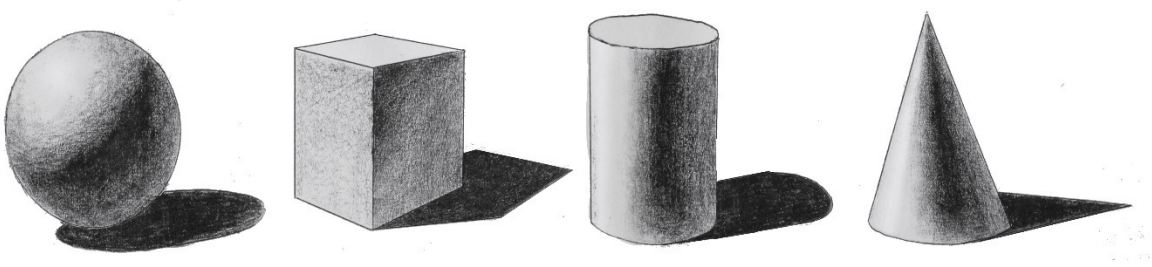
ACCELERATED MATHEMATICS

CHAPTER 15

DIMENSIONAL GEOMETRY II

TOPICS COVERED:

- Volume of Cylinders
- Volume of Cones
- Volume of Spheres
- Surface Area of Prisms
- Surface Area of Cylinders



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Accelerated Mathematics Formula Chart Name:

Linear Equations

Slope-intercept form	$y = mx + b$	Direct Variation	$y = kx$ (8^{th} grade)
Constant of proportionality	$k = \frac{y}{x}$	Slope of a line	$m = \frac{y_2 - y_1}{x_2 - x_1}$ (8^{th} grade)

Circumference Circle $C = 2\pi r$ or $C = \pi d$

Area

Rectangle	$A = bh$	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$
Parallelogram	$A = bh$	Circle	$A = \pi r^2$
Triangle	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$		

Surface Area (8^{th} grade)	**Lateral**	**Total**
Prism	$S = Ph$	$S = Ph + 2B$
Cylinder	$S = 2\pi rh$	$S = 2\pi rh + 2\pi r^2$

Volume

Triangular prism	$V = Bh$	Cylinder	$V = Bh$ or $V = \pi r^2 h$ (8^{th} grade)
Rectangular prism	$V = Bh$	Cone	$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$ (8^{th})
Pyramid	$V = \frac{1}{3}Bh$	Sphere	$V = \frac{4}{3}\pi r^3$ (8^{th} grade)

Pi $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$

Distance	$d = rt$	**Compound Interest**	$A = P(1 + r)^t$
Simple Interest	$I = prt$	**Pythagorean Theorem**	$a^2 + b^2 = c^2$ (8^{th} grade)

Customary – Length	***Customary – Volume/Capacity***	***Customary – Mass/Weight***
1 mile = 1760 yards 1 yard = 3 feet 1 foot = 12 inches	1 pint = 2 cups 1 cup = 8 fluid ounces 1 quart = 2 pints 1 gallon = 4 quarts	1 ton = 2,000 pounds 1 pound = 16 ounces
Metric – Length	***Metric – Volume/Capacity***	***Metric – Mass/Weight***
1 kilometer = 1000 meters 1 meter = 100 centimeters 1 centimeter = 10 millimeters	1 liter = 1000 milliliters	1 kilogram = 1000 grams 1 gram = 1000 milligrams

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Area/Volume/Surface Area Computation Page

EXAMPLES

$A = \frac{1}{2}(b_1 + b_2)h$

$A = \frac{1}{2}(10 + 20) \bullet 6$

$A = 90 \text{ cm}^2$

$V = \pi r^2 h$

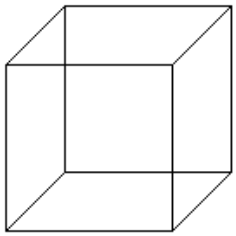
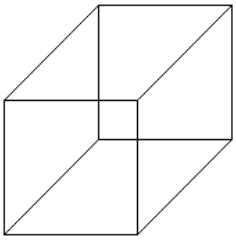
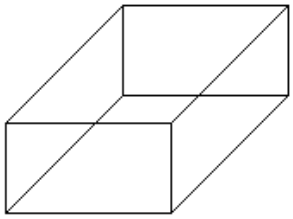
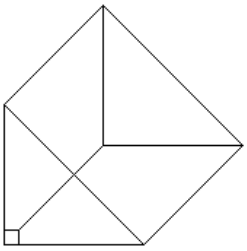
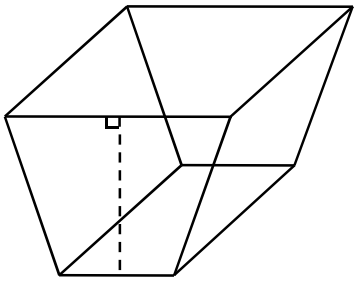
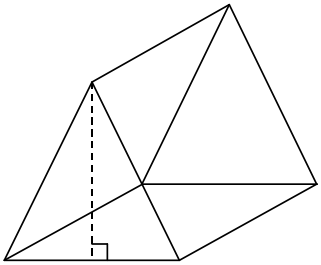
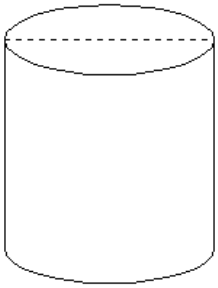
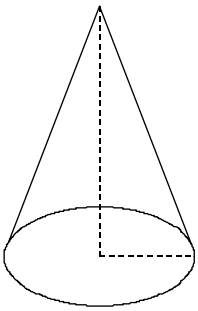
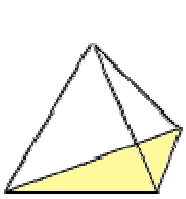
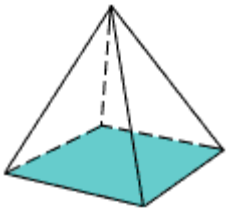
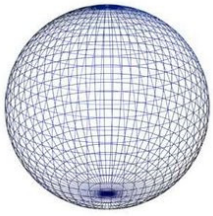
$V = 3.14 \bullet 10^2 \bullet 5$

