Warm Up- 3.3

Find the exact value for each of the following:

1. $\sin\frac{5\pi}{4}$

2. $\sin\left(-\frac{5\pi}{4}\right)$

3. $\cos\frac{2\pi}{3}$

4. $\cos\left(-\frac{2\pi}{3}\right)$

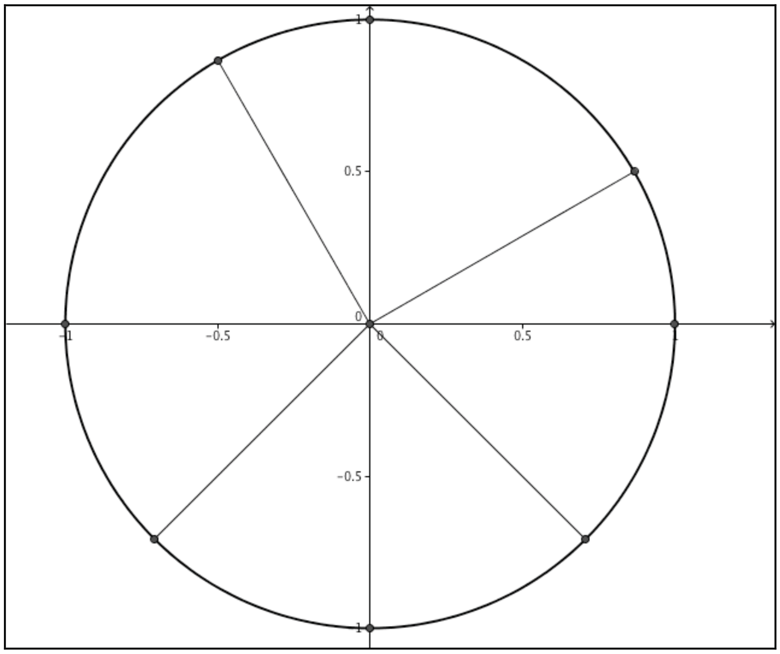
5. $\tan\frac{\pi}{6}$

6. $\tan\left(-\frac{\pi}{6}\right)$

What do you notice…..

The circle below is a unit circle (because it has a radius of 1). What is the equation of a circle centered at the origin with radius 1?

A few of the standard angles are shown (they have the good reference angles). Figure out which angles are drawn, recall that the radius is 1, and find the coordinates of the point on the unit circle where the angle intersects it. (Think about the ratio of the sides of a $30° - 60° - 90°$ or $45° - 45° - 90°$, draw reference triangles, and solve them!)



Now we need to think some. Look at the point you found in the first quadrant. What angle generated that intersection point? What are its sine and cosine? Let's do that for each angle and summarize in the chart below:

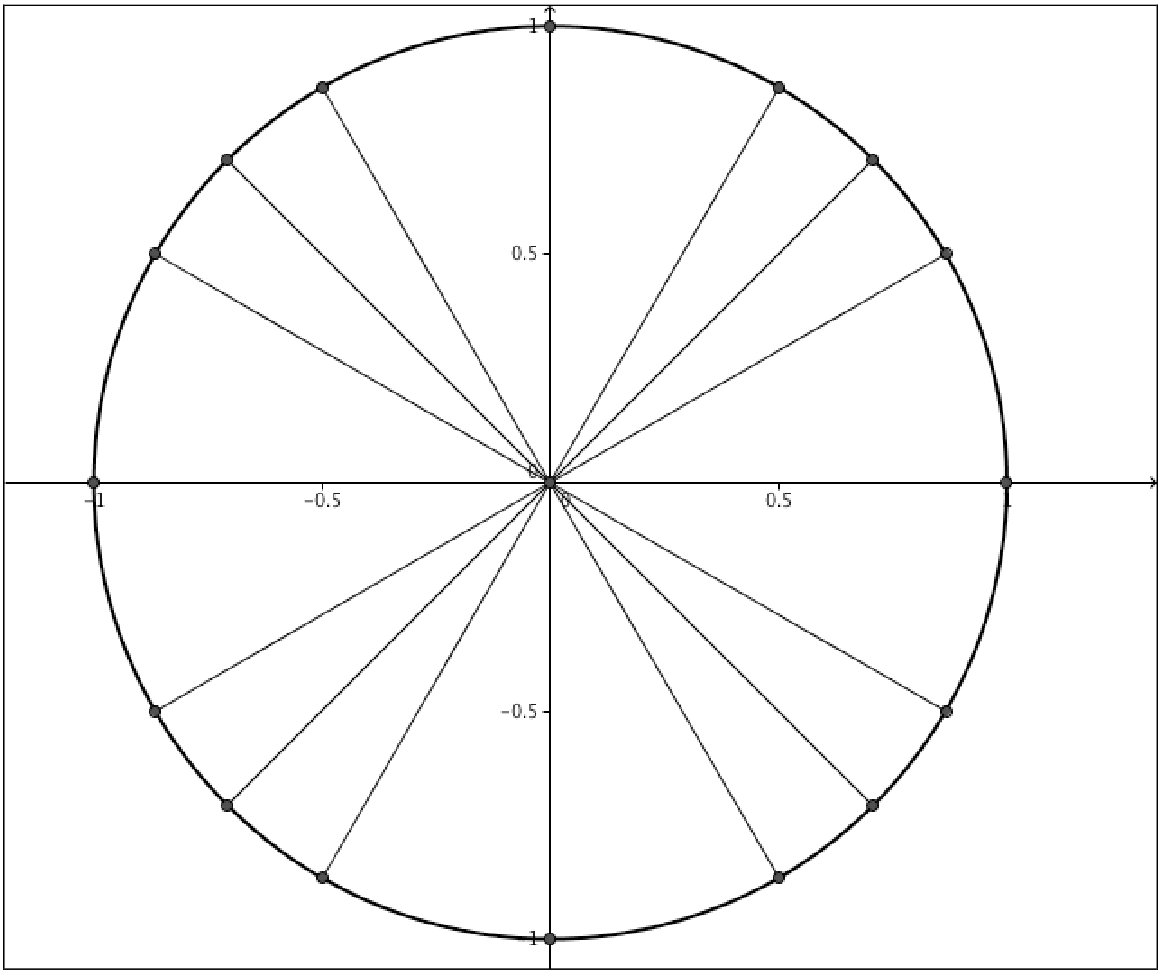
θ	$\sin(\theta)$	$\cos(\theta)$	Unit Circle Point

Notice a pattern there?

If the point (x, y) is the intersection of the terminal side of angle θ and the unit circle, then, in terms of θ, the point can be written as ___________________.

On the *Unit Circle* every ordered pair can be written as $(\quad , \quad)$ of the angle passing through the point.

Use the points that you found previously (and confirmed with the class) and symmetry to fill in the coordinates of all the intersection points with the unit circle below.



The Unit Circle is one of the more elegant (and at the same time accessible) things in mathematics. It ties together a lot of ideas: the trig functions, symmetry, intersection points, periodicity, and lots of things I'm probably not thinking of offhand. If for no other reason than that you should appreciate it a little bit.

On the Unit Circle:

$\cos(\theta) =$ $\qquad$ $\sin(\theta) =$ $\qquad$ $\tan(\theta) =$

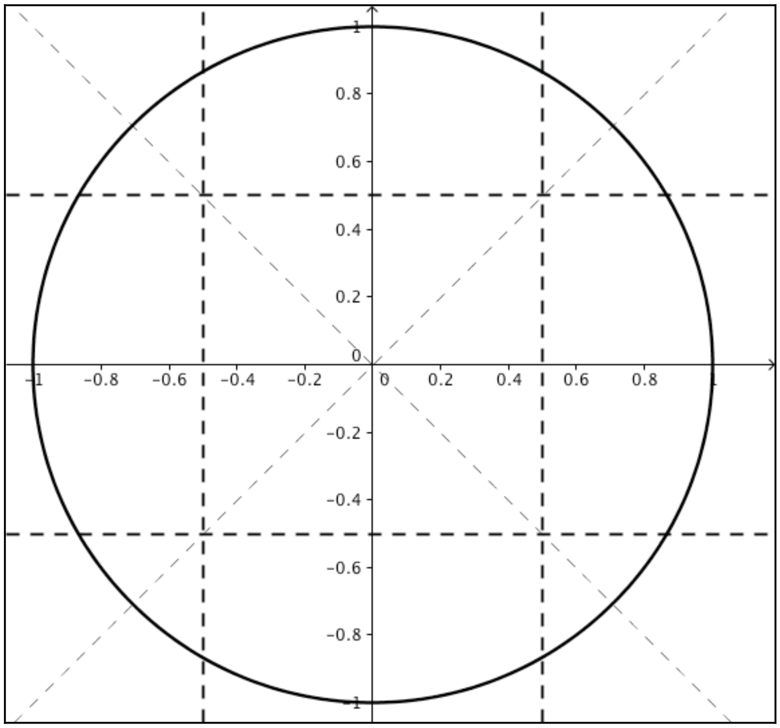
$\sec(\theta) =$ $\qquad$ $\csc(\theta) =$ $\qquad$ $\cot(\theta) =$

(this is another set of definitions of the trig functions…)

Use your calculator to approximate each of the following:

$\frac{\sqrt{2}}{2} \approx$	$\frac{\sqrt{3}}{2} \approx$

These values are important because they are the exact values of certain trig functions at certain angles and they can be found on the unit circle below. Keep them in mind as you answer the questions that follow the circle.



Use the unit circle above to put an inequality sign between each of the following pairs. You might also need to consider reference angles, cofunctions, and ASTC to make this happen. Try to have a compelling argument for your choice.

$\sin(35°)$ $\square$ $\sin(42°)$ $\sin(155°)$ $\square$ $\sin(40°)$ $\cos(70°)$ $\square$ $\sin(160°)$

$\cos(12°)$ $\square$ $\sin(20°)$ $\cos(170°)$ $\square$ $\cos(215°)$ $\sin(48°)$ $\square$ $\cos(41°)$

$\sin(145°)$ $\square$ $\cos(280°)$ $\cos(359°)$ $\square$ $\sin(178°)$ $\cos(165°)$ $\square$ $\cos(220°)$

Even and Odd Functions:

Even	Odd
A function is even if	A function is odd if
Example: $f(x)=x^2$	Example: $f(x)=x^3$

The trig functions don't work quite so easily (since we'd really have nothing algebraic to finagle with) so we'll need to do something else. We can use a whole bunch of stuff we already know to figure this out. Here's a short list of stuff that will be useful: the Unit Circle, ASTC, reference angles.

Sketch each of the following, determine their reference angle, make a generalization:

$75°$	$-75°$
$130°$	$-130°$

Generalization about θ and $-\theta$?

Since they have the same reference angle the $|\sin(\theta)|$ and $|\sin(-\theta)|$ are equal (that's the most useful feature of reference angles).

Because of that fact above, it's possible that $\sin(-\theta)=\sin(\theta)$ (which would mean sine is even) or $\sin(-\theta)=-\sin(\theta)$ (which would mean sine is odd). The same might be true for cosine, tangent, cosecant, secant, and cotangent…We need to figure out which is true for each of them.

Fill in the first two columns of this table. We'll fill in the last as a class.

Original Angle (exact value)	Opposite of Original (exact value)	Opposite of Original (in terms of original)
$\sin\left(\frac{\pi}{6}\right)=$	$\sin\left(-\frac{\pi}{6}\right)=$	$\sin\left(-\frac{\pi}{6}\right)=$
$\sin(135°)=$	$\sin(-135°)=$	$\sin(-135°)=$
$\sin\left(\frac{4\pi}{3}\right)=$	$\sin\left(-\frac{4\pi}{3}\right)=$	$\sin\left(-\frac{4\pi}{3}\right)=$
$\sin(315°)=$	$\sin(-315°)=$	$\sin(-315°)=$

Based on the evidence in the table above we can conclude that sine is what type of function and why?

