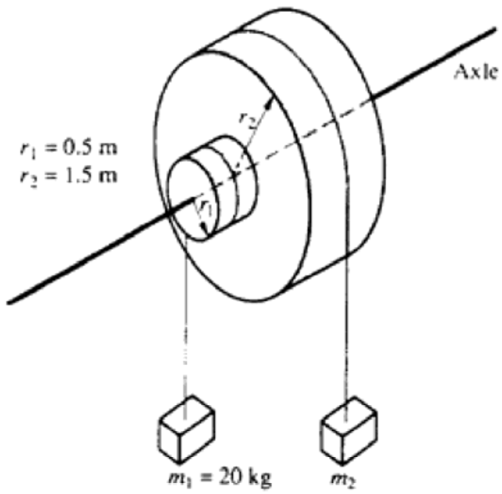
Name: ________________________ Class: ___________________ Date: __________ ID: A

Unit 4-Rotation FR Quiz

Problem

1.



1991M2. Two masses. m_1 and m_2 are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively. as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is $I = 45$ kgm^2 Also $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.

a. Determine m_2 such that the system will remain in equilibrium.

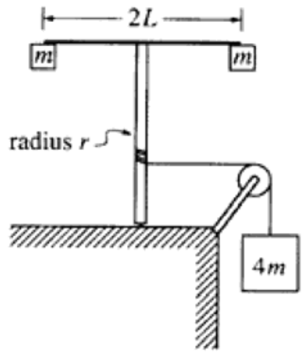
The mass m_2 is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders.

c. Determine the tension in the cable supporting m_1

d. Determine the linear speed of m_1 at the time it has descended 1.0 meter.

2.



Experiment A

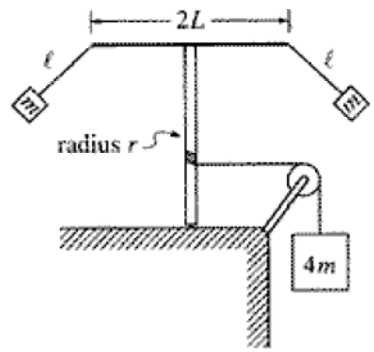
2001M3. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius *r*, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length $2L$, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.

a. Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.

b. Determine the downward acceleration of the large block.

c. When the large block has descended a distance *D,* how does the instantaneous total kinetic energy of the three blocks compare with the value *4mgD* ? Check the appropriate space below and justify your answer.

Greater than $4mgD$ _____ Equal to $4mgD$ _____ Less than $4mgD$ _____



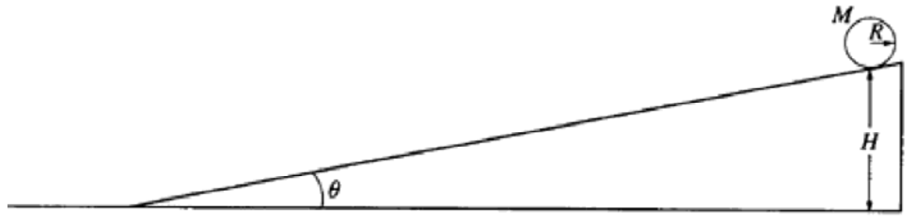
Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length *l*. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

d. When the large block has descended a distance *D*, how does the instantaneous total kinetic energy of the three blocks compare to that in part C? Check the appropriate space below and justify your answer.

Greater before _____ Equal to before _____ Less than before _____

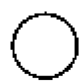
3.



1997M3. A solid cylinder with mass M, radius R, and rotational inertia $\frac{1}{2}MR^2$ rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height H. The inclined plane makes an angle θ with the horizontal. Express all solutions in terms of M, R, H, θ, and g.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane Your arrow should begin at the **point of application** of each force.



c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is $(2/3)g \sin\theta$.

d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

e. The coefficient of friction μ is now made less than the value determined in part (d), so that the cylinder both rotates and slips.

i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part (a). Justify your answer.

ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.

Unit 4-Rotation FR Quiz
Answer Section

PROBLEM

1. ANS:

Mech 2.

(a) 3 points

For recognition that $\sum$torque = 0 or equivalent — 1 point

$m_2 g r_2 = m_1 g r_1$ (or $m_2 r_2 = m_1 r_1$) — 1 point

$$m_2 = \frac{m_1 r_1}{r_2} = \frac{(20\ \text{kg})(0.5\ \text{m})}{(1.5\ \text{m})}$$

$m_2 = \frac{20}{3}$ kg — 1 point

(b) and (c) 6 points

For a correct dynamical equation for the torque:
$\tau = I\alpha$ — 1 point

For a correct application of the torque equation for the two cylinders:
$Tr_1 = (45\ \text{kg}\cdot\text{m}^2)\alpha$ — 1 point

For Newton's second law:
$F = ma$ — 1 point

For a correct application of Newton's law for mass m_1:
$(20\ \text{kg})g - T = (20\ \text{kg})a$ — 1 point