$V = 1570 \text{ m}^3$

$S = 2B + Ph$

$S = 2(8 \bullet 6) + (28) \bullet 10$

$S = 376 \text{ in}^2$

Cube	Square prism
	
Rectangular prism	Right triangular prism
	
Trapezoidal prism	Isosceles triangular prism
	
Cylinder	Cone
	
Triangular and Square Pyramids  	Sphere 

Activity 15-1: **Volume of Cylinders** Name: _____

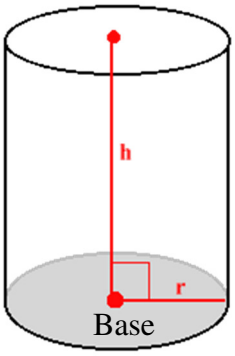
The volume of a solid is how much it can hold or the measure of the amount of space it occupies.

It is measured in cubic units.

The formula for a cylinder is $V = Bh$ or $V = \pi r^2 h$.

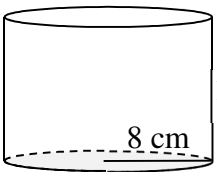
The **B** stands for the area of the base and the

h stands for the height of the cylinder.

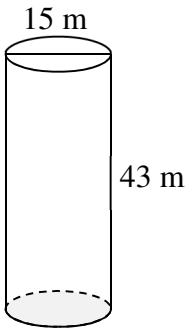


Find volume of the cylinder.

V = _____



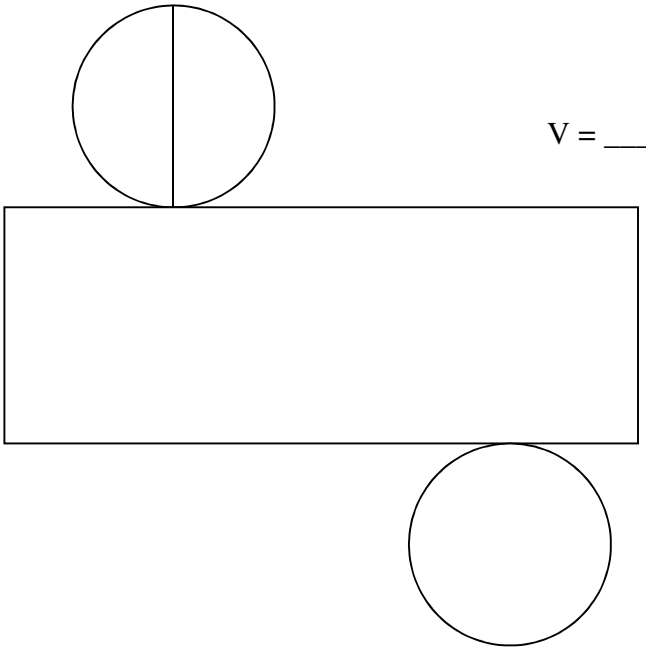
V = _____



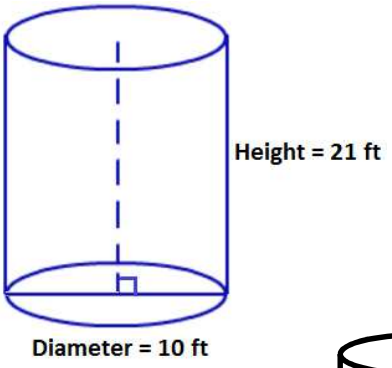
Please measure to the nearest $\frac{1}{4}$ of an inch.

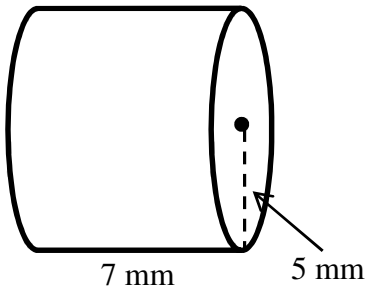
Dimensions: _____, _____, _____

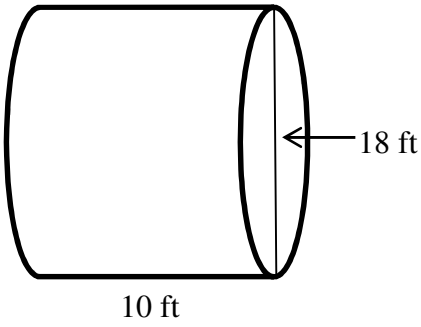
V = _____

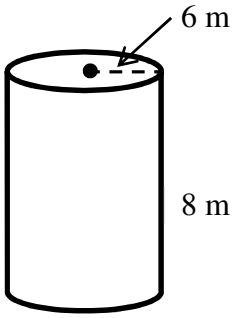


1-4. Find the volume of each cylinder.