Original Angle (exact value)	Opposite of Original (exact value)	Opposite of Original (in terms of original)
$\cos\left(\frac{\pi}{6}\right)=$	$\cos\left(-\frac{\pi}{6}\right)=$	$\cos\left(-\frac{\pi}{6}\right)=$
$\cos(120°)=$	$\cos(-120°)=$	$\cos(-120°)=$
$\cos\left(\frac{4\pi}{3}\right)=$	$\cos\left(-\frac{4\pi}{3}\right)=$	$\cos\left(-\frac{4\pi}{3}\right)=$
$\cos(180°)=$	$\cos(-180°)=$	$\cos(-180°)=$

Based on the evidence in the table above we can conclude that sine is what type of function and why?

So to summarize:

Cosine is an _______________ function. Sine is an ________________ function.

Most trig functions are odd. Only cosine and secant are even.
We can use the properties of even and odd functions to evaluate all sorts of things relatively quickly.

Evaluate each of the following:

Evaluate	Rewrite (even/odd property)	Final evaluation
$\sin\left(-\frac{5\pi}{4}\right)$		
$\sin(-270°)$		
$\csc(-\pi)$		

For your enjoyment, here are two of my favorite Unit Circle Style Questions:

An angle in standard position intersects the unit circle at the point $\left(\frac{6}{\sqrt{61}},-\frac{5}{\sqrt{61}}\right)$.

a. Confirm that this is possible (use the equation of the unit circle!).

b. Find the exact value of all six trigonometric functions at the angle.

c. Find the angle using your calculator.

Fun Fact:

Similar triangles and the unit circle allow us to really quickly figure out the exact values of the cosine and sine of any angles whose terminal sides pass through "Pythagorean triple points." (I just sort of made that quoted term up, but I think you'll get what I mean.)

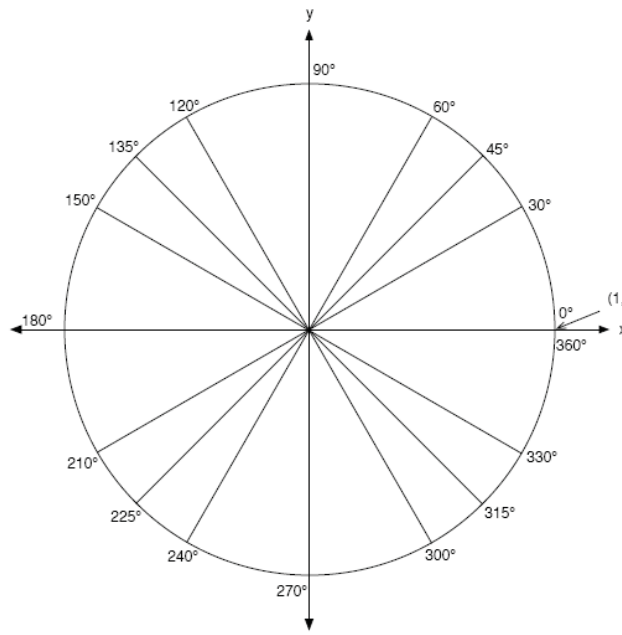
Find the point at which the terminal side of an angle passing through the given points intersects the unit circle. Do you get it?

Given Point	Some Work?	Unit Circle Point
$(5,12)$		
$(-7,24)$		
$(-3,-4)$		

The notes above roughly correlate to Section 3.3 of your textbook.

Name _______________________________ 3.3 Circular Functions Extra Practice

1. Write in the equivalent radian measures for all of the angles written in degrees.



2. Evaluate the six trigonometric functions at each real number.

a. $t = \dfrac{\pi}{3}$ b. $t = \dfrac{3\pi}{4}$

3. Can you name some functions from your past that were even and odd? Sketch a graph of the functions you name.

4. Which trigonometric functions are even and odd?

5. Use your even and odd facts to evaluate:

$\sin(-60^\circ)$

$\tan(-45^\circ)$

$\sin\left(-\dfrac{\pi}{4}\right)$

$\sec\left(-\dfrac{\pi}{3}\right)$

$\csc\left(-\dfrac{\pi}{6}\right)$