For recognizing that parts (b) and (c) are coupled (i.e., attempting to solve simultaneous equations) — 1 point

$T = (20\ \text{kg})(g - a)$

$a = \alpha r$ — 1 point

$T = (20\ \text{kg})(g - \alpha r_1)$

Substituting into torque equation:

$(20\ \text{kg})(g - \alpha r_1) = (45\ \text{kg}\cdot\text{m}^2)\alpha$

$(20\ \text{kg})gr_1 - (20\ \text{kg})\alpha r_1^2 = (45\ \text{kg}\cdot\text{m}^2)\alpha$

$\alpha[45\ \text{kg}\cdot\text{m}^2 + (20\ \text{kg})r_1^2] = (20\ \text{kg})gr_1$

$\alpha = (20\ \text{kg})(9.8\ \text{m/s}^2)(0.5\ \text{m})\ /\ [45\ \text{kg}\cdot\text{m}^2 + (20\ \text{kg})(0.5\ \text{m})^2]$

$\alpha = 2.0\ \text{rad/s}^2$ — 1 point

$$T = (20\ \text{kg})\left[9.8\ \text{m/s}^2 - \left(2.0\ \frac{\text{rad}}{\text{s}^2}\right)(0.5\ \text{m})\right]$$

$T = 180$ N — 1 point

Mech 2. (continued)

(d) 3 points

For applicable kinematic equation(s):

$v^2 = 2as$ OR	$s = \frac{1}{2}at^2$ and $v = at$ OR	$\omega^2 = 2\alpha\theta$ and $v = \omega r$	1 point
$a = \alpha r$	$t = v/a$	$v = r\sqrt{2\alpha\theta}$	
	$s = \frac{1}{2}a\frac{v^2}{a^2} = \frac{1}{2}\frac{v^2}{a}$	$\theta = \frac{s}{r}$	
	$v^2 = 2as$	$v = r\sqrt{\frac{2\alpha s}{r}}$	
	$a = \alpha r$		

$v = \sqrt{2\alpha r_1 s}$

For correct substitution: $v = \sqrt{2(2.0\ \text{rad/s}^2)(0.5\ \text{m})(1\ \text{m})}$ — 1 point

$v = 1.4$ m/s — 1 point

(Alternate solution) — (Alternate Po

Using conservation of energy:

$mgs = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ — (1 point)

$\omega = v/r$ — (1 point)

$mgs = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$

$= \frac{1}{2}v^2\left(m + \frac{I}{r^2}\right)$

$v^2 = 2mgs/(m + I/r^2) = \dfrac{2(20\ \text{kg})(9.8\ \text{m/s}^2)(1\ \text{m})}{[20\ \text{kg} + (45\ \text{kg}\cdot\text{m}^2)/(0.5\ \text{m})^2)]}$

$v = 1.4$ m/s — (1 point)

For at least one answer with correct units, and no incorrect units — 1 point

PTS: 15 DIF: AP Physics C TOP: E

2. ANS:

3. (a) 3 points	**Distributio of Points**
For a correct formula for the rotational inertia	**1 point**

$I = \sum mr^2$

For a sum containing a term of the form mL^2 (may include extra incorrect terms, but the point was not awarded if the expression does not contain an mL^2 term) — **1 point**

$I = mL^2 + mL^2$

For the correct answer — **1 point**

$I = 2mL^2$

3. (b) 6 points

For a correct expression of Newton's 2nd law — **1 point**

$F = ma$

For correct substitutions into Newton's law — **1 point**

$4mg - T = 4ma$

For a correct formula for torque — **1 point**

$\tau = I\alpha$ <u>or</u> Tr

$I\alpha = Tr$

$T = \dfrac{I\alpha}{r}$

From Newton's 2nd law equation above:

$T = 4mg - 4ma$

Substituting into the torque equation:

$\dfrac{I\alpha}{r} = 4mg - 4ma$

For substituting the expression for I from part (a) into Newton's law — **1 point**

$\dfrac{2mL^2\alpha}{r} = 4mg - 4ma$

For the expression $\alpha = a/r$ — **1 point**

Substituting this expression into the previous equation:

$\dfrac{2mL^2a}{r^2} = 4mg - 4ma$

For the correct answer — **1 point**

$a = \dfrac{2gr^2}{L^2 + 2r^2}$

<u>Note:</u> For the solution $a = \dfrac{4mg - T}{4m}$, obtained by solving $4mg - T = 4ma$ for a directly, a maximum of **3 points** was awarded for part (b) as follows; **1 point** for Newton's law, **1 point** for substitutions, and **1 point** for answer.

