NUMB3RS Activity: The Center of it All
Episode: "Pilot"

Topic: Circles   **Grade Level:** 9 - 12
Objective: Explore the circumcenter and investigate the relationship between the position of the circumcenter and types of triangles.
Time: 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator with Cabri™ Jr. App installed

Introduction

Charlie is helping his brother Don find the location of a killer. To do this, Charlie needs to find the center of an irregular region because he suspects the center may represent the location of the killer's residence. To explain the process to Don, Charlie shows him a sprinkler spraying water in a circular path. Charlie states that even if the sprinkler can't be seen, by analyzing where the water drops land and knowing the path the sprinkler follows, the position of the sprinkler can be determined.

Discuss with Students

Teachers usually present the concept of circumcenter starting directly with a triangle. In this activity students begin by observing that any three noncollinear points determine a circle. They will then extend this to locate the circumcenter of the circle using the Cabri Jr. App on their graphing calculators. This App can be downloaded fro free at **http://education.ti.com/cabrijr**. For teachers without this App or graphing calculators, the activity can also be effectively completed using a more traditional straightedge and compass. The difference is that the Cabri Jr. App makes a dynamic drawing that the students can manipulate.

Through manipulating the diagrams, students will observe that the intersection of the perpendicular bisectors is the center of the circle. Students also investigate the relationship between the position of the circumcenter and the type of triangle formed.

Before using this activity, make sure students are familiar with the following terms: Chord, Segment, Radius, Collinear, Perpendicular Bisector, Circumcenter, Circumscribe, Centroid and Congruent.

Student Page Answers:
1. *The circle degenerates into a line.* **2.** *The perpendicular bisectors are parallel.* **3.***The triangle is acute.* **4.** *The triangle is obtuse.* **5.** *The triangle is right.* **6.** *It is the in the center of the triangle.*

Name: _____   Date: _____

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Given three point, the task is to find a point equal distance from all three points. To accomplish this, construct a circle passing through the three points. This reduces the problem to finding the center of the circle given three points. You will use the Cabri™ Jr. App on your graphing calculator. Follow the instructions below and then answer the questions at the end of the activity.
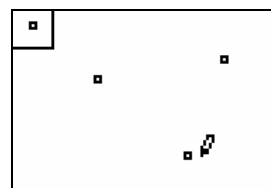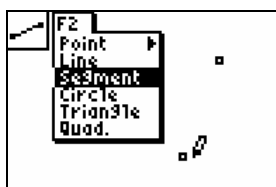
1

Press APPS and select CabriJr.

2
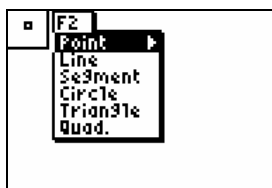
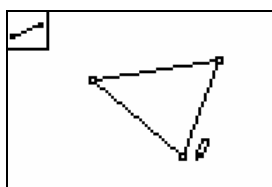To create three non-collinear points, press WINDOW ▶ and choose **Point**. Press ENTER.

3

Move the cursor around and press ENTER at each in each of the three places you want to create a point. Make sure the three points are not collinear.
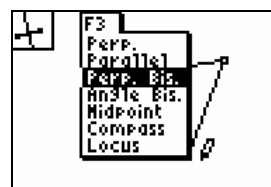
4

Next, press WINDOW and choose **Segment**.

5

Move the cursor over one of the points until it flashes, press ENTER. This marks one of the endpoints of the segment. Then move the cursor to one of the other points you marked. Press ENTER again to create the segment. Repeat the process to create the remaining line segment.
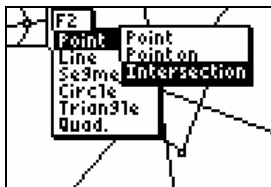
6



Press ZOOM and choose **Perp. Bis.** Press ENTER.

7

To create the perpendicular bisector, move the cursor to one of the segments until the cursor changes to an arrow and the segment flashes and press ENTER. Repeat the process to create the perpendicular bisector for the remaining segments.
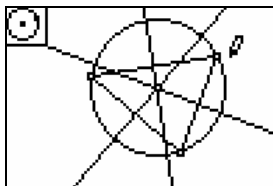
8

Notice that the three perpendicular bisectors intersect. Press WINDOW ▶ and choose **Intersection**.

9
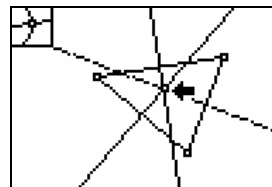
Move the cursor to the point where the perpendicular bisectors intersect. The cursor will change to an arrow and both lines will flash. Press ENTER to create the point of intersection
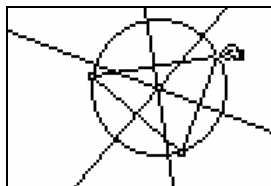
10

To make a circle with the intersection point as the center, press WINDOW and choose **Circle**.
Place the cursor on the point of intersection and press ENTER. This marks the center of the circle. Now move the cursor to one of the original three points you plotted. Press ENTER to create the circle.

11

Now that we have created the circle, use Cabri Jr. to explore the figure Press CLEAR to turn off the drawing tool. To move one of the points on the circle, move the cursor to one of the three original points and press the ALPHA key to select the point

12
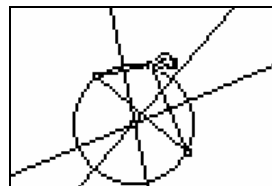
Now that the point is selected, move the cursor around and notice how the circle changes. Also note the relationship between the perpendicular bisectors and the center of the circle.

