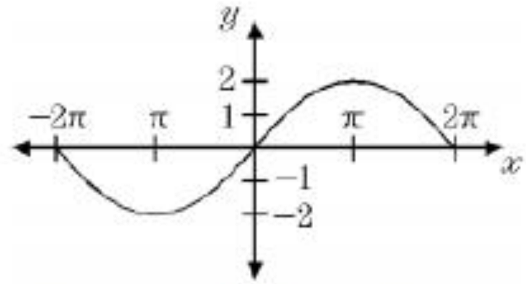
6. **What is the equation of the graph below?**



1) $y = \sin 2x$ 3) $y = \cos 2x$

2) $y = 2\cos x$ 4) $y = 2\sin x$

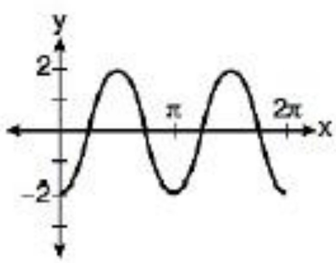
7. **What is the equation represents the graph below?**



1) $y = 2\sin 2x$ 3) $y = \frac{1}{2}\sin x$

2) $y = 2\sin \frac{1}{2}x$ 4) $y = 2\cos 2x$

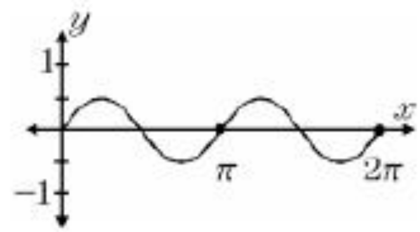
8. Which equation represents the graph below?



1) $y = -2\sin 2x$ 3) $y = -2\cos 2x$

2) $y = -2\sin \frac{1}{2}x$ 4) $y = -2\cos \frac{1}{2}x$

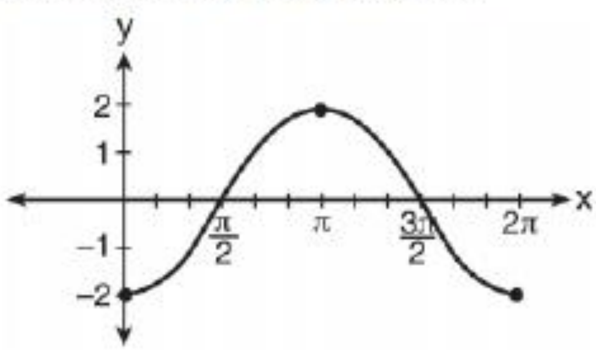
9. **Which equation is represented in the accompanying graph?**



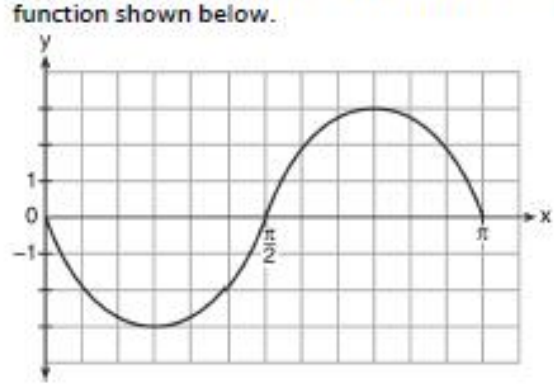
1) $y = 2\sin 2x$ 3) $y = 2\sin \frac{1}{2}x$