3. (c) **3 points**	**Distribution of Points**
For correctly checking the space in front of "Equal to $4mgD$"	**1 point**
For correct justification, such as "The kinetic energy gained by the two smaller blocks comes from the decrease in the potential energy of the $4m$ block." OR "Total energy is conserved."	**2 points**

Note: **No points awarded for part (c) if wrong box was checked.**

3. (d) **3 points**

For correctly checking the space in front of "Less"	**1 point**
For correct justification, such as "The small blocks rise and gain potential energy. The total energy available is still $4mgD$. Therefore the kinetic energy must be less than in part (c)."	**2 points**

Note: **No points awarded for part (d) if wrong box was checked.**

PTS: 15 DIF: AP Physics C TOP: E

3. ANS:

Mech. 3 (15 points)

(a) 4 points

For using conservation of energy	1 point
For including both linear and rotational energy	1 point

$U = K_{tran} + K_{rot}$

$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

For correctly substituting for ω — 1 point

$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$

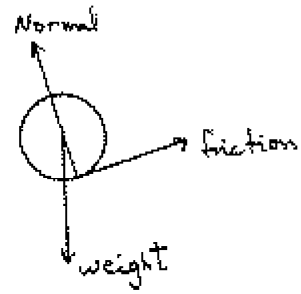
$MgH = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2$

$MgH = \frac{3}{4}Mv^2$

For the correct answer — 1 point

$v = \sqrt{\frac{4}{3}gH}$

(b) 3 points



For forces with correct direction, point of application, and label (1 point each) — 3 points

One point was deducted for any extraneous forces.

One point was awarded if all forces had correct direction and label but did not all begin at the correct point

(c) 2 points

Using an appropriate kinematic equation
$\upsilon_f^2 = \upsilon_i^2 + 2as$
For one correct substitution into this equation 1 point
For the other two correct substitutions 1 point
$\frac{4}{3}gH = 0 + 2a\left(\frac{H}{\sin\theta}\right)$
Solving for the acceleration
$a = \frac{2}{3}g\sin\theta$

(Alternate solution) *(Alternate pc*
Using Newton's second law and the equivalent rotational equation
$F_{net} = Ma$ and $\tau = I\alpha$
For proper substitutions in either of the above equations *1 point*
$Mg\sin\theta - f = Ma$ and $fR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$
Solving the rotational equation above for the frictional force
$f = \frac{1}{2}Ma$
For substituting this frictional force into the Newton's law equation *1 point*
$Mg\sin\theta - \frac{1}{2}Ma = Ma$
$Mg\sin\theta = \frac{3}{2}Ma$
$a = \frac{2}{3}g\sin\theta$

Note: Full credit was also awarded for a clear argument using the fact that the rotational energy is half the translational energy (which can be seen from part (a)). This means that the translational kinetic energy is equal to 2/3 of the total energy. The translational kinetic energy is also equal to the work done by the net force along the plane Equating 2/3 the total energy, MgH, to the work, $Ma(H/\sin\theta)$, gives an acceleration equal to $\frac{2}{3}g\sin\theta$

(d) 2 points

Applying Newton's second law to the motion of the center of mass
$Ma = Mg\sin\theta - f$

For proper use of the normal force in the expression for the frictional force — 1 point
$f = \mu N = \mu Mg\cos\theta$

Substituting for f and a in Newton's law

$$M\left(\frac{2}{3}g\sin\theta\right) = Mg\sin\theta - \mu Mg\cos\theta$$

Canceling the factors of M and g, and solving for μ

$$\frac{2}{3}\sin\theta - \sin\theta = -\mu\cos\theta$$

$$\frac{1}{3}\sin\theta = \mu\cos\theta$$

For the correct answer — 1 point

$$\mu = \frac{1}{3}\tan\theta$$

(Alternate solution) — *(Alternate p*

Determining the expression for the frictional force, in terms of the acceleration, from the torque equation (see the alternate solution to part (c))

$$f = \frac{1}{2}Ma$$

For proper use of the normal force in the expression for the frictional force — *1 point*
$f = \mu N = \mu Mg\cos\theta$

Substituting the expressions for the frictional force and the acceleration

$$\mu Mg\cos\theta = \frac{1}{2}M\left(\frac{2}{3}g\sin\theta\right)$$

For the correct answer — *1 point*

$$\mu = \frac{1}{3}\tan\theta$$

(e)

i. 2 points

For indicating that the translational speed at the bottom of the incline is greater than in part (a) — 1 point
For a reasonable explanation — 1 point
Examples:
There is less energy transferred to the rotational motion, so more goes into translational motion and thus the speed is greater.
The frictional force is less, so by Newton's second law (see the first equation in part (d)) the translational acceleration is greater. Thus the cylinder gains more speed.

ii. 2 points

For indicating that the total kinetic energy at the bottom of the incline is less — 1 point
For a reasonable explanation — 1 point
Examples:
Since the cylinder slides, some energy is dissipated as heat.
There is a non-conservative force (friction) exerted.

PTS: 15 DIF: AP Physics C TOP: G