Using your diagram and manipulating the position of the points, answer the following questions.

1. Move the points so that they are linear. What do you observe about the circle?

2. While the points are linear, what do you observe about the perpendicular bisectors?

The point where the three perpendicular bisectors intersect is the circumcenter of the triangle.

3. Move the points so that the circumcenter is in the interior of the triangle. What do you observe about the type of triangle formed?

4. Move the points so that the circumcenter is in the exterior of the triangle. What do you observe about the type of triangle formed?

5. Move the points so that the circumcenter is on the side of the triangle. What do you observe about the type of triangle formed?

6. Move the points so that the three chords are congruent. What do you observe about the location of the circumcenter?

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

The activity takes a geometric approach to solving the problem of finding the center of a circle given three non-collinear points. If you have previously studied determinates, you can also solve this problem algebraically by solving the determinant equation. This would be extremely useful to Don and the FBI, because then they would have a coordinate of the possible residence of the killer. We discovered from the activity that the points must be noncollinear for the circle to exist. If the points are plotted on a Cartesian coordinate system, one possible way of determining if three points are collinear is calculate if their determinant is zero. If it is then they are collinear, otherwise they are noncollinear. Using this concept of determinants you can extend this and find the coordinates of the center of the circle.

Given:

Point 1: (x_1, y_1)

Point 2: (x_2, y_2)

Point 3: (x_3, y_3)

Solve:

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Using minors, the determinant equation above can be simplified to

$$x = \frac{M_{12}}{2M_{11}} \qquad\qquad y = \frac{M_{13}}{-2M_{11}}$$

Or in the expanded form:

$$x = \frac{\begin{vmatrix} x_1^2 + y_1^2 & y_1 & 1 \\ x_2^2 + y_2^2 & y_2 & 1 \\ x_3^2 + y_3^2 & y_3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} \qquad\qquad y = \frac{\begin{vmatrix} x_1^2 + y_1^2 & x_1 & 1 \\ x_2^2 + y_2^2 & x_2 & 1 \\ x_3^2 + y_3^2 & x_3 & 1 \end{vmatrix}}{-2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$
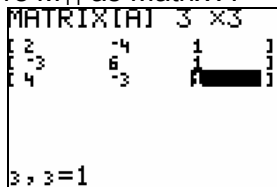
Consider this example: find the center of the circle passing through the points (2, –4), (–3, 6), and (4, –3).

You can first take a look the position of the points on a coordinate grid using the draw feature on your calculator. (Note: to find **Pt–On(**, press 2nd [DRAW], go to the **POINTS** menu, and select **1:Pt–On(**.)
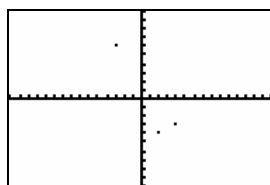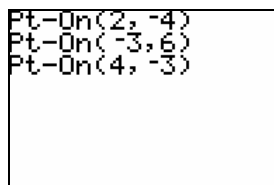
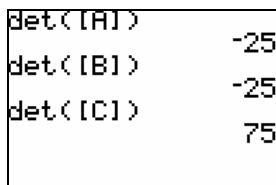Next define the three matrices:

Store M_{11} as Matrix *A*   Store M_{12} as Matrix *B*   Store M_{13} as Matrix *C*


```
MATRIX[A] 3 ×3
[ 2    -4    1    ]
[ -3   6     1    ]
[ 4    -3    1    ]

3,3=1
```


```
MATRIX[B] 3 ×3
[ 20   -4    1    ]
[ 45   6     1    ]
[ 25   -3    1    ]

3,3=1
```


```
MATRIX[C] 3 ×3
[ 20   2     1    ]
[ 45   -3    1    ]
[ 25   4     1    ]

3,3=1
```

Then find the determinant of each matrix.

```
det([A])
              -25
det([B])
              -25
det([C])
               75
```

So the coordinates of the center of the circle are:

$$x = \frac{-25}{2(-25)} = \frac{1}{2} \qquad \text{and} \qquad y = \frac{75}{-2(-25)} = \frac{3}{2}$$

You can also find the radius of the circle by finding the distance from the center to a point on the circle. For this problem, use a point on the circle, such as (2, –4), and the

center $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$: $\sqrt{\left(2 - \dfrac{1}{2}\right)^2 + \left(-4 - \dfrac{3}{2}\right)^2} = \dfrac{\sqrt{130}}{2} \approx 5.7$.

You can use your calculator to check that the circle with a center of $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$ and a radius of approximately 5.7 goes through the points (2, –4), (–3, 6), and (4, –3).

```
√(32.5)
        5.700877125
Circle(0.5,1.5,5
.7)
```
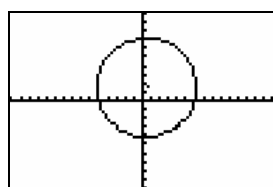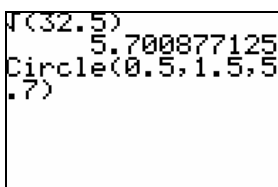

Additional Resources

*   A Java applet that demonstrates how the center of the circle changes as the points are moved can be found at: **http://web.mit.edu/linkagedemo/www/circlepage1.html**

*   A more complete development of the algebraic solution can be found at:
    http://home.att.net/~srschmitt/circle3pts.html

*   More Cabri Jr. explorations of circles can be found at:
    http://education.ti.com/exchange