2) $y = \frac{1}{2}\sin \frac{1}{2}x$ 4) $y = \frac{1}{2}\sin 2x$

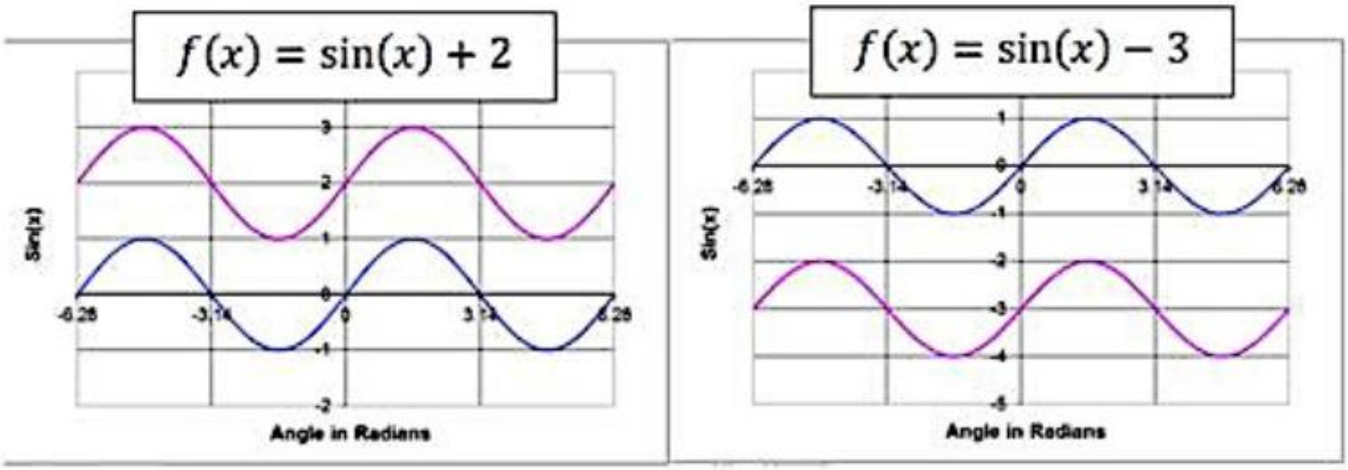
10. The accompanying graph shows a trigonometric function. Stat the equation of this function



11. Write an equation for the graph of the trigonometric function shown below.

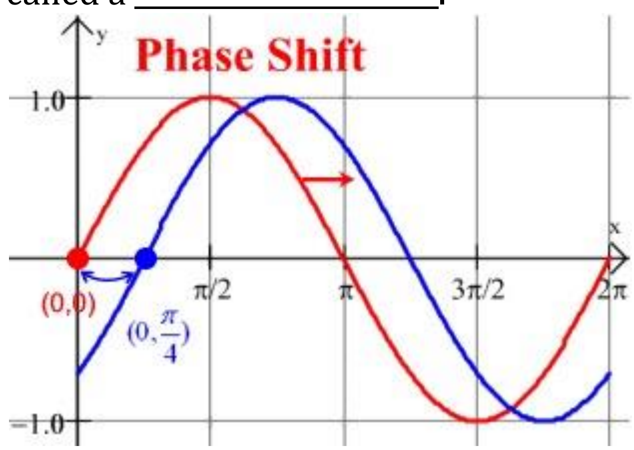


Just like any other function, adding a constant **on the end of the function** will shift the trig graph ________________________ (up if the constant is ________________, down if the constant is __________________).



To determine the midline of a graph you can add the max and min and divide by two.

Just like any other function, adding a constant on **into the function** will shift the trig graph ________________________ (left if the constant is ________________, right if the constant is __________________). This is called a _____________________.



Note: You may have to factor out B in order to determine the phase shift.