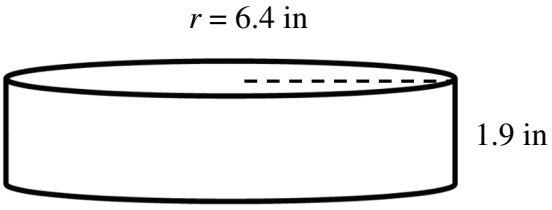
Find the volume of the cylinder with radius r and height h.

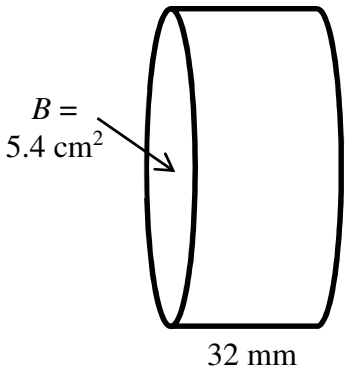
5. $r = 6$ in, $h = 12$ in

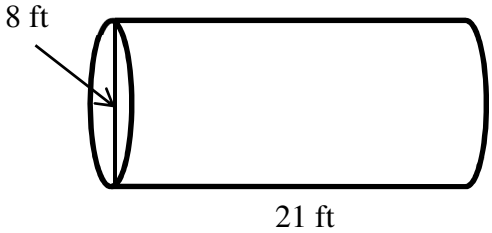
6. $r = 2$ cm, $h = 13$ cm

7. $r = 1.9$ m, $h = 8.7$ m

8-10. Find the volume of the solid. If two units of measure are used, give your answer in the smaller units. Round your answer to the nearest hundredth.



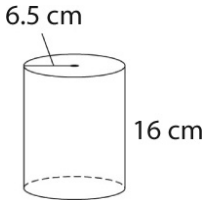




Activity 15-3: **Volume of Cylinders** Name:

Find the volume of each cylinder. Round your answer to the nearest tenth if necessary. Use 3.14 for π.

1.



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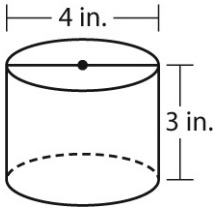
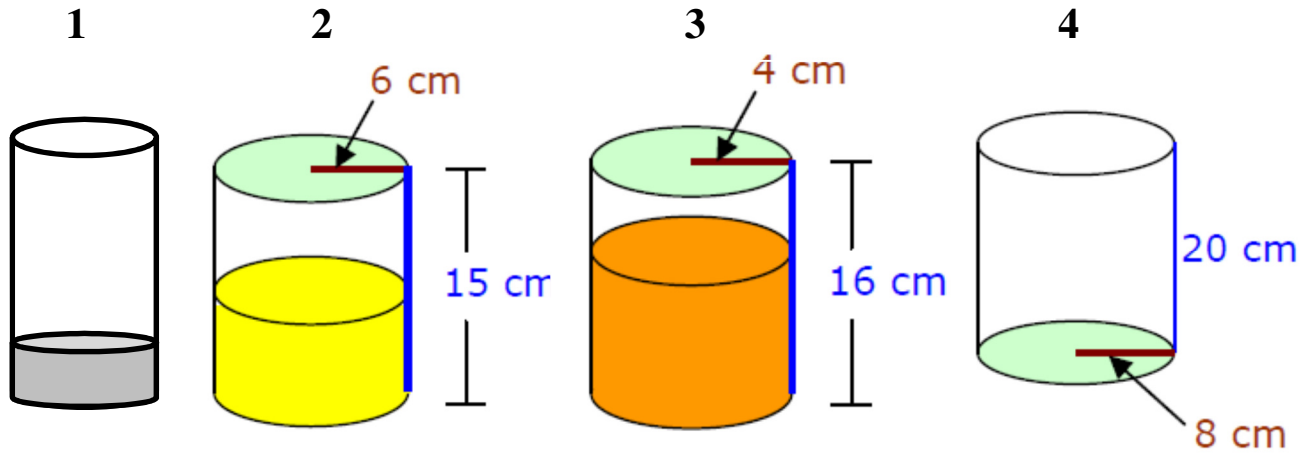


3.	A cylindrical oil drum has a diameter of 2 feet and a height of 3 feet. What is the volume of the oil drum?	
4.	New Oats cereal is packaged in a cardboard cylinder. The packaging is 10 inches tall with a diameter of 3 inches. What is the volume of the New Oats cereal package?	
5.	A small plastic storage container is in the shape of a cylinder. It has a diameter of 7.6 centimeters and a height of 3 centimeters. What is the volume of the storage cylinder?	
6.	A can of juice has a diameter of 6.6 centimeters and a height of 12.1 centimeters. What is the total volume of a six-pack of juice cans?	
7.	Mr. Macady has an old cylindrical grain silo on his farm that stands 25 feet high with a diameter of 10 feet. Mr. Macady is planning to tear down the old silo and replace it with a new and bigger one. The new cylindrical silo will stand 30 feet high and have a diameter of 15 feet. What is the volume of the old silo?	
8.	In the problem above, what is the volume of the new silo?	
9.	In the problems above, how much greater is the volume of the new silo than the old silo?	

