

Exercise 12 : Form factor of a homogeneously charged sphere

- a. Calculate the form factor of a homogeneous, spherical charge distribution:

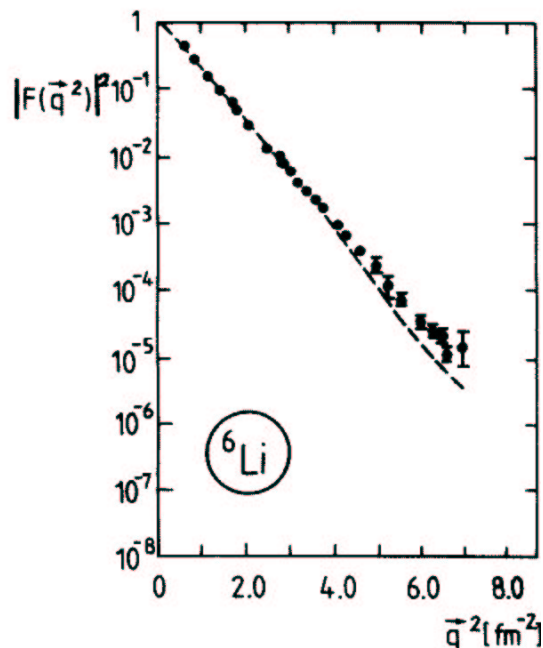
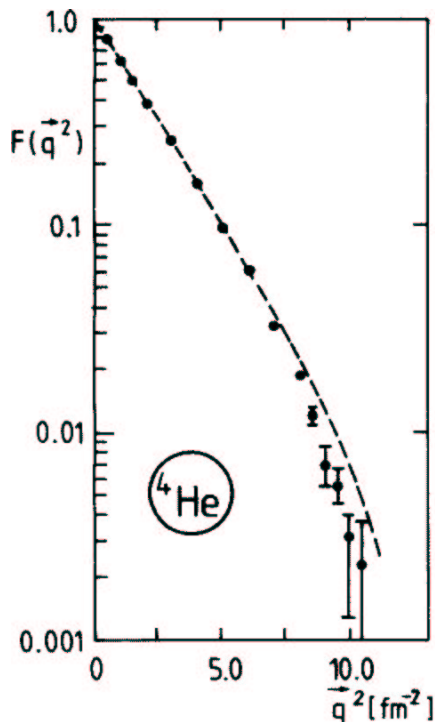
$$\frac{1}{Z}\rho(r) = \begin{cases} \frac{3}{4\pi R^3} & \text{if } r \leq R, \\ 0 & \text{if } r > R. \end{cases}$$

- b. Scattering electrons elastically from carbon, one finds a minimum of the differential cross section at $q \approx 1.8 \text{ fm}^{-1}$. What is the radius of a carbon nucleus calculated from this, assuming a homogeneous, spherical charge distribution?

Exercise 13 : Charge radii of nuclei

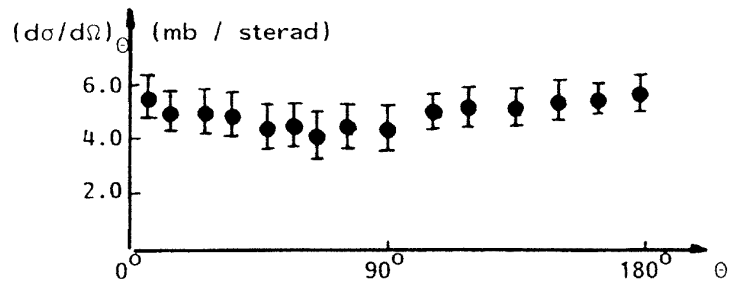
For light nuclei the charge density distribution $\rho(r)$ is well reproduced by a Gaussian distribution $\rho(r) = n \exp(-r^2/\alpha^2)$.

- a. Calculate the normalization constant n from $\int d^3x \rho(r) = Z$ (proton number).
 b. Find an expression for the mean square radius $\langle r^2 \rangle = \frac{1}{Z} \int d^3x r^2 \rho(r)$ as a function of α .
 c. Calculate the charge form factor $F(\mathbf{q}^2) = \frac{1}{Z} \int d^3x e^{-i\mathbf{q}\cdot\mathbf{r}} \rho(r)$.
 Show that $\langle r^2 \rangle = -6 \frac{d}{dq^2} F(\mathbf{q}^2) \Big|_{q^2=0}$.
 d. Estimate the charge radii $\sqrt{\langle r^2 \rangle}$ of the nuclei ${}^4\text{He}$ and ${}^6\text{Li}$ from the measured form factors, shown in the figure below.



Exercise 14 : Neutron - proton scattering

The differential cross section for the scattering of slow neutrons from protons has been measured to have the following shape:



- Show that the interaction potential between neutron and proton cannot be of the Coulomb type $V(r) = \frac{\text{const.}}{r}$.
- Estimate the range R of the neutron-proton interaction assuming that it is given by a potential

$$V(r) = \begin{cases} \infty & \text{if } r < R \\ 0 & \text{if } r \geq R \end{cases} \quad \text{Scattering from a "hard sphere"} \quad (1)$$

Comment the result.

- The differential cross sections for proton-proton and neutron-neutron scattering are found to be almost identical. What conclusions can you draw from this?
- Estimate qualitatively how the differential cross section will change at higher energies.

(Note: Units of cross sections: 1 barn = 10^{-24} cm², 1 mbarn = 10^{-27} cm² = 0.1 fm²)