Graphing Trig Functions

$$\boldsymbol{y = A\sin(B(x - C)) + D}$$

$$\boldsymbol{y = A\cos(B(x - C)) + D}$$

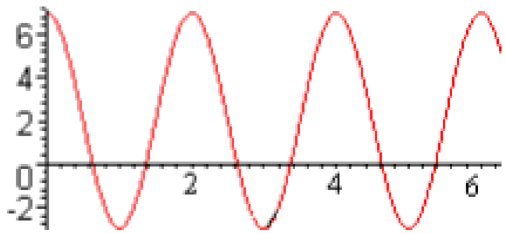
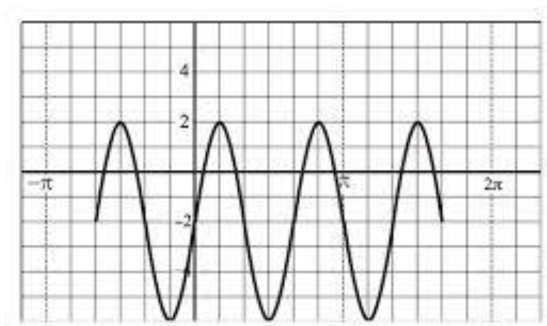
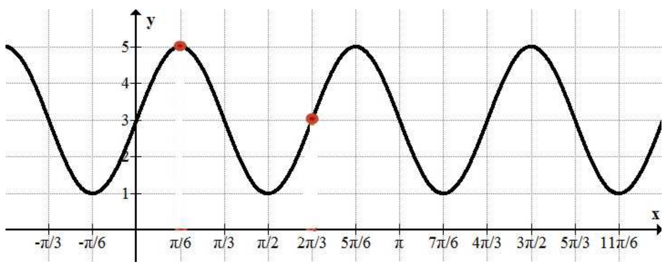
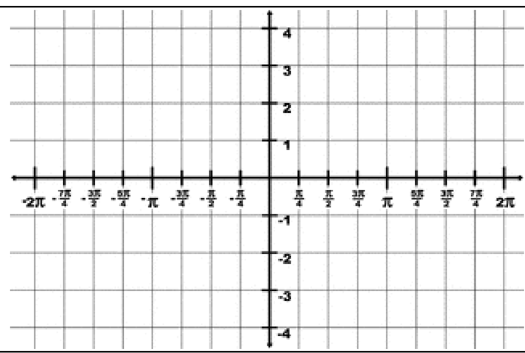
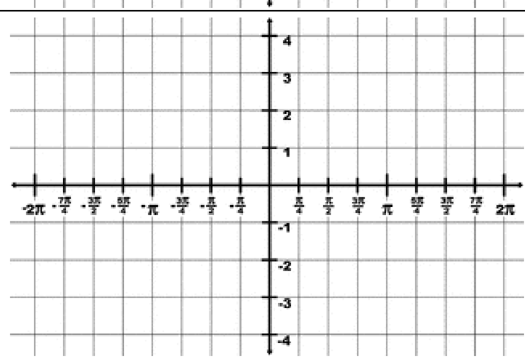
$|A| =$ amplitude

B = Horizontal Stretch, use to find period $|\frac{2\pi}{B}|$

C= Phase Shift (Horizontal Shift)

D= Vertical Shift (or "displacement")

Practice: Complete the table with the missing information. Graph when necessary.

	**Midline:** **Vertical Shift:** **Amplitude:** **Period:** **Equation:**
	**Midline:** **Vertical Shift:** **Amplitude:** **Period:** **Equation:**
	**Midline:** **Vertical Shift:** **Amplitude:** **Period:** **Equation:**
	**Equation:** $y=2\sin(1/2\ (x-3))+4$ **Midline:** **Vertical Shift:** **Amplitude:** **Phase Shift:** **Period:**
	**Equation:** $y = 3\cos(3x+6)+2$ **Midline:** **Vertical Shift:** **Amplitude:** **Phase Shift:** **Period:**

1)

The amplitude of the graph of the equation $y = 4\sin 2x$ is

A. 1 B. 2 C. $\frac{1}{2}$ D. 4

2) What is the period of the function $y = 5\sin 3x$?

1) 5
2) $\frac{2\pi}{5}$
3) 3
4) $\frac{2\pi}{3}$

3) What is the range of the function $y = 3\sin x$?

A. $y \geq 0$
B. $-1 \leq y \leq 1$
C. $y \leq 3$
D. $-3 \leq y \leq 3$

4) What is the amplitude of the graph of the equation $y = 2\sin\frac{1}{2}x$?