Activity 15-4: **Volume of Cylinders** Name:

For the four problems below use the four corresponding pictures.

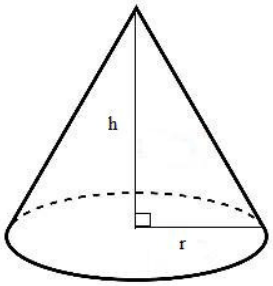


1.	A cylindrical glass vase is 6 inches in diameter and 12 inches high. There are 3 inches of sand in the vase, as shown.	
2.	Which of the following is closest to the volume of the sand in the vase? A $85\ in^2$ B $254\ in^2$ C $54\ in^2$ D $339\ in^2$	
3.	The radius of the base of a can of lemonade mix is 6 cm. The height of the can is 15 cm. The lemonade mix fills the can to a height of 7 cm. What is the volume of the lemonade mix in the can?	
4.	The radius of the base of a paint can is 4 cm. The height of the can is 16 cm. The paint in the can fills it to a height of 10 cm. How many liters of paint thinner must be added to the can in order to completely fill it to the top? Note: 1 liter of paint thinner fills 1000 cm^3. The radius of the base of a right circular cylinder is 8 cm. The height of the cylinder is 20 cm. Find the volume of the cylinder.	

Activity 15-5: **Volume of Cones** Name:

The **volume of a cone** is one third the product of the area of the base, B and the height, h.

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$



1. A jewelry maker designs a pair of cone shaped earrings out of sterling silver. How much sterling silver is needed to make a pair of earrings?

Height of the cone = 21 mm
Radius of the circle base = 5 mm

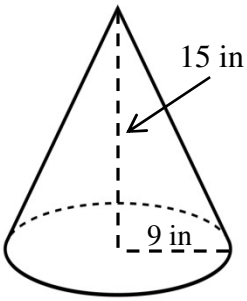
Find the volume of the cone with radius r and height h.

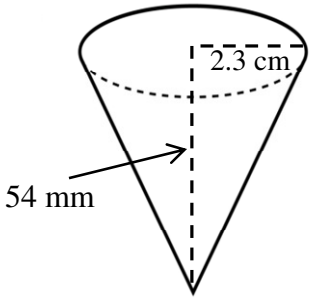
2. r = 8 in, h = 15 in

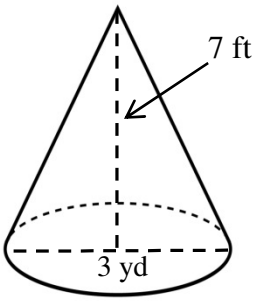
3. r = 10 m, h = 9 m

4. r = 24 mm, h = 18 mm

5-7. Find the volume of the cone. If two units of measure are used, give your answer in the smaller units. Round to the nearest tenth.







Find the volume of the cone with the given dimensions, where r = radius, d = diameter, and h = height. If two units are used, give your answer in the smaller units. Round to the nearest tenth.

8. r = 4 in, h = 12 in

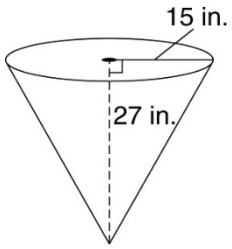
9. r = 2.1 m, h = 84 cm

10. d = 11 ft, h = 24 ft

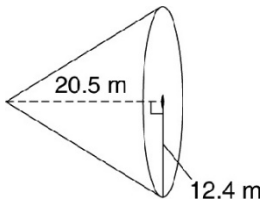
Activity 15-6: **Volume of Cones** Name:

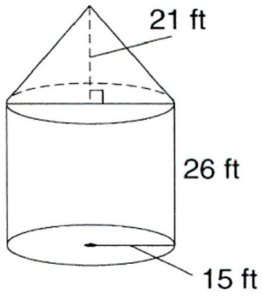
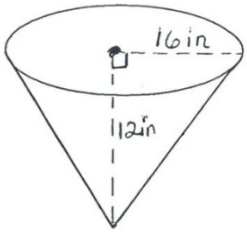
Find the volume of each cone. Round your answer to the nearest tenth if necessary. Use 3.14 for π.

1.



2.

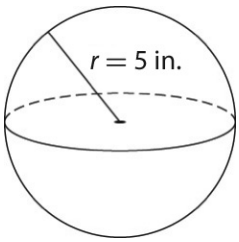


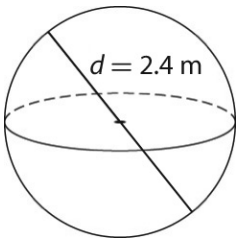
3.	The mold for a cone has a diameter of 4 inches and is 6 inches tall. What is the volume of the cone mold to the nearest tenth?	
4.	A medium-sized paper cone has a diameter of 8 centimeters and a height of 10 centimeters. What is the volume of the cone?	
5.	A funnel has a diameter of 9 in. and is 16 in. tall. A plug is put at the open end of the funnel. What is the volume of the cone to the nearest tenth?	
6.	A party hat has a diameter of 10 cm and is 15 cm tall. What is the volume of the hat?	
7.	Find the volume of the composite figure to the nearest tenth. a. Volume of cone b. Volume of cylinder c. Volume of composite figure 	
8.	**Cone Formula:** V = _____ What is the height of the cone? _____ What is the radius of the base? _____ 	
10.	A typical lodge pole pine tree found in Montana has a trunk that is about 8 inches in diameter at its base and grows to 50 feet tall. Estimate the volume of the trunk of the tree in cubic feet.	
11.	How many lodge pole pines are needed to make 1 cord of wood? (Cord = 4 feet by 8 feet by 4 feet. A typical cord has 20% air because of unused stacking space.)	

Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

Find the volume of each sphere. Round your answer to the nearest tenth if necessary. Use 3.14 for π. Show your work.

1. 

2. 

3. r = 3 inches

4. d = 9 feet

5. r = 1.5 meters

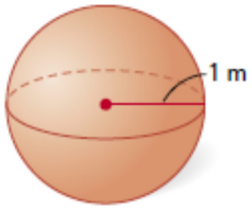
6.	A globe is a map of Earth shaped as a sphere. What is the volume to the nearest tenth of a globe with a diameter of 16 inches?	
7.	The maximum diameter of a bowling ball is 8.6 inches. What is the volume to the nearest tenth of a bowling ball with this diameter?	
8.	According to the National Collegiate Athletic Association men's rules, a tennis ball must have a diameter of more than $2\frac{1}{2}$ inches and less than $2\frac{5}{8}$ inches. What is the volume of a sphere with a diameter of $2\frac{1}{2}$ inches?	
9.	In the problem above, what is the volume of a sphere with a diameter of 2 inches?	
10.	In the problems above, write an inequality that expresses the range in the volume of acceptable tennis balls.	
11.	A regulation NBA basketball has a diameter of 9.4 inches. What is the volume of one of these basketballs? Round to the nearest tenth.	

Activity 15-8: **Volume of Spheres** Name: _____

Sphere Formula: Volume = _____

Find the volume of each solid below. Round answers to the nearest tenth.

1.



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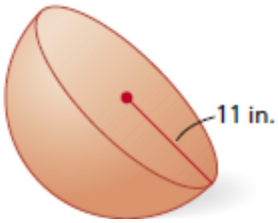

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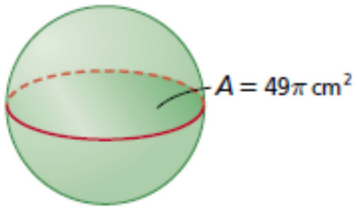
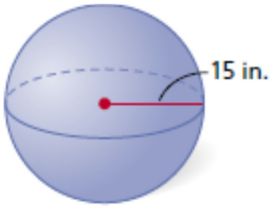
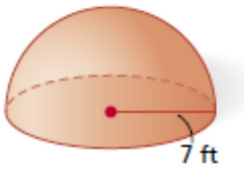


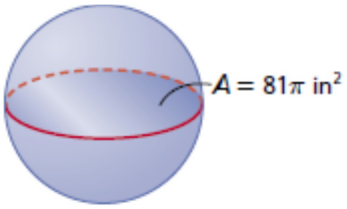
4.



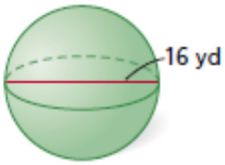
5.



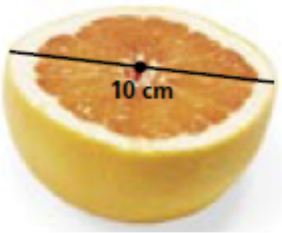
6.



7.



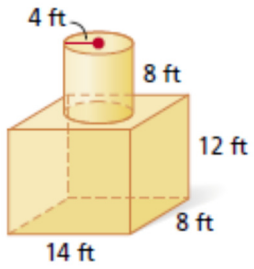
8. Approximately how many times as great is the volume of the grapefruit as the volume of the lime?

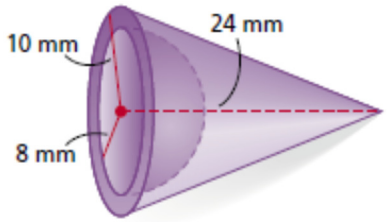


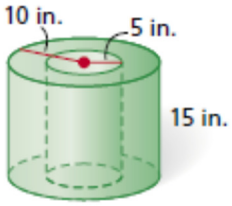
9. Find the volume of a sphere with a circumference of 36π ft.

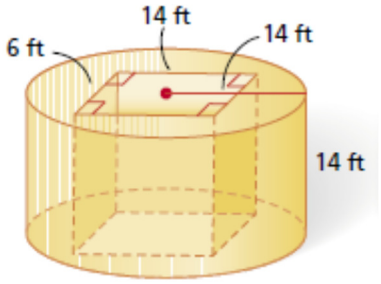
Activity 15-9: **Volume of Composite Figures** Name:

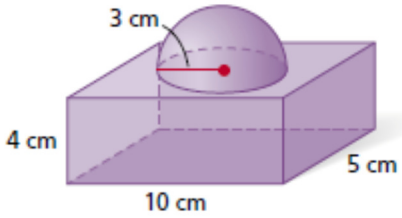
Find the volume of the composite figures below.

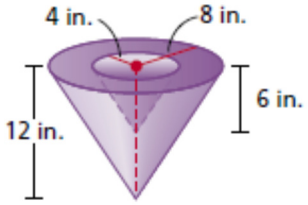
1. 

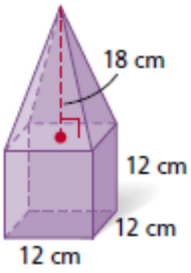
2. 

