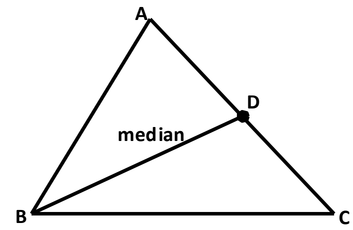
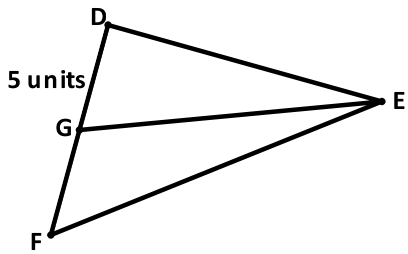
Geometry Concepts 5.4 Notes on Medians and Centroids Name ______________________________

A **median** in a triangle is a segment that connects a vertex of the triangle to the midpoint of the opposite side.



In the figure to the right, $\overline{BD}$ is a median in ΔABC.

Note how it "begins" at a vertex (point *B*). Also, note how it connects to point *D*, which appears to be the middle of side $\overline{AC}$.



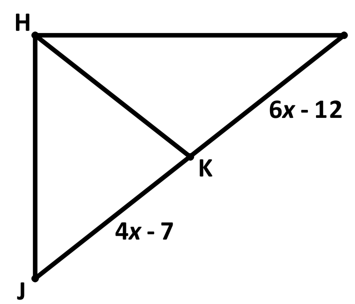
Suppose $\overline{GE}$ is a median of ΔDEF to the left.

If $DG = 5$, what is *GF*?

Since $\overline{GE}$ is a median, point *G* must be the midpoint of side $\overline{DF}$. This means it cuts the side into two equal portions.

Therefore, if $DG = 5$, then $\boxed{GF = 5}$.

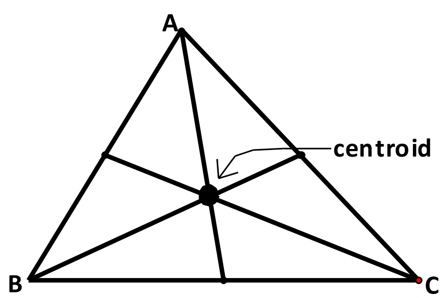
If $\overline{HK}$ is a median of ΔHIJ to the right, what is the value of *x*?



This must mean that point *K* is the midpoint of side $\overline{JI}$.

So again, point *K* divides the side into two equal pieces.

Therefore, $4x - 7$ must equal $6x - 12$:



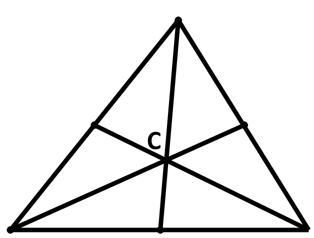
The three medians of a triangle intersect at exactly one point. This means they are <u>concurrent</u>.

The point of concurrency for the medians of a triangle is known as the **centroid**.

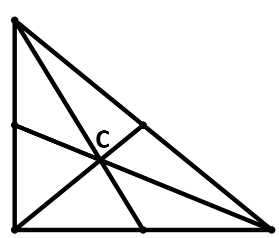
The centroid is labeled in the diagram to the left.

The location of the centroid (point *C*) has been found for three types of triangles below:

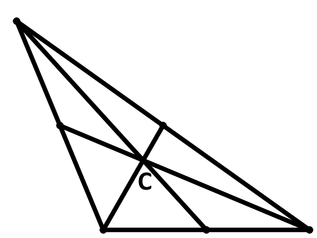
Acute



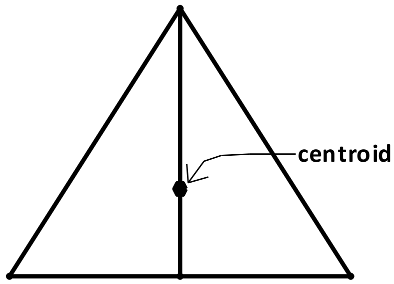
Right



Obtuse

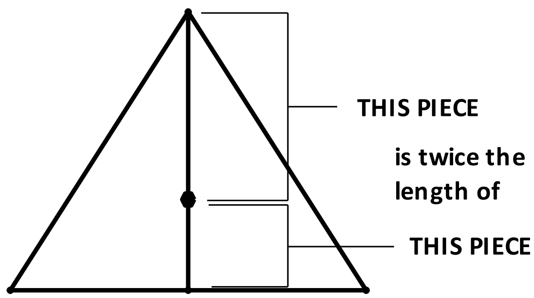


The centroid is always located **inside** of a triangle.



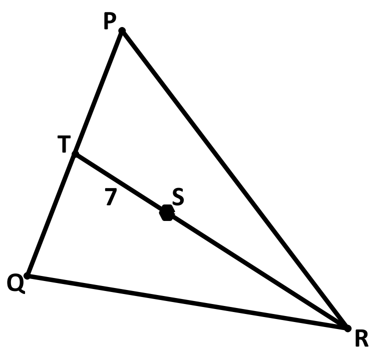
Suppose the centroid of the triangle to the left has already been located, and one of its medians is drawn.

An important property was observed in the task from the last lesson:



On any median, the distance from the vertex to the centroid is <u>two times</u> (double) the distance from the centroid to the opposite side.

In the figure to the left, this relationship is explained in visual form.

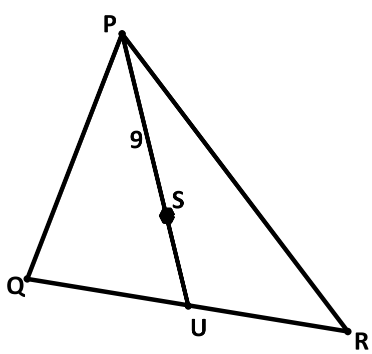


In the figure to the left, point *S* is the centroid of ΔPQR, and $\overline{TR}$ is one of its medians.