1) $\frac{1}{2}$
2) 2
3) π
4) 2π

5) If $f(x) = 2\sin 3x + C$, then the maximum value of $f(x)$ is:

1) C
2) $C+2$
3) $C+3$
4) $C+6$

6) What is the minimum value of $f(\theta)$ in the equation $f(\theta) = 3\sin 4\theta$?

1) -1
2) -2
3) -3
4) -4

7) The graph of which function has an amplitude of 2 and a period of 4π?

1) $y = 2\sin\frac{1}{2}x$
2) $y = 2\sin 4x$
3) $y = 4\sin\frac{1}{2}x$
4) $y = 4\sin 2x$

8) What is the period of the function $y = \frac{1}{2}\sin\left(\frac{x}{3} - \pi\right)$?

1) $\frac{1}{2}$
2) $\frac{1}{3}$
3) $\frac{2}{3}\pi$
4) 6π

9) What is the amplitude of the graph of the equation $y = 4\sin\frac{1}{2}x$?

10) A certain radio wave travels in a path represented by the equation $y = 5\sin 2x$. What is the period of this wave?

11) The path traveled by a roller coaster is modeled by the equation $y = 27\sin 13x + 30$. What is the maximum altitude of the roller coaster?

12) The expression $3\sin\frac{1}{2}x$ reaches its maximum value when x, expressed in radians, equals

1) $\frac{\pi}{2}$
2) $\frac{3}{2}$
3) 3
4) π

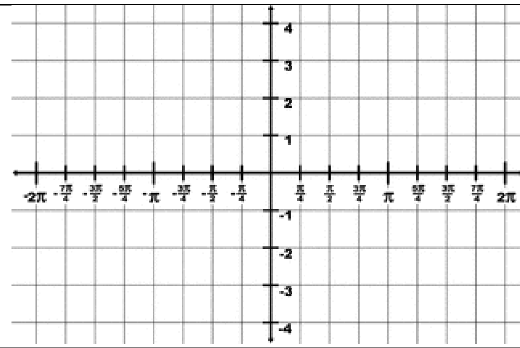
Sine and Cosine Graphing Practice

1. Equation: $y = sin\ 2x - 2$

Midline: **Vertical Shift:**

Amplitude: **Phase Shift:**

Period:

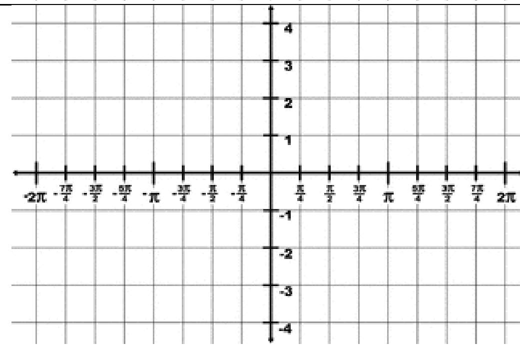


2. Equation: $y = 3\ cos\ (\pi x - 2) + 1$

Midline: **Vertical Shift:**

Amplitude: **Phase Shift:**

Period:

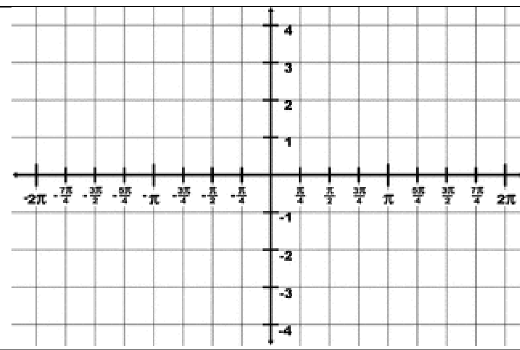


3. Equation: $y = -sin\ (\pi x + 1) - 2$

Midline: **Vertical Shift:**

Amplitude: **Phase Shift:**

Period:

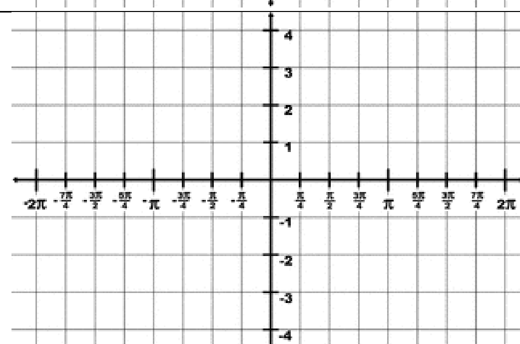


4. Equation: $y = 3\ cos \frac{1}{2} x$

Midline: **Vertical Shift:**

Amplitude: **Phase Shift:**

Period:

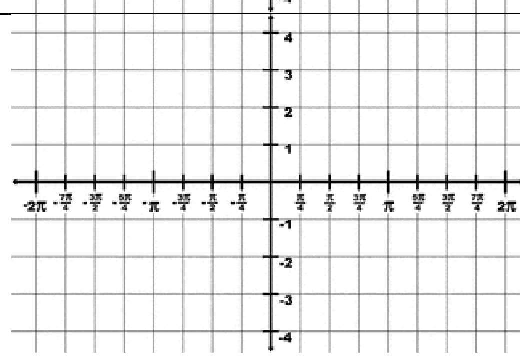


5. Equation: $y = sin\ (2x - \pi) + 1$

Midline: **Vertical Shift:**

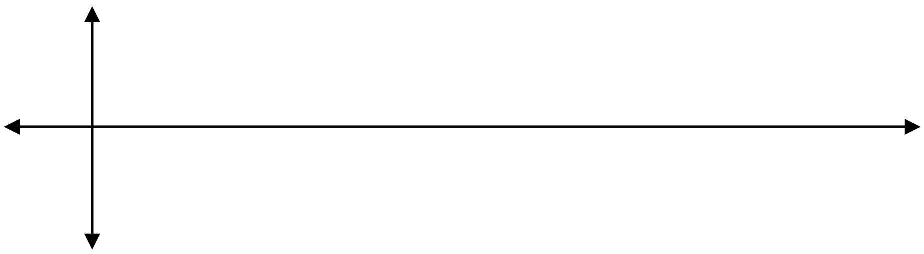
Amplitude: **Phase Shift:**

Period:



Sinusoidal Applications: Using your knowledge of sine and cosine curves, you will be able to write the equation of applications using circular patterns.

Example 1: A weight is suspended from a spring. Assuming no friction or air resistance, when the weight is pulled down a small distance, it will oscillate indefinitely about the equilibrium position. If the weight is pulled down 3 cm then after 1 second it will be back at the equilibrium position, at 2 seconds it will be at the 3 cm above the equilibrium position, and 3 seconds it will be back at equilibrium and at 4 seconds and it will be 3cm below.



a) Find the equation of a sinusoidal function that will model this movement.

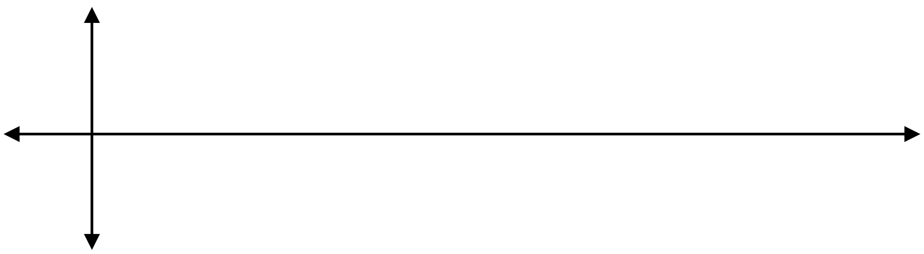
	Amplitude (height from equilibrium; A=1/2 (Max-Min) and is it a reflection?	**Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b	**Phase Shift** Did it move left/right? How much?	**Vertical Shift** Did it move up or down from the x-axis? How much?
Sine				
Cosine				

Sine: ______________________________ Cosine: __________________________________

b) Find the distance of the weight from its equilibrium position, 1.5 seconds after release and 15 seconds after release?