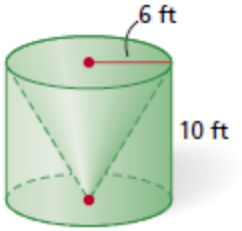
3. 

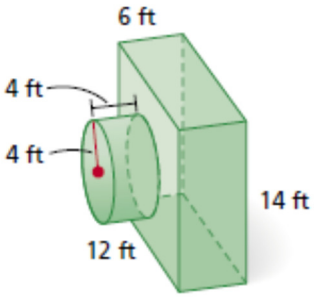
4. 

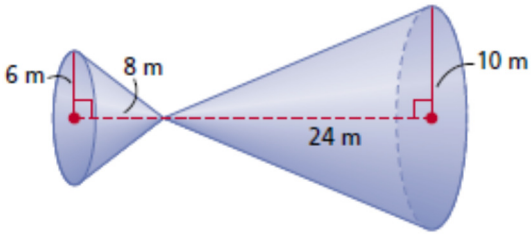
5. 

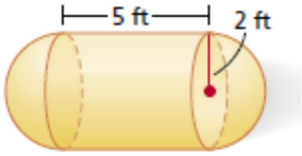
6. 

7. 

8. 

9. 

10. 

11. 

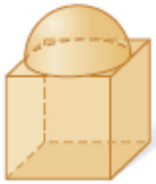
12. Which expression represents the volume of the composite figure formed by a hemisphere with radius r and a cube with side length $2r$?

A $r^3\left(\frac{2}{3}\pi+8\right)$

B $\frac{4}{3}\pi^3+2r^3$

C $2r^2(2\pi+12)$

D $\frac{4}{3}\pi^3+8r^3$



Spheres and Cones: We All Scream for Ice Cream

Round all answers to the nearest hundredths place.

It is a hot summer day. Several kids take Mrs. Bailey out for ice cream. Kathryn and Lauren wants their ice cream in a cone. Matthew and Jackson want their ice cream in a cylindrical cup. They are all getting one scoop of ice cream. Since Mrs. Bailey uses every opportunity for them to apply their knowledge, she poses a question to them. If the ice cream was packed into their container of choice, would one scoop fill that container? So as they ate their ice cream, they measured the diameter of the scoop of ice cream and the diameter and height of the cup and cone. They found that the diameter of the scoop of ice cream was 3 inches. The cup and cone both had a diameter of 3 inches and a height of 4.5 inches.

1.	What is the volume of the cone?	
2.	What is the volume of the cup?	
3.	What is the volume of the scoop of ice cream?	
4.	Will either or both of the containers hold the packed ice cream?	
5.	What is the ratio (x:1) of the cup's volume to the scoop of ice cream's volume? Explain what this means.	
6.	What is the ratio (x:1) of the cone's volume to the scoop of ice cream's volume? Explain what this means.	

Two more students join the group eating ice cream. Josh and Kevin are given an additional challenge. After the others explain their findings, Josh and Kevin are asked to find out what the height of the cone would have to be to hold two scoops of packed ice cream. Mrs. Bailey promised Josh and Kevin that they would get two scoops of ice cream if they could solve this problem. The diameter of the cone is still 3 inches.

7.	What is the volume of two scoops of ice cream?	
8.	If the radius of the cone remains the same, what must the height of the cone be so that the two scoops of packed ice cream will fill the cone without excess? Is this a reasonable height for an ice cream cone? Why or why not?	

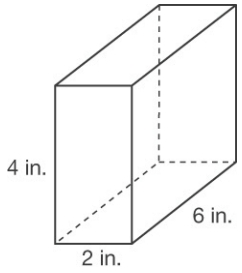
Everyone was having so much fun Mrs. Bailey promised to buy each of them a cylindrical container of their favorite ice cream to take home with them if they could answer the following questions.

9.	The cylindrical container of ice cream has a diameter of 5 inches and a height of 7.25 inches. What is its volume?	
10.	The product label claims that the container holds 14 scoops of ice cream with each scoop having a diameter of 3 inches. How many scoops of ice cream will the container really hold?	

Created by Lance Mangham, 6th grade math, Carroll ISD

Find the surface area of each prism.

1.



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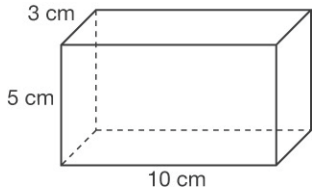
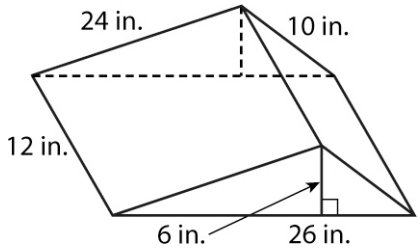


3.



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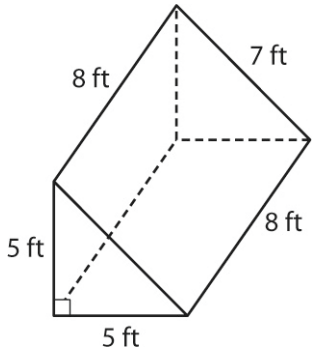
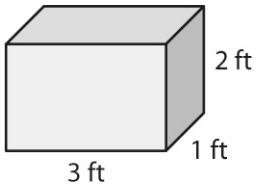


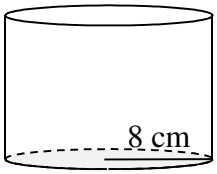
5. Marita is decorating the prism at the right with tiles. Each tile is 1 square foot. Each tile costs $0.45. How much will it cost Marita to tile the whole prism?