If *TS* = 7 units, what is *SR*?

Using the property from above, *SR* must be two times *TS*.

Therefore, SR= ______________



In the same triangle, suppose $\overline{UP}$ is another median.

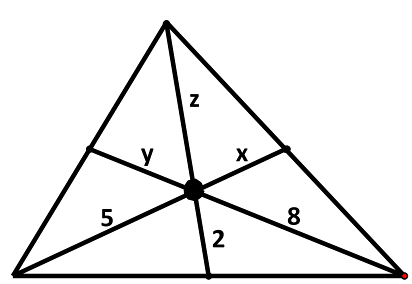
If *PS* = 9 units, what is *SU*?

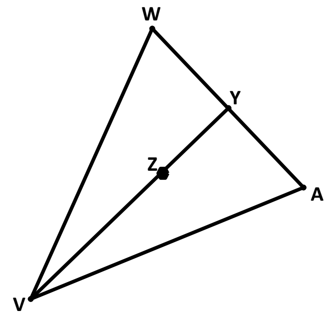
Using the same property, *PS* is two times *SU*.

So, in order to find *SU*, one would <u>divide by 2</u>.

SU =______________.

In the figure to the right, the three medians of the triangle have been drawn. Find the values of *x*, *y*, and *z*.



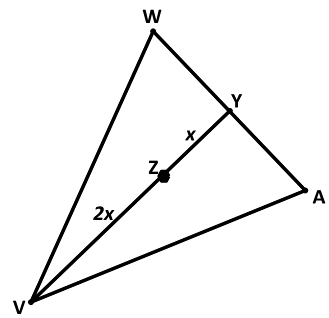


Suppose that point Z is the centroid of ΔWAV to the left. If $\overline{VY}$ is a median that measures 12 units, what is ZY?

Here is what we know:

We must find ZY. It is currently unknown, so call it x.

We know VZ is two times that length, so call it $2x$.

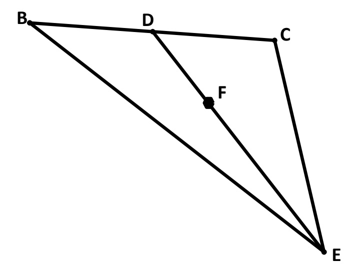


These two lengths, added together, must equal 12 (the length of the whole median, given in the problem). Thus:

$$x + 2x = 12$$
$$3x = 12$$
$$x = 4$$

Therefore, $\boxed{ZY = 4}$.

When doing problems like this, it is always best to name the shorter part of the median "x".



In the figure to the right, F is the centroid of ΔBCE.

If DE = 27, what is FE?

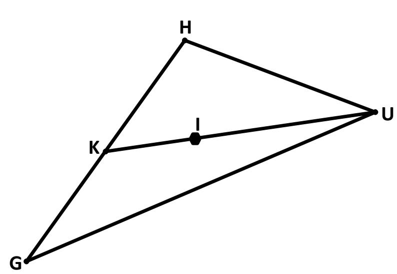
Again, let the shorter piece of the median, DF, be x.

And, let the longer piece of the median, FE, be $2x$. So:

$$x + 2x = 27$$
$$3x = 27$$
$$x = 9$$

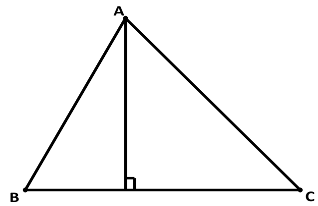
Wait! Remember that x represents the shorter piece (DF). So, that piece is 9 units.

However, the problem asks for FE (the longer piece). In order to find it, simply double the length of the shorter piece. Thus, $\boxed{FE = 18}$.



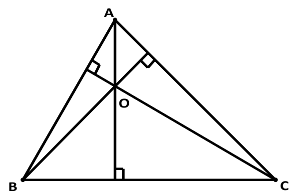
If point I is the centroid of ΔHUG, and one of its medians named $\overline{KU}$ has a length of 42 units, what is IU?

Notes on Orthocenters and Altitudes Name______________________________



An **altitude** of a triangle is a perpendicular segment from a vertex of the triangle to the line containing the opposite side.

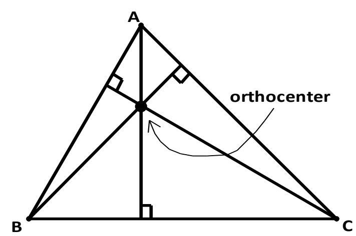
Note that an altitude has been drawn from vertex *A*.



Now, let's draw all three altitudes together in the same depiction of ΔABC. View the picture to the left.

As it turns out, all three altitudes are **concurrent** because they intersect at exactly one point - point *O*!

The **orthocenter** is the point of concurrency for the lines containing the altitudes of a triangle.

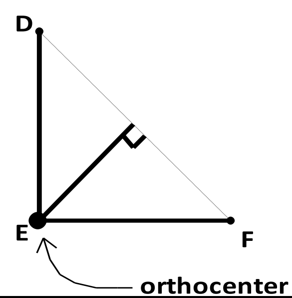


Observe ΔABC to the left. The orthocenter (the point where the altitudes meet) has been identified.

Compare the location of the orthocenter here to its location in each of triangles from Questions 3 & 4 above. This leads to the following very important conclusion:

If a triangle is an **acute** triangle, then its orthocenter is located **inside** of the triangle.

If a triangle is a **right** triangle, then the orthocenter is located on the **vertex of the right angle**.



If a triangle is a **obtuse** triangle, then the orthocenter is located on the **outside** of the triangle.