Example 2: Tarzan is swinging back and forth on a grapevine. As he swings, he goes back and forth across riverbank, going alternately over land and water. Jane decides to model his movement mathematically and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t and that y is positive when Tarzan is over water and negative when he is over land.

Jane finds that when t=2, Tarzan is at the end of his swing where y=-23. She finds that when t is 5, he reaches the other end of his swing and y is 17.



a) Find the equation expressing Tarzan's distance from the riverbank in terms of t.

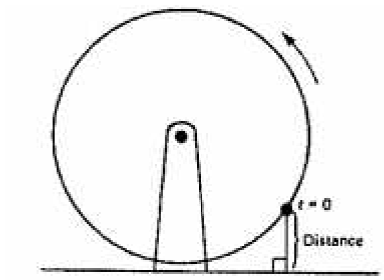
	Amplitude (height from equilibrium; A=1/2 (Max-Min) and is it a reflection?	**Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b	**Phase Shift** Did it move left/right? How much?	**Vertical Shift** Did it move up or down from the x-axis? How much?
Sine				
Cosine				

Sine: ______________________ Cosine: ______________________

b) Find y when t=2.8 and t=6.3

c) Where was Tarzan when Jane started the stopwatch?

Example 3: You've probably noticed that as you ride a Ferris wheel, your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, your seat is at the position shown in the diagram below. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet.



a) What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero?

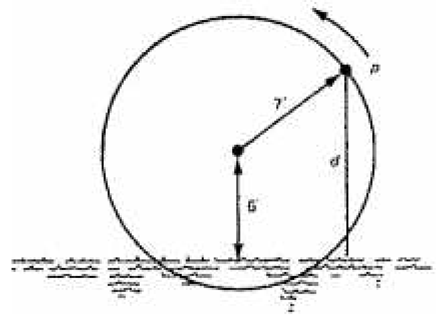
Find the equation expressing Tarzan's distance from the riverbank in terms of t.

	Amplitude (height from equilibrium; A=1/2 (Max-Min) and is it a reflection?	**Period** P=how long for one cycle, $\frac{2\pi}{b}$=P; solve for b	**Phase Shift** Did it move left/right? How much?	**Vertical Shift** Did it move up or down from the x-axis? How much?
Sine				
Cosine				

Sine: ______________________ Cosine: ________________________

c) Predict the height above the ground when t=9.

Practice 1: Suppose a waterwheel (shown in the figure) makes a complete rotation every 12 seconds. You start your stopwatch. Three seconds later, point P on the rim of the wheel is at the greatest height. You are to model the distance of point P from the surface of the water in terms of the number of seconds, t, the stopwatch reads.



a. Make a sketch of the graph of this sinusoid.

b. Write an equation (sine or cosine) that models this sinusoid.

c. How far is point P from the water after 5.5 seconds?

Practice 2: As an oil well pumps, the height of its cathead varies sinusoidally with time. Suppose that the pump is started at time t=0 seconds. Two seconds later, it is at its highest point above the ground, 22 feet. It is at its next low point (8 feet) 2.5 seconds after that.

a. Make a sketch of the graph of this sinusoid.

b. Write an equation (sine or cosine) that models this sinusoid.

c. What is the height of the cathead after 10 seconds?

