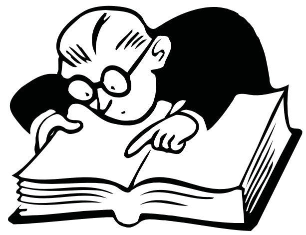
Name: ______________________________ Date: ______________________________
Period: ______ Trigonometry- ***3.1 Warm Up***

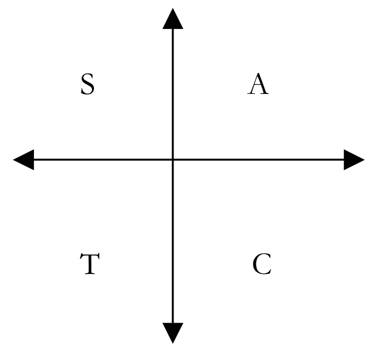


Let's Recall from Chapter 1!

A reference angle (for any angle in standard position) is the positive acute angle made with the x-axis and the terminal side of the angle.

The Chart *(must be memorized!)*

	0	**30**	**45**	**60**	**90**
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined



1. Draw the following angles in standard position and name the reference angle.

$a.\ 317^\circ$ $b.\ 99.9^\circ$

$c.\ 190^\circ$ $d.\ -400^\circ$

2. Find the exact value for each of the following.

$a.\ \cos 315^\circ$ $b.\ \sin 240^\circ$

$c.\ \tan 150^\circ$ $d.\ \sec 120^\circ$

$e.\ \sin 225^\circ$ $f.\ \cot 330^\circ$

Steps:

1. Identify the quadrant in which the angle terminates. (This will determine the sign of the exact value)
2. Identify the reference angle.
3. Look up the trig value for the reference angle.
4. Using ***A***ll ***S***tudent ***T***ake ***C***alculus (or ***C***lasses), determine the appropriate sign.

Reference Angle:

The *reference angle* for any angle θ in standard position is the positive acute angle between the terminal side of θ and the *x*-axis. We will use θ' (theta prime) to denote a reference angle. The textbook uses $\hat{\theta}$ (theta hat).

Reference angles are pretty easy to "see" when you look at an angle. Find the reference angle for each of the following angles by drawing the angle and looking (with a little arithmetic).

$\theta = 35°$	$\theta = 125°$
$\theta = 235°$	$\theta = 335°$

There are formulas for how to find reference angles. We'll write them down, but you're much better off just understanding.

Quadrant I: $\theta' =$
Quadrant II: $\theta' =$
Quadrant III: $\theta' =$
Quadrant IV: $\theta' =$

Again, you can use these formulas, but you don't really need them if you understand what a reference angle is…just think about the picture!

A useful thing about reference angles…

The Reference Angle Theorem
A trigonometric function of an angle and its reference angle differ at most in sign.

Use your calculator to first evaluate each of the following and then to find the reference angle for each. Write trig functions to three decimals and the reference angle in DMS.

θ	$\sin(\theta)$	$\csc(\theta)$	$\cot(\theta)$	Ref Angle
$15°25'37"$				
$164°34'23"$				
$195°25'37"$				
$344°34'23"$				

Now restate the reference angle theorem in your own words.

We can use the reference angle theorem and all the values that you have memorized to find the exact value of a lot of different angles.

θ	Quadrant	θ'	$\sin(\theta')$	$\sin(\theta)$	$\cos(\theta')$	$\cos(\theta)$
$150°$						
$210°$						
$120°$						
$300°$						
$225°$						
$315°$						

It's important to instantly recognize the angles between 0 and $360°$ that have the key reference angles $30°$, $45°$, and $60°$. List them below:

θ'	QI	QII	QIII	QIV
$30°$				
$45°$				
$60°$				

Because you have memorized the sine, cosine, and tangent of each of those specific reference angles, you've also memorized the sine, cosine, and tangent of each angle you just listed in that table; you just need to realize it.

Now that we have an understanding of reference angles we can work backwards to find the answers to all sorts of questions. Most notably, we can find angles from certain given information.

Let's start by writing down those formulas for reference angles that I told you you're better off not memorizing (did I lie? Oops…). Then we'll solve them for θ rather than θ'.

Quadrant	Solved for θ'	Solved for θ
I		
II		
III		
IV		

We'll need those.

Here's a little fact you might not know (nor do you really have any reason to know it before this…):

> Getting Reference Angles on Calculators
> If you use the inverse functions on your calculator with the absolute value of any given trig ratio, the calculator always returns the reference angle!

That's a big deal. If you don't use the absolute value you are (potentially) entering a world of pain.

Find the angle/angles between 0 and $360°$ in the indicated quadrant satisfying the given information.

Ratio	Quadrant	Reference Angle	Angle(s)
$\sin(\theta) = 0.5388$	I		
$\sin(\theta) = 0.5388$	II		
$\cos(\theta) = -0.5736$	III		
$\tan(\theta) = -1.8041$	IV		
$\sec(\theta) = 3.9939$	IV		
$\cot(\theta) = -2.3559$	II		
$\sin(\theta) = -.4578$	???		
$\lvert\cos(\theta)\rvert = .6582$	???		

An infinite number of solutions? Yeah…it's true.

> If two angles are coterminal, then the values of the trig functions are the same at those angles.

Compare $\sin(22°)$ to $\sin(382°)$ to $\sin(742°)$.
What's true about these values?

What's true about the angles?

How would you write every single angle for which $\sin(\theta)$ is equal to $\sin(22°)$?

Find every possible angle satisfying the given information.

Ratio	Quadrant	Reference Angle	Angle(s)
$\sin(\theta) = 0.4688$	???		
$\csc(\theta) = 6.3287$	II		
$\cos(\theta) = -0.4523$	???		
$\sec(\theta) = 2.8119$	IV		

Summary:
- We can find the reference angle for any angle.
- You're better off thinking of the picture of the angle than memorizing formulas.
- We can find the exact values of trig functions for any angle in the 30, 45, or 60 camp.
- We can work backwards (and often do) to find angles in any quadrant given ratios.
- Unless otherwise restricted, there are an infinite number of angles that fit the given information.

All of these notes roughly correlate with Section 3.1 in the textbook.