Activity 15-11: **Surface Area of Cylinders** Name: _____

There are two _____ _____of a solid. The _____ surface area is the amount of surface on the _____ _____of the solid. This does NOT include the _____. The _____ surface area is the amount of surface on _____ faces.

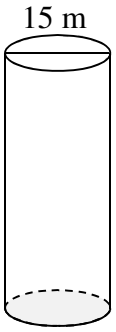
*The formulas for a **cylinder** are* _____ and _____.



LSA = _____

SA = _____

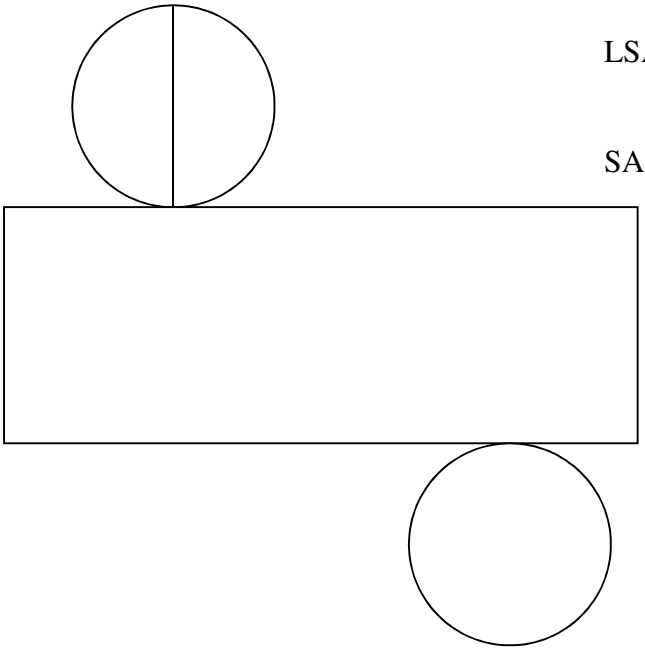
Your turn: Find lateral surface area and total surface area of the cylinder.



LSA = _____

SA = _____

Please measure to the nearest $\frac{1}{4}$ of an inch.



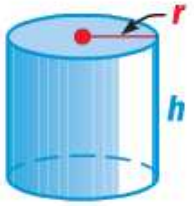
Dimensions: _____, _____, _____

LSA = _____

SA = _____

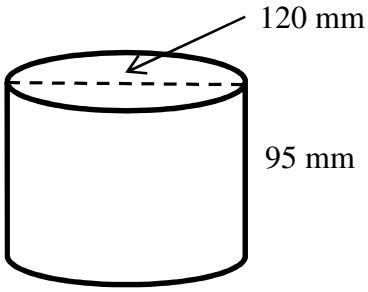
Activity 15-12: **Surface Area of Cylinders** Name:

The **surface area** of a polyhedron is the sum of the areas of its faces. The **surface area of a cylinder** is the sum of twice the area of the base and the product of the base's circumference and the height.



$$S = 2\pi r^2 + 2\pi rh$$

1. Find the surface area of a stack of CDs.



2. Find the surface area of a cylinder that has a radius of 5 feet and a height of 8 feet.

Sketch a cylinder with radius r and height h. Then find its surface area. Use 3.14 for pi.

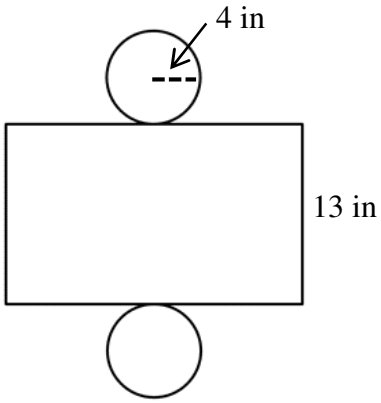
3. $r = 4$ cm, $h = 8$ cm

4. $r = 10$ cm, $h = 12$ cm

5. $r = 3$ ft, $h = 21$ ft

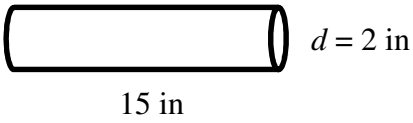
Identify the solid shown by the net. Then find the surface area. Use 3.14 for pi.

6.

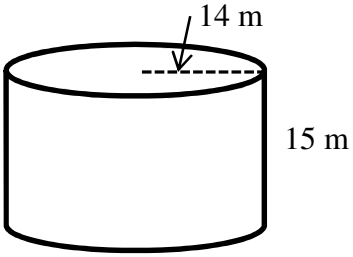


Draw a net for the solid. Then find the surface area of the solid. Use 3.14 for pi.

7.



8.



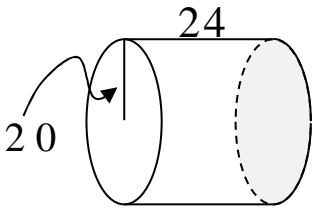
Activity 15-13: **Volume and Surface Area of Cylinders** Name:

Solve the following application problems. Draw a picture to help you.