AFM Study Guide for Unit 8 Test Name ____________________

I. Convert each degree measure to radian measure.

1. 150^o 2. 210^o 3. 45^o 4. 240^o

II. Each radian measure to degree measure.

5. $\frac{\pi}{6}$ 6. $\frac{\pi}{4}$ 7. $\frac{5\pi}{6}$ 8. $\frac{7\pi}{6}$

III. In which quadrant, or on which axis, does the terminal side of the each angle lie? Draw the angle in standard position.

9. 150^o 10. 210^o 11. -60^o

12. -240^o 13. 540^o 14. 2π

15. $\frac{3\pi}{4}$ 16. $\frac{7\pi}{3}$ 17. $\frac{5\pi}{4}$

IV. Find the coterminal angle between 0 and 360 or 0 and 2π for the following measures.

18. $-24°$ 19. -330^o 20. $750°$ 21. $\frac{7\pi}{3}$ 22. $-\frac{17\pi}{3}$

23. $400°$ 24. $-280°$ 25. $940°$ 26. $1059°$ 27. $-624°$

28. $\frac{6\pi}{3}$ 29. $\frac{9\pi}{2}$ 30. $\frac{11\pi}{3}$ 31. $\frac{-\pi}{3}$ 32. $\frac{36\pi}{13}$

V. A point on the terminal side of angle θ is given. Find the exact value of each of the six trig functions of θ

33. (-4, 3) 34. (2, 3) 35. (5, -5) 36. (-1, -3)

VI. Arc Length and Area of a Sector

37. Find the length of an arc of a circle of radius 8 m if the arc subtends a central angle of 1 radian.

38. Find the measure of a central angle θ (in degrees) in a circle of radius 5 ft if the angle is subtended by an arc of length 7 ft.

39. A circular arc of length 100 ft subtends a central angle of 70°. Find the radius of the circle.

40. Find the area of a sector with central angle 52° in a circle of radius 200 ft.

41. A sector in a circle of radius 25 ft has an area of 125 ft^2. Find the central angle of the sector in radians.

VII. Find the exact values of the following.

42. $\sin(315^\circ)$ 43. $\cot(-135^\circ)$ 44. $\csc\left(\frac{5\pi}{6}\right)$ 45. $\sec(405^\circ)$ 46. $\cos\left(-\frac{22\pi}{3}\right)$ 47. $\tan(4\pi)$

48. $\cos 225^\circ$ 49. $\tan 210^\circ$ 50. $\tan 420^\circ$ 51. $\sin\frac{2\pi}{3}$

52. $\csc\frac{7\pi}{6}$ 53. $\tan\frac{9\pi}{4}$ 54. $\sin(-240)$ 55. $\tan\left(-\frac{\pi}{4}\right)$

VIII. Find the value of the SIX trigonometric functions of θ from the information given.

56. $\tan\theta = 4$, $\sin\theta < 0$

57. The point (-4,5) is on the terminal side of θ

58. If (-1, -5) is a point on the terminal side of angle θ, find the exact value of each of the six trig functions.

59. If $\cos\theta = 2/5$, and $\sin\theta < 0$, find the remaining trig functions.

IX. Let θ be an angle in standard position. Name the quadrant in which θ lies.

60. $\sin\theta < 0,\ \cos\theta > 0$

61. $\tan\theta > 0,\ \sin\theta < 0$

62. $\sin\theta > 0,\ \cos\theta > 0$

63. $\tan\theta < 0,\ \sin\theta < 0$

64. $\tan\theta < 0,\ \cos\theta < 0$

X. Linear and Angular Speed

65. A phonograph record has a radius of 3 inches and revolves at 45 RPM. Find the linear speed of the outside edge of the record in inches per second.

66. The propeller of an airplane has a radius of 3 ft. The propeller is rotating at 2250 revolutions per minute. Find the linear (in feet per second) and angular speed (in radians per second) of the tip of the propeller.

67. Given a central angle of 125°, find the length of its intercepted arc in a circle of radius 7 cm. Round to the nearest tenth.