1.	Campbell's soup company is having a contest for students at DIS to redesign the label for the chicken noodle soup. If the diameter of the can is 3 in, and the height is 4 in, how much paper do students need to create their design?	
2.	Susan has a fish tank in the shape of a cylinder that is 26 inches tall. The diameter of the tank is 12 inches. If there are 2 inches of rocks in the bottom, how much water is needed to fill the tank?	

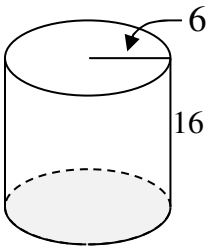
3. V = _____

SA=_____



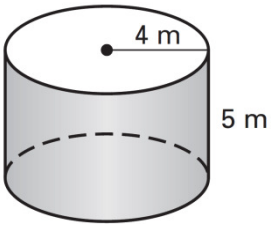
4. V = _____

SA=_____



Find the surface area of each figure. Don't forget to include units!

5.



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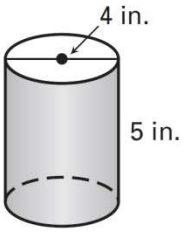
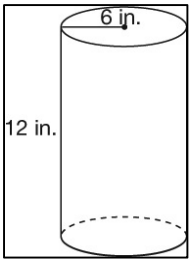


#7-9: Find the lateral surface area of each cylinder.
#10-12 Find the total surface area of each cylinder.
Round your answers to the nearest tenth, if necessary. Use 3.14 for π.

7.



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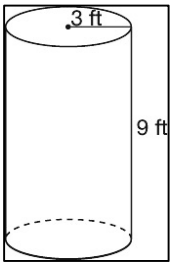

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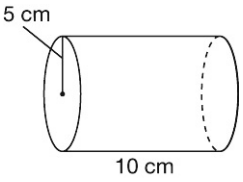

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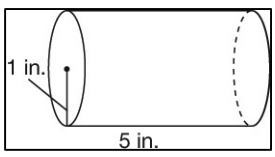

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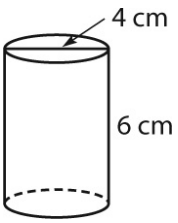
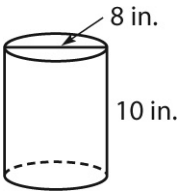


12.



The Buffalo snowstorm has buried the Bills' Ralph Wilson Stadium

In November 2014 Buffalo, NY was hit with multiple snowstorms that ended up dumping about 5 feet of snow on Ralph Wilson Stadium. The Buffalo Bills were to play the New York Jets that following Sunday and the ground crew had no idea if they would be able to get the stadium ready in time for the game. Here are some headlines about the event:

The Bills estimate they will need to remove 220,000 tons of snow to clear the stadium for Sunday's game. The team is seeking at least 500 fans -- working on three shifts -- to shovel out the stadium. In return, the Bills will pay fans $10 per hour and offer free game tickets.

Assuming one person was shoveling at a rate of two scoops per minute, it'd take about 33.5 years to entirely clear the stadium.

If 500 people show up to shovel, it'd take them each about 35,200 scoops to clear the whole stadium. But at that rate, it'll still take three and a half weeks to clear the stadium.

Let's use some math to determine if the statements above make sense.

Conversions we need to know:

1 acre = 4840 square yards	1 square yard = 9 square feet
$A = \pi r^2$	1 cubic yard = 27 cubic feet
$V = \pi r^2 h$ (cylinder)	$V = lwh$ (rectangular prism)

You may use a calculator for all math involved with this project.

1.	**How big is Ralph Wilson Stadium?** Ralph Wilson Stadium is an oval. However, you can estimate the area of the stadium using a circle with a radius of 365 feet. What is the area inside the stadium in square feet? In square yards? In acres?	
2.	**How much is one ton of snow?** One ton of snow can vary quite a bit depending on how wet or dry the snow is. For our purposes we will estimate one ton on snow to be a block of snow 10 ft. by 20 ft. by 1 ft. deep. What is the volume of one ton of snow?	
3.	**What is the volume of the snow that fell?** We need to determine the entire volume of snow inside the stadium. They estimated that 5 feet of snow fell. What is the volume of snow inside the stadium in cubic feet? In cubic yards?	
4.	**In tons, how much does all the snow inside the stadium weigh?**	
5.	How does your estimate compare to the Bills' estimate on the front page?	
6.	**How fast can people shovel?** Let's assume an average person can shovel twice per minute with each shovel holding 10 pounds of snow. How many pounds of snow can one person shovel in an hour? How long would it take one person to shovel the entire stadium (months)? How long would it take 500 people to shovel the entire stadium (hours)? How long would it take 3,000 volunteers to shovel the entire stadium (hours)?	
7.	How does your one-person estimate compare to the Bills' estimate on the front page?	
8.	How does your 500-person estimate compare to the Bills' estimate on the front page?	
9.	**What about the entire Ralph Wilson Stadium Complex?** Ralph Wilson Stadium is only one part of the entire area that would need to be cleared. Volunteers would also need to clear the parking lots, nearby street, sidewalks, etc. The entire Ralph Wilson Stadium Complex is 197 acres. Based on this information, what would be the total weight of the snow the volunteers would need to remove?	
10.	How does your new estimate compare to the Bills' estimate on the front page?	
11.	How long would it take one person to shovel the entire complex (years)?	
12.	How does your new estimate compare to the Bills' estimate on the front page?	
13.	The Bills offered to pay fans $10 an hour. How much money would the Bills have paid in total to clear the entire complex?	