68. Find the area of a sector if the central angle measures 3π/7 radians and the radius of the circle is 11 cm.

69. Determine the angular velocity if 5.8 revolutions are completed in 9 seconds.

70. A circular serving table in a buffet has a radius of 3 feet. It makes 2.5 revolutions per minute. Determine the angular velocity in radians per second of a bowl of peaches sitting on the serving table.

71. The fastest human on a bicycle was John Howard, who achieved an incredible speed of 152.3 mph in 1985. If the tires on John's bicycle have a diameter of 30 inches and turn at rate of 141 revolutions per minute in a warm-up, what is the bicycle's speed in mph

XI. State the amplitude and period, phase shift and midline.

72. $y=-\cos\left(\frac{1}{2}\left(\theta-\frac{\pi}{2}\right)\right)-2$

73. $y=\cos(\theta+\pi)-2$

74. $y=\frac{1}{2}\sin 2\left(\theta+\frac{\pi}{6}\right)-1$

75. y= –sin(2x)+4

76. $y=\cos\frac{1}{2}\theta$

77. $2\sin(\frac{1}{2}x)-1$

78. There are many rides at the amusement park whose movement can be described using trigonometric functions. The Ferris Wheel is a good example of periodic movement.

Sydney wants to ride a Ferris wheel that has a radius of 60 feet and is suspended 10 feet above the ground. The wheel rotates at a rate of 2 revolutions every 6 minutes. (Don't worry about the distance the seat is hanging from the bar.) Let the center of the wheel represents the origin of the axes.

a. Write a function that describes a Sydney's height above the ground as a function of the number of seconds since she was ¼ of the way around the circle (at the 3 o'clock position).

b. How high is Sydney after 1.25 minutes?

c. Sydney's friend got on after Sydney had been on the Ferris wheel long enough to move a quarter of the way around the circle. How would a graph of her friend's ride compare to the graph of Sydney's ride? What would the equation for Sydney's friend be?

79. The pedals of a bicycle are mounted on a bracket whose center is 29 cm above the ground. Each pedal is 16.5 cm from the center of the bracket. Assume that the bicycle is pedaled at 12 revolutions per minute. With the starting position of the pedal in a horizontal position at $t = 0$ seconds.

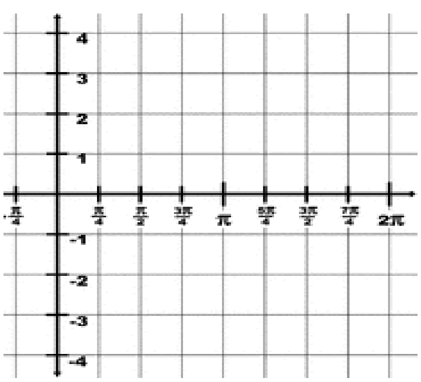
(a) Sketch the graph of this sinusoidal function of the height of one pedal in cm (with respect to **seconds**) for the first three cycles. Label and number the axes appropriately.

(b) Write an equation for this function expressing the height y as a function of time t.

(c) When is the pedal 40 cm above the ground for the first time?

(d) How high is the pedal after 23 seconds? At this time, is the pedal going up or down?

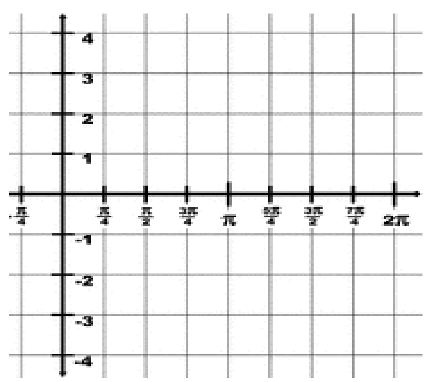
80. $y = -2\sin\theta - 1$



Amplitude: ________ Period: _________

Phase shift: _________ Midline: ______

81. 3cos(x) -1



Amplitude: ______ Period: _________

Phase shift: ______ Midline: ______